High School Math Contest

Prepared by the Mathematics Department of

Rose-Hulman Institute of Technology

Terre Haute, Indiana

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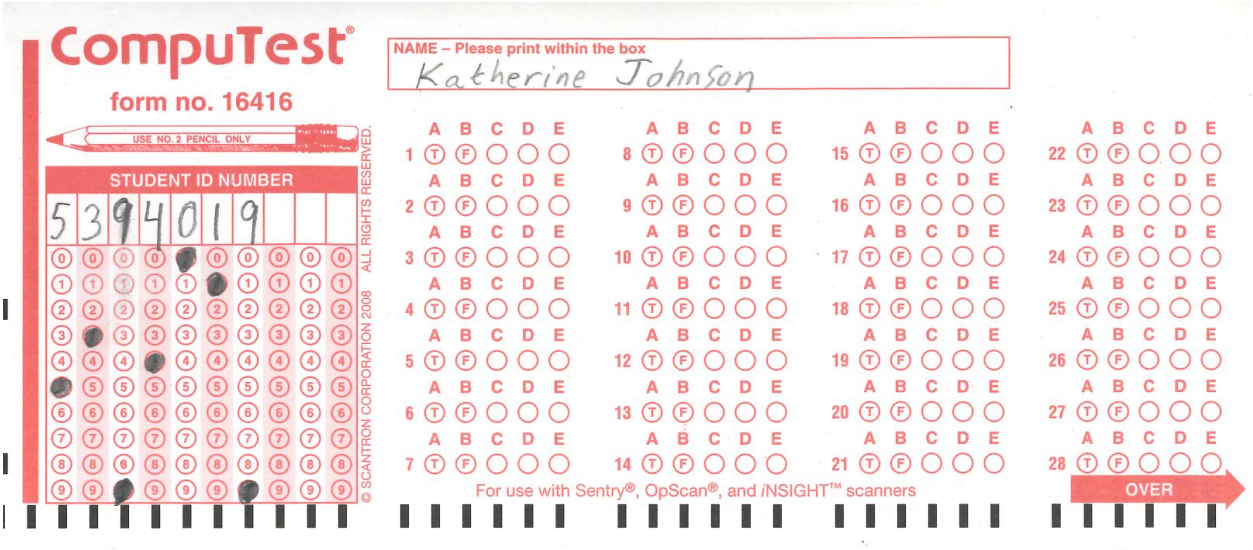
Instructions: Put your name and home address on the back of your Scantron card. Make sure that your Contest Student ID number is recorded in positions 1 through 7 of the ID section. Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

Good luck!



1. What is the smallest odd prime that divides 2018?

A. 2 B. 3 C. 5 D. 7 E. None of these

2. Solve for x if $ax = ay + bx$.

A. $\frac{a}{b-a}$ B. $\frac{ay}{a-b}$ C. $\frac{ax}{b-a}$ D. $\frac{a}{a-b}$ E. None of these

3. A purse contains twice as many dimes as nickels and twice as many quarters as dimes for a total of \$5.00. If the purse contains only nickels, dimes and quarters then how many coins are in the purse?

A. 22 B. 24 C. 26 D. 28 E. None of these

4. The average of two numbers is $\frac{7}{8}$. One of the numbers is $\frac{2}{3}$. Find the other number.

A. $\frac{5}{4}$ B. $\frac{3}{2}$ C. $\frac{12}{13}$ D. $\frac{13}{12}$ E. None of these

5. Given five points in a plane with no three in a straight line. How many different triangles can be formed by connecting any three points chosen from the given five points?

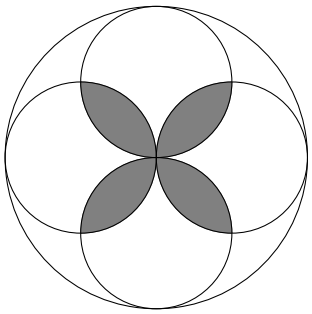
A. 8 B. 9 C. 10 D. 11 E. None of these

6. Zelma can mow the lawn in 2 hours and Herb can mow the lawn in 3 hours. How many hours would it take them to mow the lawn together?

A. 1.3 B. 1.2 C. 0.9 D. 0.8 E. None of these

7. Four identical circles of radius 10 are inscribed in a circle of radius 20 as shown below. Determine the area of the shaded region.

