High School Math Contest

Prepared by the Mathematics Department of

Rose-Hulman Institute of Technology
Terre Haute, Indiana

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Instructions: Put your name and home address on your answer sheet. Make sure that your *Contest Student ID number* is recorded in positions 1 through 7 of the Student ID number section.

Record all your answers to the problems on the front of the answer sheet. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

Good luck!

1. What is the largest prime factor of 2022?

A. 3 B. 5 C. 7 D. 11 E. None of these

2. If $i = \sqrt{-1}$ then $i^{157} =$

A. $\sqrt{-1}$ B. -1 C. $-\sqrt{-1}$ D. 1 E. None of these

3. If $\frac{1}{2}x - \frac{1}{3} = \frac{1}{3}x - \frac{1}{4}$ then x is

A. -4 B. $-\frac{1}{4}$ C. $\frac{1}{4}$ D. 4 E. None of these

4. Find the value of $(x+7)/(z-2)$ when $x = 2$ and $z = 1$

A. -16 B. -9 C. 9 D. 16 E. None of these

5. After selling 20% of their oranges the grocer still had 60 remaining. How many oranges did the grocer sell?

A. 10 B. 12 C. 15 D. 18 E. None of these

6. How many ordered pairs (x, y) of nonnegative digits are such that the nine digit number $137x853y2$ is divisible by 9?

A. 8 B. 9 C. 10 D. 11 E. None of these

7. A certain bakery sells 10 different types of donuts. Each time Zelma visits this bakery she purchases one of these donuts. The largest possible number of times Zelma can do this without getting the same type of donut five times is

A. 40 B. 41 C. 50 D. 51 E. None of these

8. Herb took a trip and traveled 50 miles per hour (mph) for the first two hours and then 60 mph for the last three hours. So his average speed for the trip in mph was

A. 55 B. 57 C. 59 D. 60 E. None of these

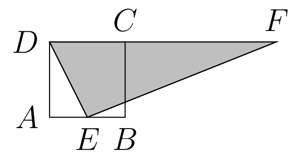
9. The number of positive integers less than 101 that are divisible by either 4 or 6 is

A. 33 B. 34 C. 41 D. 49 E. None of these

10. Five teenagers play the game of "*odd person out*" in order to determine who gets the last slice of pizza remaining from the pizza pie they ordered for their dinner. Each teenager flips a fair coin. If one of the flips is different from the other four then the teenager who flipped the lone result gets the last slice. The probability the last slice gets eaten by one of these teenagers after the coins are flipped just once is

A. $\frac{1}{16}$ B. $\frac{5}{8}$ C. $\frac{5}{16}$ D. $\frac{5}{32}$ E. None of these

11. Point E is the midpoint of side AB on square $ABCD$ with $AB = 4$. Right triangle DEF is drawn so that DCF is collinear as shown in the diagram. The area of triangle DEF is



A. 10 B. 20 C. 30 D. 40 E. None of these

12. The sum of all the real-valued solutions of $|x^2 - 5\,x + 4| = 2$ is
A. 5 B. 10 C. 15 D. 20 E. None of these

13. If each interior angle, measured in degrees, of a regular n-gon is $162°$, then n =

A. 16 B. 18 C. 20 D. 22 E. None of these

14. There are 100 orchards in Mason County where apples, cherries, and peaches are grown, and each of these orchards has at least one of these three types of trees. If 77 have apple trees, 57 have cherry trees, 50 have peach trees, and 5 orchards have all three types of these trees, then the number of orchards that have exactly two of the three types of these trees is
A. 15 B. 37 C. 42 D. 74 E. None of these

15. A right triangle has sides with integer lengths. These three lengths form an arithmetic sequence. One leg of the right triangle has length 2020. The length of the other leg is
A. 1515 B. 2020 C. 2022 D. 2525 E. None of these

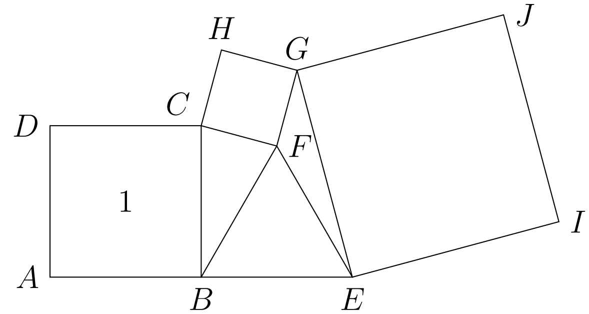
16. If n is a positive integer and
$n + (n+1) + (n+2) + \cdots + 135 + 136 + 137 = 9453$, then $n =$

A. 26 B. 27 C. 37 D. 38 E. None of these

17. Suppose that C is an angle in the first quadrant, with $\sin(C) = c$, and D is an angle in the second quadrant, with $\sin(D) = d$. Then $\sin(C + D) =$

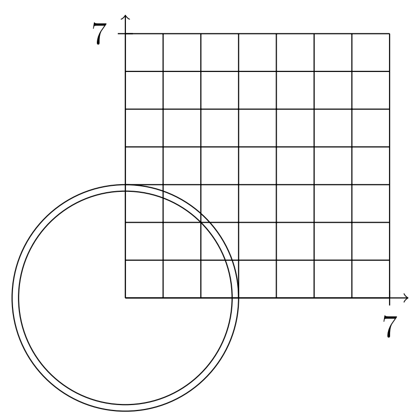
A. $c + d$ B. $c\sqrt{1-d^2} + d\sqrt{1-c^2}$ C. $c\sqrt{1-d^2} - d\sqrt{1-c^2}$ D. $-c\sqrt{1-d^2} + d\sqrt{1-c^2}$
E. None of these

18. Square $ABCD$ with area 1 is adjacent to equilateral triangle BEF. The side lengths of the equilateral triangle are all 1. Square $CFGH$ is constructed above square $ABCD$ and triangle BEF. Square $EIJG$ is constructed to the right of the squares and triangle as shown. The area of square $EIJG$ is



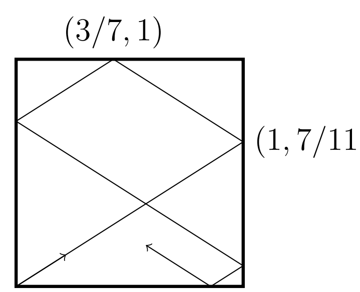
A. 3/2 B. $\sqrt{3}$ C. 2 D. $\sqrt{5}$ E. None of these

19. A 7-by-7 grid of squares with area 1 is placed in the first quadrant. A circle with center $(0, 0)$ is drawn through each lattice point, $(0, 1)$, $(1, 0)$, $(1, 1)$, $(2, 0)$, $(0, 2)$, $(2, 1)$, $(1, 2)$, etc. Two example circles are show in the diagram. The sum of the areas of the 64 circles is



A. 1960π B. 2022π C. 2222π D. 2240π E. None of these

20. A beam of light reflects off the interior sides of a unit square so that the incoming angle the beam makes with the side is the same as the outgoing angle. The beam starts at $(0, 0)$ and first hits a side of the square at $(1, 7/11)$. The beam next hits the side of the square at $(3/7, 1)$ and continues in this way until it hits one of the corners and is stopped. How many times does the beam of light hit the sides of the square before it is stopped?



A. 16 B. 17 C. 18 D. 19 E. None of these