HIGH SCHOOL MATHEMATICS CONTEST

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November 14, 2015

Instructions: Put your name and home address on the back of your answer card. Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use any calculator other than your brain and fingers.

All students will answer the same 20 questions. Each question is worth 5 points for the correct answer, 0 points for no answer and -1 point for the wrong answer. You will find that the more difficult problems are at the end of the test.

Good Luck!

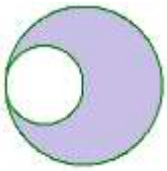
Prob 1. Evaluating $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}$ gives

a) $\frac{47}{60}$ b) $\frac{77}{60}$ c) $\frac{67}{60}$ d) $\frac{89}{120}$ e) none of these

Prob 2. If $x+y=8$ and $x-y=7$, then $x-2y=$

a) 1 b) -1 c) 4 d) -4 e) none of these

Prob 3. The outer circle shown in the figure has diameter *D*. The inner circle passes through the center of and is tangent to the outer circle. The area of the shaded region is



a) $3\pi D^2/16$ b) $\pi D^2/2$ c) $3\pi D^2/8$ d) $5\pi D^2/8$
e) none of these

Prob 4. Let $\cap$ denote intersection and $\cup$ denote union. If A and B are the sets A = {1,2,4,6,8} and B = {2,3,4,5,6} then $(A\cap B)\cup B$ =

a) empty b) {1,2,4,6,8} c) {2,3,4,5,6} d) {1,2,3,4,5,6,8}
e) none of these

Prob 5. How many cubic inches are in a rectangular block with dimensions 2 feet by 3 inches by 1 yard?

a) 3584 b) 10624 c) 3622 d) 2592 e) none of these

Prob 6. A dealer bought an article for \$7, sold it for \$8, bought it back for \$9, and sold it for \$10. How much profit did he make?

a) \$3 b) \$4 c) \$1 d) \$5 e) none of these

Prob 7. If the symbol $\otimes$ is defined so that $X \otimes Y = X + 2Y + 1$, then $\{2 \otimes 3) \otimes 2 =$

a) 13 b) 14 c) 15 d) 16 e) none of these

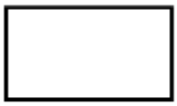
Prob 8. Using only quarters, dimes and nickels, how many different ways are there to make change for half a dollar?

a) 6 b) 10 c) 8 d) 9 e) none of these

Prob 9. If x is a real number then $x^2 > 4x$

a) for all x b) only for $x < 0$ c) only for $x > 4$ or $x < 0$ d) only for $x > 2$ or $x < 0$
e) none of these

Prob 10. A rectangular field is three times as wide as it is long. If its perimeter is 800 feet then its area in square feet is

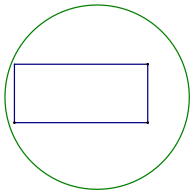


a) 30,000 b) 60,000 c) 40,000 d) 80,000
e) none of these

Prob 11. Amy walks at 2 mph. Joe rides his bicycle at 8 mph. If they start at the same time and travel in opposite directions, in how many minutes will they be 22 miles apart?

a) 2.2 b) 142 c) 122 d) 32 e) none of these

Prob 12. A rectangle is covered by a circle if every point of the rectangle is also a point of the circle. The figure is an example. The area (in square inches) of the smallest circle that will cover a rectangle with sides 2 inches by 3 inches is



a) 3π b) $13\pi/4$ c) $15\,\pi/4$ d) 4π
e) none of these

Prob 13. Sally ate 483 grapes in 21 days. Each day she ate 2 more than on the day before. How many did she eat on the fifteenth day?
a) 34 b) 28 c) 29 d) 31 e) none of these

Prob 14. A box contains 2 red balls and 3 green balls. Jim randomly picks one of the 5 balls and sets it aside. He then randomly picks one of the remaining 4 balls. What is the probability that he has chosen exactly one ball of each color?

a) 3/5 b) 2/5 c) 2/3 d) 1/3 e) none of these

Prob 15. How many three digit numbers satisfy the following three conditions: a) the product of the three digits is 72, b) the units digit is equal to or greater than the tens digit and c) the tens digit is equal to or greater than the hundreds digit?

a) 5 b) 6 c) 7 d) 8 e) none of these

Prob 16. Consider the equation $i\,|z|^2 + z = 3 + 29i$, where z is complex, $|z|$ is the absolute value of z and $i = \sqrt{-1}$. The sum of the squares of all possible z's satisfying the equation is

a) -25 - 8i b) -23 - 6i c) -27 - 4i d) -22 - 9i e) none of these

Prob 17. The symbol $n!$ (n factorial) is the product of the integers 1 through n. Trailing zeros are those at the end of a positive integer. For example 165,000 has three trailing zeros. How many trailing zeros does 2015! have?

a) 502 b) 499 c) 582 d) 524 e) none of these

Prob 18. A number of objects is represented by 43 in base b and by 38 in base B. If B^* is the smallest value of B such that these conditions can be satisfied and b^* the corresponding value of b, then the product of b^* and B^* is

a) 48 b) 84 c) 72 d) 56 e) none of these

Prob 19. We call the finite sequence $\{a_0, a_1, a_2, \ldots a_n\}$ *curious* if a_i is the number of i's in the sequence for each i = 0,1,.. . ,n. For $n = 3$, the sequences {1,2,1,0} and {2,0,2,0}are examples. How many distinct curious sequences are possible with n = 4?

a) 0 b) 2 c) 3 d) 1 e) none of these

Prob 20. Let [x] represent the greatest integer which is less than or equal to x. For example, [3] = 3 and [2.6] = 2. Let x be positive and k a positive integer with $1 \le k \le 169$. For how many values of k does the equation $x[x] = k$ have a solution?

a) 78 b) 74 c) 79 d) 75 e) none of these