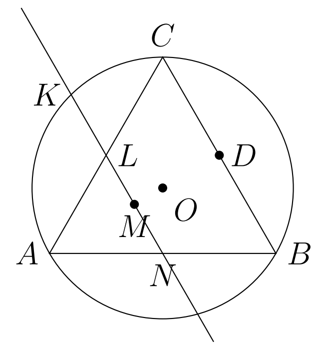
| | | | |
|---|---|---|---|
| 1 | 6 b | 11 c | 16 c |
| 2 a | 7 d | 12 b | 17 c |
| 3 e | 8 d | 13 a | 18 e |
| 4 e | 9 e | 14 e | 19 e |
| 5 c | 10 d | 15 a | 20 c |

**11** Triangle $ANL$ is similar to triangle $ABC$ so $LM/DC = AM/AD = 1/2$. Because $O$ is the center of the circle $AO/AD = 2/3$. Therefore $MO/AD = 1/6$. The circumscribed circle is centered at $O$ so $OK = AO = \frac{2}{3}AD = \frac{2}{\sqrt{3}}AD$. Because $AD \perp BC$, $AD \perp ML$ so $OK^2 = MO^2 + MK^2$. This is $\frac{4}{9}AD^2 = \frac{1}{36}AD^2 + MK^2$ hence $MK = \frac{\sqrt{15}}{6}AD$. Thus $LK = MK - ML = \frac{\sqrt{15}}{6}AD - \frac{\sqrt{3}}{6}AD$. We have $AL = \frac{1}{2}AC = \frac{1}{2}CB = DC = AD/\sqrt{3}$ so $\frac{AL}{LK} = \frac{1/\sqrt{3}}{\frac{\sqrt{15}}{6} - \frac{\sqrt{3}}{6}} = \frac{1}{\frac{3\sqrt{5}}{6} - \frac{1}{2}} = \frac{2}{\sqrt{5}-1} = \boxed{\frac{\sqrt{5}+1}{2}}$.

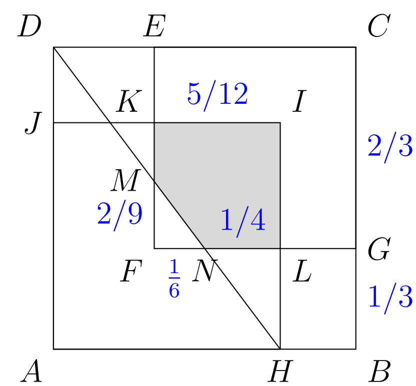



**12** The probability that seed 1 wins in the first round is 4/5. The probability that seed 2 wins in the first round is 3/5 and the probability that seed 1 beats seed 2 is 2/3. The probability that seed 3 wins in the first round is 2/5 and the probability that seed 1 beats seed 3 is 3/4.

Hence the probability that seed 1 beats seed 2 to win the tournament is $\frac{4}{5} \cdot \left(\frac{3}{5} \cdot \frac{2}{3}\right) = \frac{8}{25}$ and the probability that seed 1 beats seed 3 to win the tournament is $\frac{4}{5} \cdot \left(\frac{2}{5} \cdot \frac{3}{4}\right) = \frac{6}{25}$.

These are mutually exclusive events so the probability that seed 1 wins the tournament is $\frac{8}{25} + \frac{6}{25} = \frac{14}{25} = \boxed{0.56}$.

**13** The sides of square $ABCD$ have length 1. Therefore $IJ = 3/4$, $FG = 2/3$, and $FL = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$. Hence the area of square $KFLI$ is 25/144. Triangles $\triangle DAH$, $\triangle HLN$, and $\triangle MFN$ are similar right triangles with $\frac{AD}{AH} = \frac{4}{3}$. We have $HL = GB = BC - GC = 1 - \frac{2}{3} = \frac{1}{3}$ and $LN = \frac{3}{4}HL = \frac{1}{4}$. Thus $FN = FL - LN = \frac{5}{12} - \frac{1}{4} = \frac{1}{6}$ and $MF = \frac{4}{3}FN = \frac{2}{9}$. The area of $\triangle MFN$ is $\frac{1}{2} \cdot \frac{1}{6} \cdot \frac{2}{9} = \frac{1}{54}$ so the are of $KMNLI$ is $\frac{25}{144} - \frac{1}{54} = \boxed{\frac{67}{432}}$.




