Review of Conservation of Energy

(by P. Cornwell)

From ES201 conservation of energy can be written as:

$$\frac{dE_{sys}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m}_i \left(h + \frac{v^2}{2} + gz \right)_i - \sum_{out} \dot{m}_o \left(h + \frac{v^2}{2} + gz \right)_o$$

In contrast to conservation of linear and angular moment, which are vector equations, conservation of energy is a scalar equation. In this class we'll usually be using the finite time form for a closed system as shown below.

$$\boxed{\Delta E_{sys} = W}$$

where the energy of the system is

$$E_{sys} = E_K + E_G + E_S + U$$

where

	Comments
$E_K = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G w^2$	(you will always need to use kinematics to relate v_G and ω.)
$E_G = mgz$	(*z* is the distance the center of mass is from the datum)
$E_S = \frac{1}{2} k x^2$	(*x* is measured from the free length of the spring)
U = internal energy	(this is usually zero in this class unless there is an impact)

and work is defined to be

$$W = \int_1^2 \vec{F} \cdot d\vec{r} \qquad \text{or} \qquad W = \int_1^2 M d\theta$$

<u>Special Cases</u>:

constant force: $$W = \int_1^2 \vec{F} \cdot d\vec{r} = \int_1^2 F ds = F \int_0^s ds \quad \Rightarrow \quad W = Fs$$

constant moment: $$W = \int_1^2 M d\theta = M \int_0^\theta d\theta \quad \Rightarrow \quad W = M\theta$$

rolling friction (on a fixed surface): $$W = \int_1^2 \vec{F} \cdot d\vec{r} = \int_1^2 F ds_c \left(\frac{dt}{dt} \right) = \int_1^2 F \underbrace{v_c}_{\text{velocity of the point of contact}} dt = 0 \quad \Rightarrow \quad W = 0$$