

# High School Math Contest

Prepared by the Mathematics Department of

*Rose-Hulman Institute of Technology*

Terre Haute, Indiana

November 11, 2017

**Instructions:** Put your name and home address on the back of your Scantron card. Make sure that your Contest Student ID number is recorded in positions 1 through 7 of the ID section. Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

Good luck!

**CompuTest**  
form no. 16416

USE NO. 2 PENCIL ONLY

NAME - Please print within the box  
*Katherine Johnson*

STUDENT ID NUMBER  
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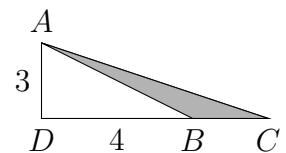
1. If three times a number is 15, then what is five times the number?  
A. 9                      B. 15                      C. 25                      D. 75                      E. None of these

2. Compute the area of a circle with a diameter of 10.  
A. 25                      B.  $25\pi$                       C. 100                      D.  $100\pi$                       E. None of these

3. If  $a \star b = a + (a/b)$  then what is the value of  $3 \star (2 \star 1)$ ?  
A. 2                      B.  $7/3$                       C.  $15/4$                       D. 9                      E. None of these

4. One coin was tossed ten times. More heads were tossed than tails. What is the smallest possible number of heads that was tossed?  
A. 0                      B. 4                      C. 5                      D. 10                      E. None of these

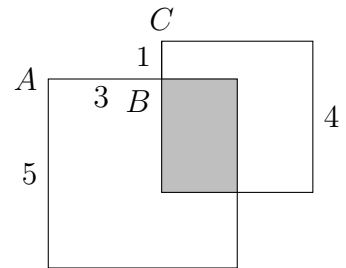
5. Determine the area of triangle  $ABC$  if  $AD = 3$ ,  $DC = 10$ , and  $DB = 4$ .



- A. 6                      B. 9                      C. 12                      D. 15                      E. None of these

6. The first three terms of an arithmetic sequence are  $3x + 2$ ,  $5x + 3$ , and  $8x + 5$ . What is the value of  $x$ ?
- A.  $-1$                       B.  $0$                       C.  $1$                       D.  $2$                       E. None of these

7. Squares of side lengths 4 and 5 have parallel sides. They intersect as shown in the diagram. Line segment  $AB$  has length 3 and line segment  $BC$  has length 1. Compute the area of the shaded region.



- A. 2      B. 4      C. 6      D. 10      E. None of these

8. If the perimeter of a square is increased by 10% then by what percent is the area increased?
- A.  $2\frac{1}{2}$                       B. 5                      C. 10                      D. 21                      E. None of these

9. Casey answered 40 questions correctly. Casey's score was 80%. How many questions did Casey miss?
- A. 8      B. 12      C. 32      D. 50      E. None of these

10. What is the angle between the hour hand and minute hand of a clock at 1:10?
- A.  $23\frac{1}{13}^\circ$                       B.  $25^\circ$                       C.  $27\frac{8}{11}^\circ$                       D.  $30^\circ$                       E. None of these

11. A *Niven Number* is a number that is divisible by the sum of its digits. Find the smallest number larger than 2017 that is a Niven Number.

- A. 2021      B. 2022      C. 2023      D. 2024      E. None of these

12. For how many integers less than 2017 is the smallest odd prime factor either 3 or 5?

- A. 403      B. 672      C. 941      D. 1075      E. None of these

13. If the two sentences below are true, which conclusion must also be true?

Either Maggie has a hamster or Bobby has a dog.

Maggie and Bobby are not both dog owners.

- A. Bobby has a dog or Maggie has a dog.  
B. Maggie has a hamster and Bobby has a dog.  
C. Maggie has a hamster or Maggie does not have a dog.  
D. Maggie does not have a dog.  
E. None of these

14. How many palindromes with an even number of digits are prime numbers?

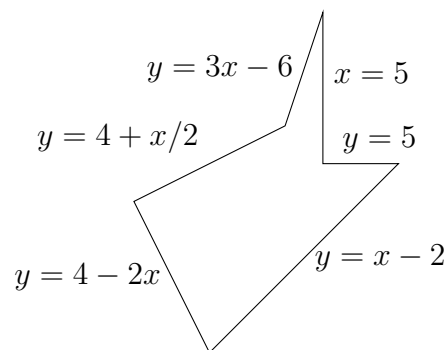
- A. 0      B. 1      C. 2      D. Infinitely many      E. None of these

15. A six-sided die has the numbers 1,2,3,4,5,6. Herb rolls the die twice and adds the values. A four sided die has the numbers 1,2,3,4. Zelma rolls this die three times and adds the values. What is the probability that they have the same total?

- A. 0      B.  $\frac{1}{24}$       C.  $\frac{251}{2304}$       D.  $\frac{47}{384}$       E. None of these

16. How many positive integers less than 1000 have at least one digit that is a prime?  
 A. 784                      B. 800                      C. 875                      D. 936                      E. None of these

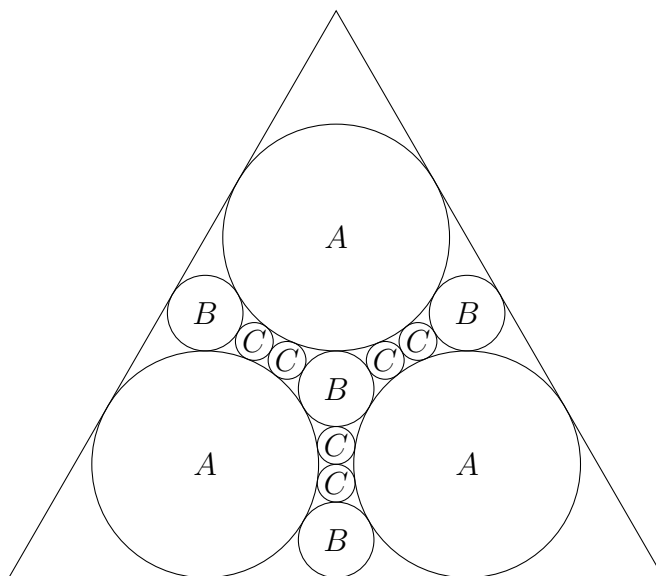
17. The region shown in the diagram is bounded by the lines  $y = 4 - 2x$ ,  $y = 4 + x/2$ ,  $y = 3x - 6$ ,  $x = 5$ ,  $y = 5$ , and  $y = x - 2$  as shown. Determine the area of the region.  
 A. 21                      B. 28                      C. 31.5                      D. 42                      E. None of these



18. How many numbers less than 1,000,000,000 (one billion) are divisible by 7 and have the property that the non-zero digits are 5, 1, and 1?  
 A. 0                      B. 21                      C. 28                      D. 42                      E. None of these

19. Suppose that  $f(x)$  is a one-to-one function with  $x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Suppose further that for each value of  $x$ ,  $f(x) \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ . Determine the number of such one-to-one functions for which  $f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)$  is a multiple of 3.  
 A.  $94 \cdot 10!$                       B.  $95 \cdot 10!$                       C.  $285 \cdot 10!$                       D.  $286 \cdot 10!$                       E. None of these

20. Circles are inscribed in an equilateral triangle as shown at the right. The circles labeled  $A$  are the same size as each other. The circles labeled  $B$  are the same size as each other. The circles labeled  $C$  are also the same size as each other. The radius of the circles labeled with  $A$  is 30. Determine the radius of the circles labeled  $C$ .



- A. 3                      B. 4.5                      C.  $3\sqrt{3}$                       D. 5                      E. None of these