HIGH SCHOOL MATHEMATICS CONTEST

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Instructions: Put your name and home address on the back of your answer card. Record all your answers to the problems on the front of the card. Use the backs of the question sheets for scratch paper. You may not use any calculator other than your brain and fingers.

All students will answer the same 20 questions. Each question is worth 5 points for the correct answer, 0 points for no answer and -1 point for the wrong answer. You will find that the more difficult problems are at the end of the test.

Good Luck!

Prob. 1. If $3y+4=5y+6$ then $4y$ is

a) -4 b) 4 c) 2 d) -2 e) none of these

Prob. 2. The intersection of the sets $A=\{2,5,7,8,9\}$ and $B=\{1,2,3,6,7,8\}$ is the set

a) {1,2,3,4,5,6,7,8,9} b) {1,2,3,5,7,8,9} c) {2,7,8} d) {2,8} e) none of these

Prob. 3. The number of edges plus the number of faces of a cube is

a) 10 b) 12 c) 14 d) 16 e) none of these

Prob. 4. The decimal .00125 is equal to the fraction

a) 1/8 b) 1/80 c) 1/800 d) 1/8000 e) none of these

Prob. 5. $(4^0)(4)^4 =$

a) 4^0 b) 16 c) 0 d 256 e) none of these

Prob. 6. The number of cents in n nickels and q quarters is

a) $5n + q/4$ b) $n/5 + q/4$ c) $5n + 25q$ d) $n/5 + 25q$ e) none of these

Prob. 7. If x and y satisfy the equations $x + 2y = 3$ and $2x - y = 7$, then $x + y =$

a) 14/6 b) 17/5 c) 15/7 d) 16/5 e) none of these

Prob. 8. In a computer language, a variable is denoted either by a single capital letter (A through Z) or by a capital letter followed by a single digit (A0, A1, …,Z9). How many different variables are possible in this language?

a) 286 b) 282 c) 284 d) 280 e) none of these

Prob. 9. The figure shows a rectangle surmounted by a semicircle If the areas of the semicircle and the rectangle are both equal to 4π, then the height h of the rectangle is

a) $\pi/2$ b) $\pi/\sqrt{2}$ c) $2\sqrt{2}$ d) $2\sqrt{\pi}$ e) none of these

Prob. 10. If the operation * is defined so that $a * b = ab/(a + b)$, then $3 * (3 * 3) =$

a) 9/8 b) 9/4 c) 4/9 d) 2/9 e) none of these

Prob. 11. Sally rows 12 miles per hour in still water. In a river with constant flow rate, she rows a certain distance upstream in two hours and rows back the same distance downstream in one hour. At what rate was the river flowing in miles per hour?

a) 1 b) 3 c) 5 d) 7 e) none of these

Prob 12. If a and b are integers greater than 0, then the number of pairs (a,b) that satisfy the equation $12a + b = ab$ is

a) 2 b) 4 c) 6 d) 8 e) none of these

Prob. 13. If we call an integer boring if all of its digits are the same, then how many integers greater than 1 and less than 10000 are both boring and prime?

a) 4 b) 5 c) 7 d) 6 e) none of these

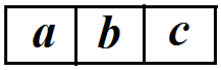
Prob. 14. One car goes around a lap in 40 seconds and a second car goes around the lap in 45 seconds. How many seconds will it take the faster car to gain one lap?

a) 300 b) 320 c) 340 d) 360 e) none of these

Prob 15. Memory jog for 'geometric progression', (2, 4, 8, 16 is an example). If the sides of a right triangle form a geometric progression and the short side has length 1, then the hypotenuse has length

a) $(1+2\sqrt{5})/2$ b) $\sqrt{5}-1$ c) $(1+\sqrt{5})/2$ d) $2\sqrt{5}-1$ e) none of these

Prob. 16. The three boxes shown in the figure contain three positive integers (zero is not a positive integer). The three integers are all different and their sum is 8. How many different ways can the boxes be filled? (order counts, thus $a=2,\ b=5,\ c=1$ is not the same as $a=1,\ b=2,\ c=5$)



a) 9 b) 11 c) 13 d) 15 e) none of these

Prob. 17. The figure shows an addition problem with each letter representing a distinct digit. The leading digits are not zero. Find the digit represented by each letter. The sum of the digits F + R + S + X + T is

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  F O R T Y
      T E N
      T E N
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  S I X T Y
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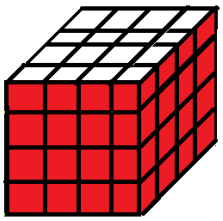
a) 23 b) 24 c) 25 d) 26 e) none of these

Prob. 18. A cube has edges of length *n*, where *n* is an integer. The figure shows the cube with $n=4$. Two faces with an edge in common are painted grey. The cube is then cut into n^3 smaller cubes. Let n_0, n_1 and n_2 be the number of cubes with exactly 0, 1 and 2 grey faces, respectively. If $n_0 / n_2 = 1600$ then n_1 is

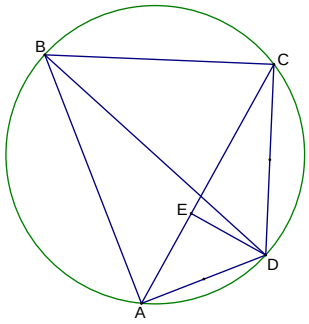


a) 3280 b) 3360 c) 3240 d) 3320 e) none of these

Prob. 19. I have twenty 3 cent stamps and twenty 5 cent stamps. Using one or more of these stamps, how many different amounts of postage can I make?

a) 148 b) 152 c) 163 d) 142 e) none of these

Prob. 20. Quadrilateral *ABCD* (shown, but not to scale) can be inscribed in a circle in such a way that *BD* is a diameter of the circle. Let *E* be the point on *AC* such that *AC* and *DE* are perpendicular. If $AE=6$, $EC=12$ and $DE=5$, then the perpendicular distance from *B* to *AC* is



a) 13.8 b) 14.4 c) 14.2 d) 14.6 e) none of these