High School Math Contest

Prepared by the Mathematics Department of

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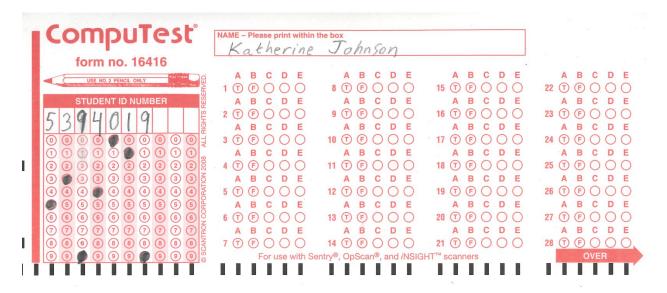
November 16, 2019

Instructions: Put your name and home address on the back of your Scantron card. Make sure that your *Contest Student ID number* is recorded in positions 1 through 7 of the Student ID number section.

Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

Good luck!



1. What is the integer closest to $12/13 + 7/8$?				
A. 1	B. 2	C. 19	D. 21	E. None of these
per gallon. His ga	as expenses will de	ecrease by what	percent?	that now gets 30 miles
A. 50%	B. $33\frac{1}{3}\%$	C. 30%	D. 20%	E. None of these
v				ce was cut by 10%. In aputer after the March
A. 750	B. 756	C. 770	D. 840	E. None of these
4. A square field one acre each. Ho A. 16				divided into squares of actly one side? E. None of these
5. At 6:00 the m When will this fir A. 6:55		nands of a clock C. 7:05 4/9	are 180° apart (c D. 7:06	opposite in directions). E. None of these

6. Zelma drove 600 miles. For the first 300 miles she averaged 50 miles per hour. For the second 300 miles she averaged 60 miles per hour. What was her average speed, in miles per hour, for the 600 mile trip?

A. 600/10

B. 550/10

C. 600/11

D. 550/11

E. None of these

7. If x = 101001 in base 2, and y = 2021 in base 3, then in base 5, x + y =

A. 102

B. 202

C. 402

D. 103022

E. None of these

8. A quarter circle of radius 4 meters is inscribed in a square with side length 4 meters. Determine the area of the shaded region.



A. $8 - 2\pi \text{ m}^2$ B. $16 - \frac{11\pi}{2} \text{ m}^2$ C. $\pi \text{ m}^2$ D. $\frac{\pi}{2} \text{ m}^2$ E. None of these

9. The value of x satisfies the equation $\sqrt{x+7} + \sqrt{x} = 7$. What is the value of x? C. 4 D. 7

A. 0

B. 3

E. None of these

10. Which of the following answers gives all the solutions of the equation |x| + x = 0?

A. x < 1

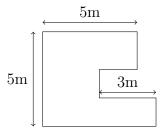
B. x = 0

C. $x \ge 0$ D. x = -7

E. None of these

- 11. If x = 3 then the arithmetic sequence x, x + 10, x + 20 consists of three prime numbers, namely, 3, 13, 23. What is the next smallest value of x so that x, x + 10, x + 20 are all prime numbers?
 - A. 7
- B. 47
- C. 127
- D. 2017
- E. None of these

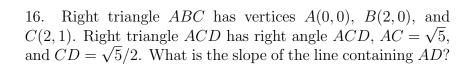
12. Determine the perimeter of the polygon.

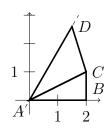


- A. Not enough information
- B. 21
- C. 24
- D. 26
- E. None of these
- 13. The number 5/7 is written in decimal form starting as 0.714... What is the sum of the first 2019 digits after the decimal point?
 - A. 9082
- B. 9084
- C. 9086
- D. 9088
- E. None of these
- 14. The line y = x + 6 intersects the parabola $y = x^2$ at points A and B. Point C is the vertex of the parabola $y = x^2$. Compute the area of triangle ABC.



- A. 15
- B. 33/2
- C. 20
- D. 65/2
- E. None of these
- 15. What is the sum of all three digit numbers whose digits sum to 9?
 - A. 9,324
- B. 10,224
- C. 17,487
- D. 17,820
- E. None of these





A. 0.5

B. 1

C. 1.5

D. 2

E. None of these

17. The digital sum of a number is found by taking the sum of the digits. If the sum is at least ten then take the sum of these digits. Continue this process until the sum is less than ten. For example, 2+0+1+9=12 and 1+2=3 so the digital sum process applied to 2019 ends at 3. Let p be the smallest prime larger than 2019 for which p+2 is also prime. Where does the digital sum applied to p(p+2) end?

A. 3

B. 5

C. 8

D. 9

E. None of these

18. Let the sum of the squares of the digits of a positive integer s_0 be represented by s_1 . In a similar way, let the sum of the squares of the digits of s_1 be represented by s_2 , and so on. If $s_i = 1$ for some $i \ge 1$, then the original integer s_0 is said to be *happy*. For example, 2019 is happy because $2^2 + 0^2 + 1^2 + 9^2 = 86$, $8^2 + 6^2 = 100$, and $1^2 + 0^2 + 0^2 = 1$.

What is the next year after 2019 that is a happy number?

A. 2020

B. 2021

C. 2022

D. 2023

E. None of these

19. Isosceles triangle ABC has AC = BC and the area of triangle ABC is the same as the area of the semi-circle with diameter AB. Let x be the measure of angle BAC. Compute $\tan(x)$ to the nearest one-hundredth.

A. 0.87

B. 1.57

C. 1.73

D. 3.14

E. None of these

20. A spinner has the numbers 1,2,3,4 which are formed by wedges of angles x° , $2x^{\circ}$, $3x^{\circ}$, and $4x^{\circ}$, respectively. The probability that any of the numbers is spun is proportional to the angle of the corresponding wedge. The spinner is spun four times. What is the probability that the sum of the numbers is a perfect square?



A. 181/2000

B. 21/128

C. 81/125

D. 3/13

E. None of these