

Signal Processing First

Lecture 18 3-Domains for IIR

1/22/2004

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LECTURE OBJECTIVES

- SECOND-ORDER IIR FILTERS
 - TWO FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^2 b_k x[n-k]$$

- H(z) can have COMPLEX POLES & ZEROS
- THREE-DOMAIN APPROACH
 - BPFs have POLES NEAR THE UNIT CIRCLE

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THREE DOMAINS

Use H(z) to get
Freq. Response

Z-TRANSFORM-DOMAIN: poles & zeros

POLYNOMIALS: H(z)

$$H(z) = \frac{\sum b_k z^{-k}}{1 - \sum a_l z^{-l}}$$

$$z = e^{j\hat{\omega}}$$

$\{a_l, b_k\}$

TIME-DOMAIN

FREQ-DOMAIN

$$y[n] = a_1 y[n-1] + \sum_{k=0}^M b_k x[n-k]$$

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Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$	\iff	$X(z)$
1.	$ax_1[n] + bx_2[n]$	\iff	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	\iff	$z^{-n_0} X(z)$
3.	$y[n] = x[n] * h[n]$	\iff	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	\iff	1
5.	$\delta[n - n_0]$	\iff	z^{-n_0}
6.	$a^n u[n]$	\iff	$\frac{1}{1 - az^{-1}}$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$H(z) =$$

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MORE POLES

- Denominator is QUADRATIC

- 2 Poles: REAL
- or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 - a_1 z - a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

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TWO COMPLEX POLES

- Find Impulse Response ?

- Can OSCILLATE vs. n

- "RESONANCE"

$$(p_k)^n = (re^{j\theta})^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**

- Depends on Pole Location

- Close to the Unit Circle?

- Make **BANDPASS FILTER**

$$\text{pole} = re^{j\theta}$$

$$r \rightarrow 1?$$

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2nd ORDER EXAMPLE

$$h[n] = (0.9)^n \cos\left(\frac{\pi}{3}n\right)u[n]$$

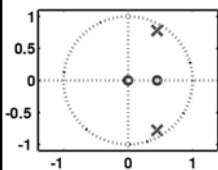
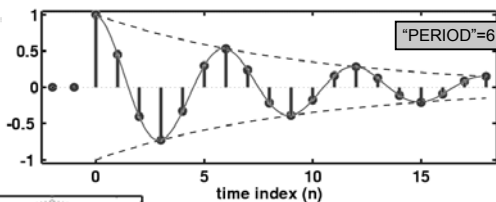
$$H(z) = \frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

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$h[n]$: Decays & Oscillates



$$h[n] = (0.9)^n \cos\left(\frac{\pi}{3}n\right)u[n]$$

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

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2nd ORDER Z-transform PAIR

$$h[n] = r^n \cos(\theta n)u[n]$$

GENERAL ENTRY for
z-Transform TABLE

$$H(z) = \frac{1 - r \cos \theta z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

$$h[n] = Ar^n \cos(\theta n + \varphi)u[n]$$

$$H(z) = A \frac{\cos \varphi - r \cos(\theta - \varphi)z^{-1}}{1 - 2r \cos \theta z^{-1} + r^2 z^{-2}}$$

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2nd ORDER EX: n-Domain

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

$$y[n] = 0.9y[n-1] - 0.81y[n-2] + x[n] - 0.45x[n-1]$$

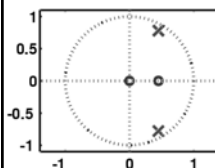
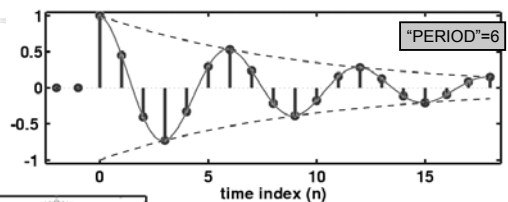
```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```

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$h[n]$: Decays & Oscillates



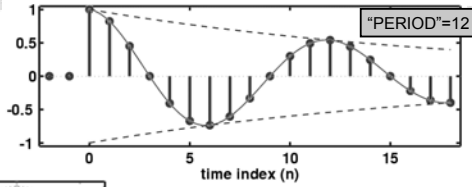
$$h[n] = (0.9)^n \cos\left(\frac{\pi}{3}n\right)u[n]$$

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

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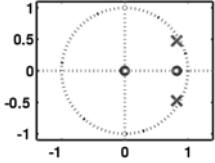
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h[n]: Decays & Oscillates



$$h[n] = (0.95)^n \cos\left(\frac{\pi}{6}n\right)u[n]$$

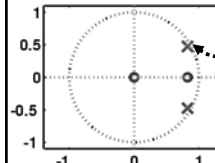
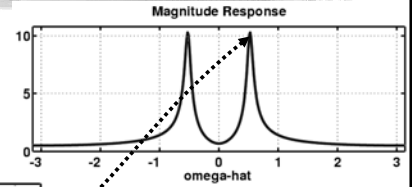
$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$



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Complex POLE-ZERO PLOT



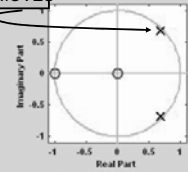
$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

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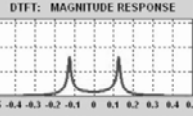
3 DOMAINS MOVIE: IIR

POLE MOVES



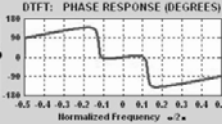
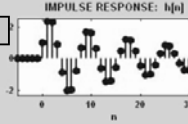
$$\frac{1 + z^{-1}}{1 - 1.36z^{-1} + 0.918z^{-2}}$$

H(z)



H(\omega)

h[n]



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