**14** The 2016 contest takes place on November 12, 2016. There are 52 weeks plus 1 day in each non-leap year so the 2017 contest takes place on November 11, the 2018 on November 10, and the 2019 on November 9. Hence Nov 8, 2019 is a Friday, not a date of the contest. The year 2020 is a leap year so November 7 is a Saturday, but it is the first Saturday in November so the 2020 contest takes place on November 14, not November 9. The 2021 contest takes place on November 13, and the 2022 contest takes place on November 12. Hence $\boxed{\text{none}}$ of these dates is the next sum date on which the contest takes place. The next time the second Saturday of November is a sum date is November 9, 2120.

**15** We use the double angle formula to see that $6\sin(\alpha) = 10\sin(\alpha)\cos(\alpha)$. Therefore $\cos(\alpha) = 3/5$ and $\sin(\alpha) = 4/5$. Thus $\sin(2\alpha) = 24/25$ and $\cos(2\alpha) = -7/25$. Therefore $\sin(3\alpha) = \sin(2\alpha)\cos(\alpha) + \sin(\alpha)\cos(2\alpha) = \frac{24}{25} \cdot \frac{3}{5} + \frac{4}{5} \cdot \frac{-7}{25} = \boxed{\frac{44}{125}}$

**16** $2016 = 2^5 \cdot 3^2 \cdot 7$ so 2016 divides $8!$. The sum $8! + 9! + \cdots + 2015!$ leaves no remainder upon division by 2016 so the only terms that contribute to the non-zero reminder is $1! + 2! + \cdots 7! = 1 + 2 + 6 + 24 + 120 + 720 + 5040 = 5913 = 2 \cdot 2016 + \boxed{1881}$

**17** A chip numbered 1 is drawn $\frac{4}{10}$ of the time, in which case the probability of drawing a chip with a 3 on the second draw is $\frac{2}{14}$.

A chip numbered 2 is drawn $\frac{3}{10}$ of the time, in which case the probability of drawing a chip with a 3 on the second draw is $\frac{2}{13}$.

A chip numbered 3 is drawn $\frac{2}{10}$ of the time, in which case the probability of drawing a chip with a 3 on the second draw is $\frac{4}{12}$.

A chip numbered 4 is drawn $\frac{1}{10}$ of the time, in which case the probability of drawing a chip with a 3 on the second draw is $\frac{2}{11}$.

The probability that a chip numbered 3 is drawn on the second draw is $\frac{4}{10} \cdot \frac{2}{14} + \frac{3}{10} \cdot \frac{2}{13} + \frac{2}{10} \cdot \frac{4}{12} + \frac{1}{10} \cdot \frac{2}{11} = \boxed{\dfrac{565}{3003}}$

**18** Instead of thinking of the beam bouncing off the wall and reflecting, think of the beam traveling in a straight line and the room reflecting off the wall each time. Then the beam leaves the room when it reaches a point of the form $(5r, 3s)$ for some integer $k$. The total distance traveled by the bouncing light beam is then $\sqrt{25r^2 + 9s^2}$ meters.