A. 80π B. 250 C. $200\pi - 400$ D. 100π E. None of these



8. If $a * b = ab + 1$ and $a \circ b = a + b$, then what is the value of $4 * [(6 \circ 8) \circ (3 * 5)]$?

A. 88 B. 121 C. 89 D. 120 E. None of these

9. One digit in 16! has been replaced by the letter A. Determine the value of the digit replaced by the letter A. $16! = 2092278988A000$.

A. 1 B. 2 C. 3 D. 4 E. None of these

10. Volunteering at your local animal shelter, you meet 4 strangers Amir, Brit, Cho and Dot. None of you know each other. Trying to strike up a conversation, you ask "Do all of you own cats?" In the following order, Amir answers "I don't know," and then Brit answers "I don't know." Next, Cho answers "I don't know." Finally, Dot answers "No." Which of the 4 people own cats?

A. None of them B. Amir

C. Amir and Brit D. Amir, Brit and Cho E. None of these

11. If $i^2 = -1$ then compute the value of $(1+i)^{50}$.

A. $2^{25}i$ B. $-2^{25}i$ C. 2^{25} D. 2^{50} E. None of these

12. What is the sum of the reciprocals of the divisors of 672?

A. 2 B. $\frac{5}{2}$ C. $\frac{8}{3}$ D. 3 E. None of these

13. You have three six-sided dice in your pocket. Two are fair dice and one has 6 dots on every side. You take one out of your pocket at random and roll it. Given that you roll a 6, what is the probability that this is a fair die?

A. 1/8 B. 1/4 C. 2/3 D. 3/4 E. None of these

14. In a survey there are three possible responses, A,B, and C. Rounded to the nearest percentage 33% of the people responded A, 13% of the people responded B and 53% of the people responded C. What is the smallest possible number of people who responded to the survey?

A. 3 B. 8 C. 45 D. 100 E. None of these

15 Given a sequence of numbers $a_1, a_2, a_3, \ldots$ such that the next one is the sum of the previous two, i.e. $a_n = a_{n-1} + a_{n-2}$ when $n > 2$. Note that the sequence depends on a_1 and a_2. If $a_7 = 11$ then compute the value of

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}.$$

A. 118 B. 119 C. 120 D. 121 E. None of these

16. What is the sum of the digits of 2^{30}?

A. 28 B. 30 C. 37 D. 46 E. None of these

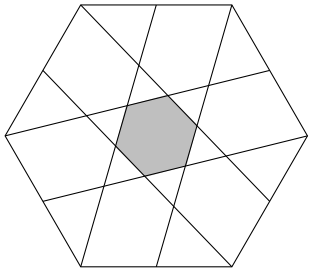
17. Write $(2+\sqrt{3})^{2017}$ as $A+B\sqrt{3}$. where A and B are integers. What is the remainder when A is divided by 5?

A. 0 B. 1 C. 2 D. 3 E. None of these

18. What is the number of positive integers less or equal to 900 that have exactly one prime factor in common with 900? For example, 28 is such a number as it shares the prime factor 2 with 900, but they have no other prime factors in common. The number 12 is not such a number because it has multiple prime factors in common with 900; the prime factors 2 and 3.

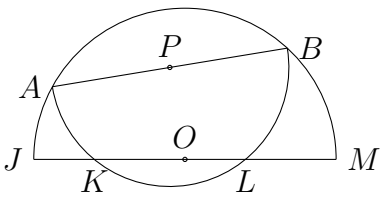
A. 180 B. 240 C. 300 D. 450 E. None of these

19. The large regular hexagon has an area of 1716. Lines are drawn joining the midpoints of the sides to vertices opposite those sides as shown. Compute the area of the shaded hexagon.



A. 132 B. 143 C. 156 D. 171.6 E. None of these

20. $\overline{AB}$ is the diameter of a semi-circle drawn on top of the semi-circle of radius 5 centered at O as shown. Points A and B lie on the circle of radius 5. If $JK = 2$, $KL = 5$, and $LM = 3$ then what is the length AB?



A. $\sqrt{62}$ B. $\sqrt{63}$ C. 8 D. $\sqrt{65}$ E. None of these