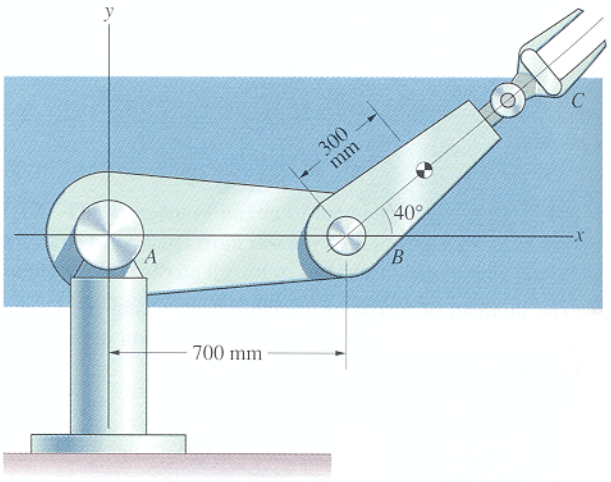
Example Problem - Le 23

7.27 Arm BC has a mass of 12 kg and the mass moment of inertia about its center of mass is 3 kg-m^2. If arm AB has a constant clockwise angular velocity of 2 rad/s and arm BC has a counterclockwise angular velocity of 2 rad/s and a clockwise angular acceleration of 4 rad/s^2,
determine:

a) the couple exerted on arm BC at B,
b) the reaction at B

(*taken from Dynamics, 2nd Edition by Bedford & Fowler*)

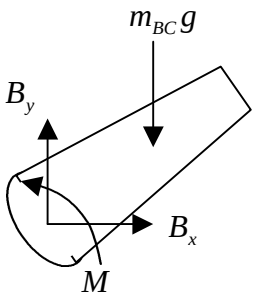


Strategy: Isolate system (arm BC)
Kinetics (COLM, COAM RF)
Kinematics (relative acceleration)

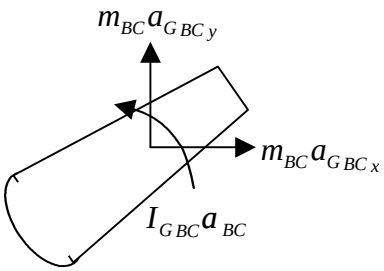
Given: $\overline{w}_{AB} = -2\hat{k}$, $\overline{a}_{AB} = 0\hat{k}$, $\overline{w}_{BC}\, 2\hat{k}$, $\overline{a}_{BC} = -4\hat{k}$, $I_{GBC} = 3$, $m_{BC} = 12$

Kinetics:

FBD



KD



unk	eqs
M	(1)
B_x	(2)
B_y	(3)
a_{GBCx}	(4)
a_{GBCy}	(5)
a_{Bx}	(6)
a_{By}	(7)
a_{Ax}	(8)
a_{Ay}	(9)

COLM(RF)

x dir: $$B_x = m_{BC}a_{GBCx} \quad (1)$$

y dir: $$B_y - m_{BC}g = m_{BC}a_{GBCy} \quad (2)$$

COAM(RF) about B

$$M - m_{BC}g(0.3\cos 40) = I_{GBC}a_{BC} + m_{BC}a_{GBCy}(0.3\cos 40) - m_{BC}a_{GBCx}(0.3\sin 40) \quad (3)$$

Kinematics:

Arm BC:

$$\bar{a}_{G\,BC} = \bar{a}_B + \bar{a}_{G_{BC}/B} = \bar{a}_B + \bar{a}_{BC} \times \bar{r}_{G_{BC}/B} - w_{BC}^2 \bar{r}_{G_{BC}/B}$$

$$a_{G\,BC\,x}\hat{i} + a_{G\,BC\,y}\hat{j} = a_{B\,x}\hat{i} + a_{B\,y}\hat{j} + a_{BC}\hat{k} \times \left(r_{G_{BC}/B_x}\hat{i} + r_{G_{BC}/B_y}\hat{j}\right) - w_{BC}^2 \left(r_{G_{BC}/B_x}\hat{i} + r_{G_{BC}/B_y}\hat{j}\right)$$

$$= a_{B\,x}\hat{i} + a_{B\,y}\hat{j} + a_{BC} r_{G_{BC}/B_x}\hat{j} - a_{BC} r_{G_{BC}/B_y}\hat{i} - w_{BC}^2 r_{G_{BC}/B_x}\hat{i} - w_{BC}^2 r_{G_{BC}/B_y}\hat{j}$$

$$\begin{aligned} i: \quad & a_{G\,BC\,x} = a_{B\,x} - a_{BC} r_{G_{BC}/B_y} - w_{BC}^2 r_{G_{BC}/B_x} \\ j: \quad & a_{G\,BC\,y} = a_{B\,y} + a_{BC} r_{G_{BC}/B_x} - w_{BC}^2 r_{G_{BC}/B_y} \end{aligned} \tag{4,5}$$

Arm AB:

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A} = \bar{a}_B + \bar{a}_{AB} \times \bar{r}_{B/A} - w_{AB}^2 \bar{r}_{B/A}$$

$$a_{B\,x}\hat{i} + a_{B\,y}\hat{j} = a_{A\,x}\hat{i} + a_{A\,y}\hat{j} + a_{AB}\hat{k} \times \left(r_{B/A_x}\hat{i} + r_{B/A_y}\hat{j}\right) - w_{BC}^2 \left(r_{B/A_x}\hat{i} + r_{B/A_y}\hat{j}\right)$$

$$= a_{A\,x}\hat{i} + a_{A\,y}\hat{j} + a_{AB} r_{B/A_x}\hat{j} - a_{AB} r_{B/A_y}\hat{i} - w_{AB}^2 r_{B/A_x}\hat{i} - w_{AB}^2 r_{B/A_y}\hat{j}$$

$$\begin{aligned} i: \quad & a_{B\,x} = a_{A\,x} - a_{AB} r_{B/A_y} - w_{AB}^2 r_{B/A_x} \\ j: \quad & a_{B\,y} = a_{A\,y} + a_{AB} r_{B/A_x} - w_{AB}^2 r_{B/A_y} \end{aligned} \tag{6,7}$$

Geometry and Constraints:

$$\bar{r}_{G_{BC}/B} = 0.3\cos 40\hat{i} + 0.3\sin 40\,\hat{j}$$

$$r_{B/A} = 0.7\hat{i} + 0.0\,\hat{j}$$

$$\bar{a}_A = 0.0\hat{i} + 0.0\,\hat{j} \tag{8,9}$$

Solving:

$$\boxed{\begin{aligned} B_x &= 35.4N \leftarrow \\ By &= 97.4N \uparrow \\ M &= 17.2NmCCW \end{aligned}}$$