We find the exit points corresponding to each possible point where the beam first hits the wall.

| First hit | Exit point | Distance |
|---|---|---|
| $(5, 1/2)$ | $(30, 3)$ | $\sqrt{909}$ |
| $(5, 2/2)$ | $(15, 3)$ | $\sqrt{234}$ |
| $(5, 3/2)$ | $(10, 3)$ | $\sqrt{109}$ |
| $(5, 4/2)$ | $(15, 6)$ | $\sqrt{261}$ |
| $(5, 5/2)$ | $(30, 15)$ | $\sqrt{1125}$ |

| | | |
|---|---|---|
| $(1/2, 3)$ | $(5, 30)$ | $\sqrt{925}$ |
| $(2/2, 3)$ | $(5, 15)$ | $\sqrt{250}$ |
| $(3/2, 3)$ | $(15, 30)$ | $\sqrt{1125}$ |
| $(4/2, 3)$ | $(10, 15)$ | $\sqrt{325}$ |
| $(5/2, 3)$ | $(5, 6)$ | $\sqrt{61}$ |
| $(6/2, 3)$ | $(15, 15)$ | $\sqrt{450}$ |
| $(7/2, 3)$ | $(35, 30)$ | $\sqrt{2125}$ |
| $(8/2, 3)$ | $(20, 15)$ | $\sqrt{625}$ |
| $(9/2, 3)$ | $(45, 30)$ | $\sqrt{2925}$ |

The greatest distance traveled is $\sqrt{2925} = 15\sqrt{13}$ so $\boxed{\text{none}}$ of the provided numbers is correct.

**19** The total number of rearrangements with `3` in position 1 and `1` in position 6 is $4! = 24$.

A `2` is in the position 2 for 3! rearrangements. Similarly, `4` and `5` are in the correct positions for 3! of the arrangements. The digits `1`, `3`, and `6` cannot be in the correct position.

If we take the total to be $4! - 3 \cdot 3! = 6$ we are mistaken as we have eliminated values with multiple $n$ in position $n$ multiple times.

The pair (`2`, `4`) are in positions 2 and 4, respectively for 2! arrangements. The same number of arrangements applies to the pairs (`2`, `5`) and (`4`, `5`).

The trio (`2`, `4`, `5`) are in positions 2,4, and 5 respectively for the 1! arrangement.

Thus there are $4! - 3 \cdot 3! + 3 \cdot 2! - 1! = 11$ such derangements.

An alternate solution is to carefully list all permissible derangements:

`342561 345261 345621 346521 352641 354261 354621 356241 362541 364521 365241`

**20** There are 12 letters, two lone letters and five pairs. Thus there are $\frac{12!}{(2!)^5}$ arrangements of the letters.

To see how many have `A`s treat `AA` as a single letter and observe that there are $\frac{11!}{2^4}$ such arrangements.

The number of arrangements with AA and EE is $\frac{10!}{2^3}$. There are $\binom{5}{2}$ ways to choose two of the letters to have next to each other.

The total number of arrangements in which no consecutive letters are the same is

$\frac{12!}{(2!)^5} - \binom{5}{1}\frac{11!}{2^4} + \binom{5}{2}\frac{10!}{2^3} - \binom{5}{3}\frac{9!}{2^2} + \binom{5}{4}\frac{8!}{2} - \binom{5}{5} \cdot 7!$

$= \frac{7!}{2^5}\left(12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 - 5 \cdot 2 \cdot 11 \cdot 10 \cdot 9 \cdot 8 + 10 \cdot 4 \cdot 10 \cdot 9 \cdot 8 - 10 \cdot 8 \cdot 9 \cdot 8 + 5 \cdot 16 \cdot 8 - 32\right)$

$= 7!\left(3 \cdot 11 \cdot 10 \cdot 9 - 5 \cdot 11 \cdot 5 \cdot 9 + 10 \cdot 10 \cdot 9 - 10 \cdot 2 \cdot 9 + 5 \cdot 4 - 1\right)$

$= 7!\left(99 \cdot [30 - 25] + 90 \cdot [10 - 2] + 20 - 1\right) = 7!\left(495 + 720 + 19\right) = 5040 \cdot 1234 = \boxed{6,219,360}$