Signal Processing First

Lecture 18
3-Domains for IIR

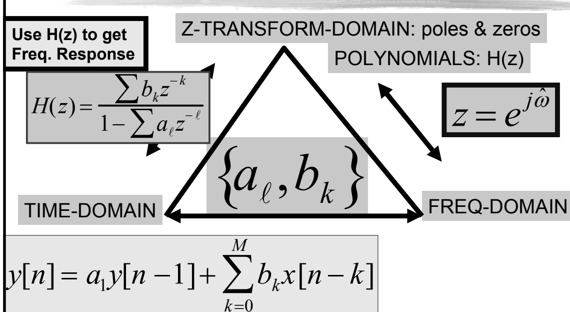
LECTURE OBJECTIVES

- SECOND-ORDER IIR FILTERS
  - TWO FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

- H(z) can have COMPLEX POLES & ZEROS
- THREE-DOMAIN APPROACH
  - BPFs have POLES NEAR THE UNIT CIRCLE

THREE DOMAINS



Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$		$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n-n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n-n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1-az^{-1}}$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2]$$
$$+ b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$H(z) =$$

MORE POLES

- Denominator is QUADRATIC
  - 2 Poles: REAL
  - or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

TWO COMPLEX POLES

- Find Impulse Response ?
  - Can OSCILLATE vs. n
  - "RESONANCE"

$$(p_k)^n = (re^{j\theta})^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**
  - Depends on Pole Location
  - Close to the Unit Circle?
    - Make **BANDPASS FILTER**

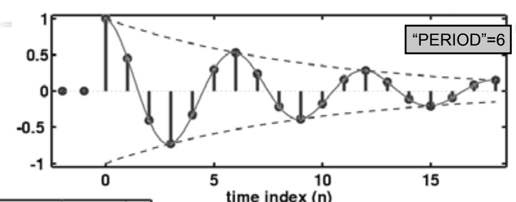
$$\text{pole} = re^{j\theta}$$
$$r \to 1\,?$$

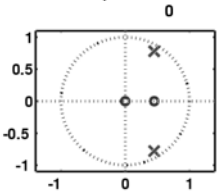
2nd ORDER EXAMPLE

$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3}n)u[n] =$$

$$H(z) = \frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

h[n]: Decays & Oscillates





$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3}n)u[n]$$

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

2nd ORDER Z-transform PAIR

GENERAL ENTRY for z-Transform TABLE

$$h[n] = r^n \cos(\theta n)u[n]$$

$$H(z) = \frac{1 - r\cos\theta z^{-1}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

$$h[n] = Ar^n \cos(\theta n + \varphi)u[n]$$

$$H(z) = A\frac{\cos\varphi - r\cos(\theta - \varphi)z^{-1}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

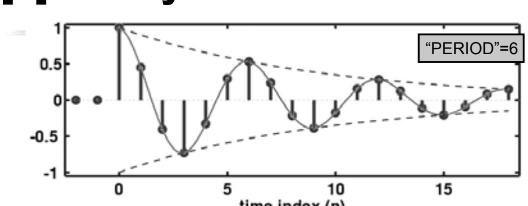
2nd ORDER EX: n-Domain

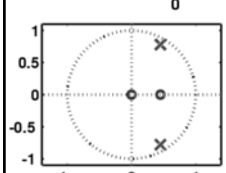
$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - 0.45x[n-1]$$

```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```


h[n]: Decays & Oscillates





$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3}n)u[n]$$

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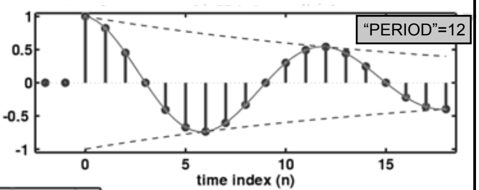
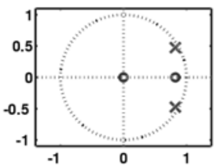
h[n]: Decays & Oscillates

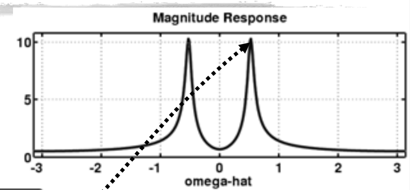


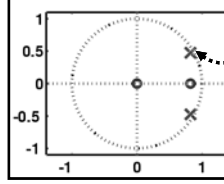


$$h[n] = (0.95)^n \cos(\tfrac{\pi}{6} n) u[n]$$

$$\frac{1 - 0.8227 z^{-1}}{1 - 1.6454 z^{-1} + 0.9025 z^{-2}}$$

Complex POLE-ZERO PLOT





$$\frac{1 - 0.8227 z^{-1}}{1 - 1.6454 z^{-1} + 0.9025 z^{-2}}$$

3 DOMAINS MOVIE: IIR