High School Math Contest

Prepared by the Mathematics Department of

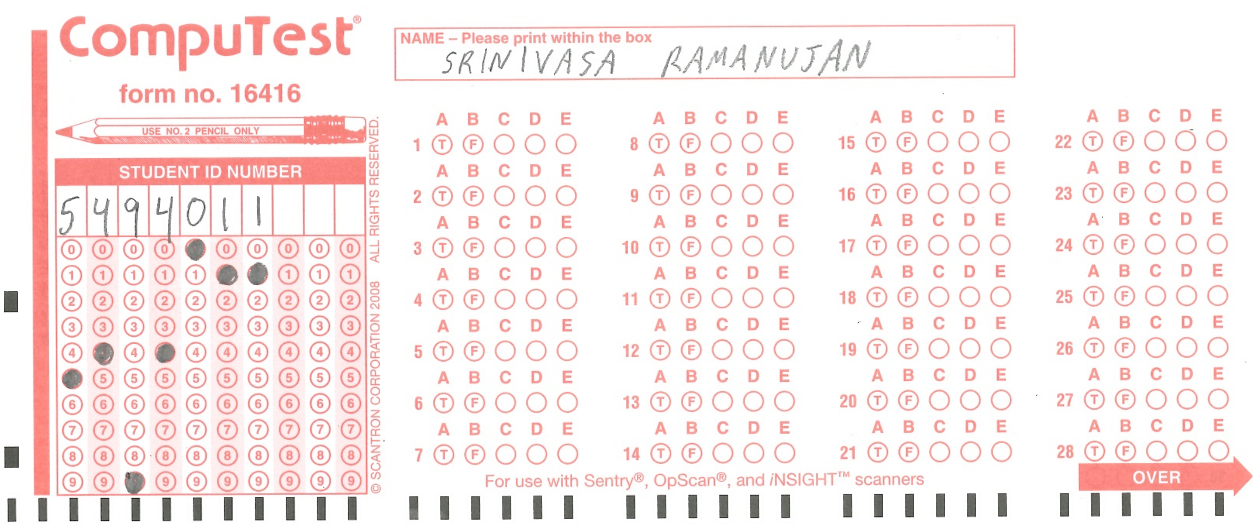
Rose-Hulman Institute of Technology
Terre Haute, Indiana

November 12, 2016

Instructions: Put your name and home address on the back of your Scantron card. Make sure that your Student ID number is recorded in positions 1 through 7 of the ID section. Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use a calculator other than your brain and fingers!

All students will answer the same 20 questions. Each question is worth 5 points for a correct answer, 0 points for no answer, and -1 point for a wrong answer. You will find that the more difficult problems are at the end of the test.

Good luck!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1. What is the smallest odd prime divisor of 2016?

A. 3 B. 5 C. 7 D. 9 E. None of these

2. What is the remainder when 2016 is divided by the sum of its digits?

A. 0 B. 3 C. 6 D. 9 E. None of these

3. Herb can paint a fence in 20 minutes. Zelma can paint a fence in 16 minutes. How many minutes does it take them to paint the fence together?

A. 4 B. 9 C. 18 D. 36 E. None of these

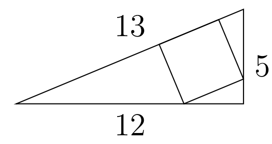
4. Let $a * b = a^b + a + b$. Compute the value of $(1 * 3) - (3 * 1)$.

A. -4 B. 0 C. 2 D. 6 E. None of these

5 The pocket of your backpack contains 5 blue pens and 3 black pens. You reach into your backpack without looking and grab a bunch of pens. What is the smallest number of pens you must grab to ensure you grab at least 3 pens of the same color?

A. 3 B. 4 C. 5 D. 6 E. None of these

6. A square is inscribed in a 5, 12, 13 right triangle as shown. Determine the length of a side of the inscribed square.



A. 780/228 B. 780/229 C. 780/230 D. 780/231 E. None of these

7. How many ordered pairs (x, y) of real numbers satisfy the system of equations?

$$x^2 + xy + y^2 = 2016$$
$$x^2 - 3xy - y^2 = 2016$$

A. 1 B. 2 C. 3 D. 4 E. None of these

8. Let A be the set of points no more than one unit from the point $(-1/2, 0)$. Let B be the set of points no more than one unit from the point $(1/2, 0)$. Find the area of $A \cap B$.

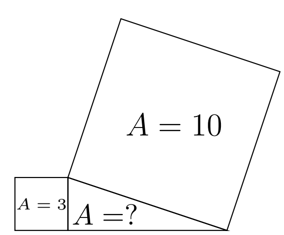
A. $\frac{\sqrt{3}}{4} - \frac{\pi}{8}$ B. $\sqrt{3} - \frac{\pi}{3}$ C. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ D. $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ E. None of these

9. Two unmarked containers hold 10 and 7 ounces of water, respectively. The containers may be filled, emptied, or poured into each other. It is possible to measure 3 ounces of water by filling the 10 ounce container and using that water to fill the 7 ounce container leaving 3 ounces of water in the large container. Thus it requires 10 ounces of water to measure out 3 ounces of water. What is the minimum number of ounces of water required to measure out 4 ounces of water using only these two unmarked containers?

A. 4 B. 7 C. 10 D. 20 E. None of these

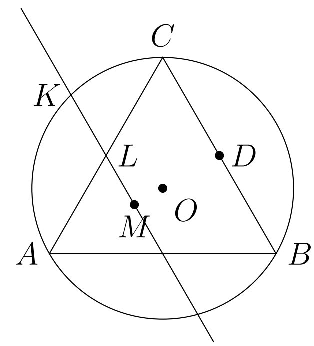
10. Two squares and a right triangle are arranged as shown in the figure. The squares have areas 10 and 3, as shown. What is the area of the triangle?

