

## High School Math Contest

Prepared by the Mathematics Department of

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Terre Haute, Indiana

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**Instructions:** Put your name and home address on your answer sheet.

Record all your answers to the problems on the front of the answer sheet. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

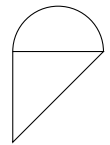
Good luck!

1.  $(\frac{2}{3} - \frac{1}{2})^3 =$   
A.  $1/216$       B.  $1/27$       C.  $1/64$       D.  $1/125$       E. None of these

2.  $\sqrt{80} =$   
A.  $8\sqrt{10}$       B.  $10\sqrt{8}$       C.  $16\sqrt{5}$       D.  $5\sqrt{8}$       E. None of these

3. The price of a toy horse is  $1/9$  of a dollar. If the price is decreased by 10% and then increased by 10% what is the final price in dollars?  
A.  $10/100$       B.  $11/100$       C.  $10/90$       D.  $11/90$       E. None of these

4. A right triangle is topped with a semicircle as show in the figure. The two legs of the triangle are each 4 units long. What is the area of the figure?



A.  $\pi + 16$       B.  $2\pi + 16$       C.  $\pi + 8$       D.  $2\pi + 8$       E. None of these

5. If  $y = x + 1$  and  $z = 2y + 3$  then  $x + y + z =$ .  
A.  $3x + 4$       B.  $3x + 6$       C.  $4x + 4$       D.  $4x + 6$       E. None of these

6. The sum of the digits of a two digit number is 10. If the digits are reversed, the new number exceeds the old number by 36. Find the new number.
- A. 28                      B. 37                      C. 46                      D. 73                      E. None of these

7. There are two distinct prime numbers that divide 2023. The sum of the two prime numbers is
- A. 24                      B. 124                      C. 1024                      D. 2024                      E. None of these

8. Bluey and Bingo have hidden their dad Bandit's car keys in the room they share. They make the following three TRUE statements as Bandit searches for his keys:

Bluey: If the keys are not under the rug, then they must be under Bingo's bed.

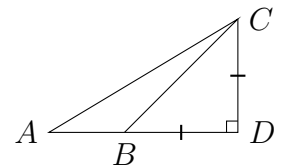
Bingo: The keys are NOT under my bed! The keys are under the rug, or in the closet.

Where are the keys?

- A. In the closet.    B. Under the rug.    C. Under Bingo's bed.    D. Under Bandit's hat.  
E. None of these

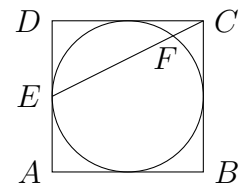
9. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be defined by  $f(x) = ax^2 + bx + c$ , where  $a, b$ , and  $c$  are constants. If  $f(-2) = 17$ ,  $f(-1) = 9$ , and  $f(2) = 21$ , then  $a + b + c =$
- A. 9                      B. 11                      C. 21                      D. 51                      E. None of these

10. Triangle  $ABC$  is drawn adjacent to isosceles right triangle  $CDB$ . If  $AC = 5$  and  $BC = \sqrt{18}$  then what is the area of triangle  $ABC$ ?



- A.  $3/2$                       B.  $3\sqrt{2}/2$                       C. 3                      D.  $3\sqrt{2}$                       E. None of these

11. A circle is inscribed in square  $ABCD$  with side length 10. The circle is tangent to side  $AD$  at  $E$ . Line  $EC$  intersects the circle at  $F$ . The length of segment  $EF$  is

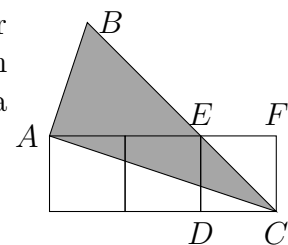


- A.  $4\sqrt{5}$       B. 9      C.  $7\sqrt{2}$       D. 10      E. None of these

12. The values of  $x$  that satisfy the equation  $\frac{4^{x^2}}{2^x} = 8$  are

- A.  $(1 \pm \sqrt{23})/2$     B.  $1/2$  and  $1/2$     C.  $-1$  and  $3/2$     D.  $(1 \pm \sqrt{13})/2$     E. None of these

13. The area of each of the three squares is 16. The upper left corner of the left-most square is  $A$ . Line  $BC$  passes through the point  $E$  on square  $CDEF$ . Triangle  $ABC$  is a right triangle. Compute the area of triangle  $ABC$ .



- A. 32      B.  $12\sqrt{10}$       C.  $81/2$       D.  $20\sqrt{5}$       E. None of these

14. The number of subsets of  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  that contain three or more elements is

- A. 120      B. 484      C. 848      D. 968      E. None of these

15. The line  $\ell_1$  has slope  $\frac{1}{2}$  and is perpendicular to the line  $\ell_2$  which passes through the points  $(a, 3a)$  and  $(8, 14)$ . Consequently, the value of  $a$  is

- A. -2      B. 4      C.  $\frac{36}{7}$       D. 6      E. None of these

16. The number of ways one can arrange eight 0's and ten 1's so that every 0 is followed by a 1 is

- A. 44                      B. 45                      C. 55                      D. 153                      E. None of these

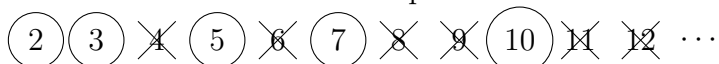
17. The equation  $(5x - 2)^2 = 4x^3 - 16x + 109$  has three solutions  $a$ ,  $b$ , and  $c$ . Compute the value of the sum  $a + b + c$ .

- A. -5                      B.  $25/4$                       C.  $5\sqrt[3]{5}$                       D. 10                      E. None of these

18. On a digital clock times are displayed using the digits 0 through 9 with hours running from 1 to 12 and minutes running from 00 to 59. The hours do not display leading zeros the minutes do display leading zeros. Seconds are not displayed. For times such as 12:15 and 3:23 some of the digits are repeated. For times such as 3:07 and 12:34 no digit is repeated. To the nearest percent for what percent of the time is no digit repeated in the displayed time?

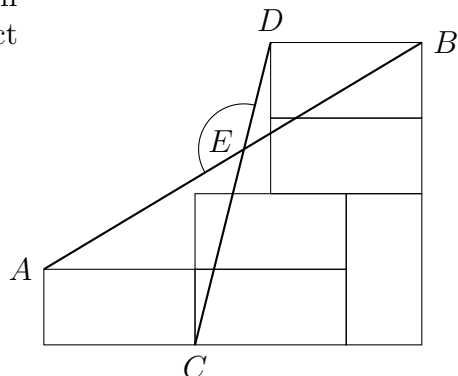
- A. 39                      B. 46                      C. 54                      D. 61                      E. None of these

19. The numbers from 2 to 100 are written. The two is circled and all other numbers whose sum of digits is divisible by 2 are crossed out. The smallest remaining number is 3. The three is circled and all other numbers whose sum of digits is divisible by 3 are crossed out. Continue circling the smallest remaining number and crossing out all other numbers whose sum of digits is divisible by that number. This is continued until all the numbers are either circled or crossed out. After the process is done how many numbers are circled?



- A. 15                      B. 20                      C. 22                      D. 25                      E. None of these

20. Six identical  $1 \times 2$  rectangles are arranged as shown along with lines  $AB$  and  $CD$ . Lines  $AB$  and  $CD$  intersect at  $E$ . Determine the measure of angle  $AED$  in degrees.



- A. 120                      B. 135                      C. 144                      D. 150                      E. None of these