A. $\frac{\sqrt{19}}{3}$ B. $\frac{\sqrt{21}}{3}$ C. $\frac{\sqrt{19}}{2}$ D. $\frac{\sqrt{21}}{2}$ E. None of these



11. Equilateral triangle ABC is inscribed in circle O. Point D is the midpoint of BC and M is the midpoint of AD. Line ML is parallel to line BC, intersects AC at L, and intersects circle O at K. Determine the ratio of the length of AL to the length of LK.

A. $\frac{\sqrt{3}}{2}$ B. $\frac{2}{\sqrt{33}}$ C. $\frac{(\sqrt{5}+1)}{2}$ D. $\frac{(\sqrt{5}-1)}{2}$ E. None of these

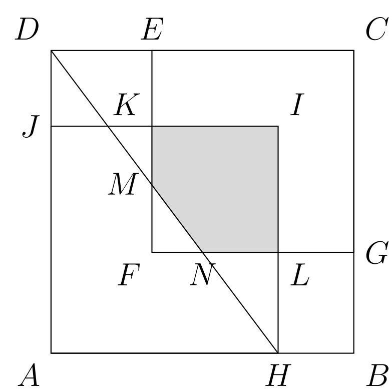


12. In a tournament with players seeded 1,2,3,4 the probability that seed a beats seed b is $b/(a+b)$. In the first round of the tournament seed 1 plays seed 4 and seed 2 plays seed 3. The two winners of the first round matches play each other for the championship. To the nearest hundredth what is the probability that seed 1 wins the tournament?

A. 0.50 B. 0.56 C. 0.60 D. 0.67 E. None of these

13. The length of the sides of square $AHIJ$ are 3/4 the length of the sides of square $ABCD$. The length of the sides of square $CEFG$ are 2/3 the length of the sides of the square $ABCD$. Squares $AHIJ$ and $CEFG$ intersect in rectangle $KFLI$. Line segment DH intersects rectangle $KFLI$ at points M and N. The area of square $ABCD$ is 1. Determine the area of the polygon $KMNLI$.

A. $\frac{67}{432}$ B. $\frac{1}{6}$ C. $\frac{25}{144}$ D. $\frac{9}{48}$ E. None of these



14. November 5, 2016 was a sum date because the sum of the month (11) and day of the month (5) is equal to the last two digits of the year (16). The Rose-Hulman high-school mathematics contest takes place each year on the second Saturday of November. When is the next time that the Rose-Hulman high-school mathematics contest will take place on a sum date?

A. Nov. 8, 2019 B. Nov. 9, 2020 C. Nov. 10, 2021 D. Nov. 11, 2022 E. None of these

15. The angle α lies between $0°$ and $180°$. If $6\sin(\alpha) = 5\sin(2\alpha)$ then what is $\sin(3\alpha)$?

A. 44/125 B. 9/25 C. $\sqrt{11}/(3\sqrt{5})$ D. 1/2 E. None of these

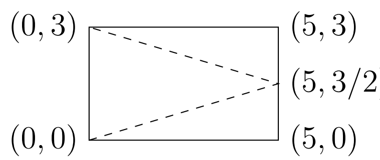
16 What is the remainder when the sum $1! + 2! + 3! + 4! + 5! + \cdots + 2015!$ is divided by 2016?

A. 0 B. 873 C. 1881 D. 2015 E. None of these

17. Ten numbered chips are placed in a bowl. Four chips are numbered 1, three are numbered 2, two are numbered 3, and one is numbered 4. A chip is drawn at random from the bowl. If the number on the chip is n then the chip is replaced in the bowl and $5-n$ chips numbered n are added to the bowl for a second drawing. For example, if the first chip drawn has a 1 then the second drawing will have eight 1s, three 2s, two 3s, and one 4. What is the probability that a chip numbered 3 is drawn on the second draw?

A. 1/15 B. 1/25 C. 565/3003 D. 4649/30030 E. None of these

18. A room is in the shape of a rectangle with vertices at the points $(0,0)$, $(5,0)$, $(5,3)$ and $(0,3)$ meters. A beam of light starts at $(0,0)$ and moves in a straight line until it hits a wall at which point the light reflects off the mirror placed on the wall so that the outgoing angle is equal to the incoming angle. If the beam hits a corner it stops there. Thus if the beam starts at $(0,0)$ and hits the opposite wall at $(5,3/2)$ then it will bounce off and hit the corner at $(0,3)$, traveling a total distance of $\frac{\sqrt{109}}{2} + \frac{\sqrt{109}}{2} = \sqrt{109}$ meters.



The point where the beam first hits the wall is either a point $(5, n/2)$ for $1 \leq n \leq 5$ or $(m/2, 3)$ for $1 \leq m \leq 9$. What is the greatest distance that any such beam of light travels?

A. $\sqrt{61}$ B. $\sqrt{109}$ C. $5\sqrt{37}$ D. $15\sqrt{15}$ E. None of these

19. A *derangement* of the digits `123456` is a rearrangement of the digits so that digit n does not appear in position n. Thus `654321` and `246513` are derangements but `653124` is not because the digit `3` appears in position 3. How many derangements of `123456` have a `3` in position 1 and a `1` in position 6?

A. 8 B. 10 C. 12 D. 14 E. None of these

20. In how many ways may the letters `RHAAEEIIOOUU` be arranged in a line so that no two consecutive letters are the same? The arrangements `AEIOURHAEIOU` and `RAEIOUHAEIUO` are two such arrangements.

A. 5040 B. 4147200 C. 6219360 D. 14968800 E. None of these