Signal Processing First

Lecture 17
IIR Filters: H(z) and Frequency Response

LECTURE OBJECTIVES

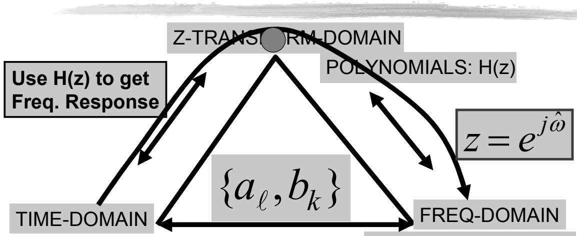
- SYSTEM FUNCTION: H(z)
- H(z) has **POLES** and ZEROS
- FREQUENCY RESPONSE of IIR
  - Get H(z) first

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

THREE DOMAINS



$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{\sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}{1 - \sum_{\ell=1}^{N} a_\ell e^{-j\hat{\omega}\ell}}$$

H(z) = z-Transform{ h[n] }

- FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$h[n] = b_0 (a_1)^n u[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

First-Order Transform Pair

$$h[n] = b a^n u[n] \quad \leftrightarrow \quad H(z) = \frac{b}{1 - a z^{-1}}$$

DELAY PROPERTY of X(z)

- DELAY in TIME<-->Multiply X(z) by z^{-1}

$$x[n] \leftrightarrow X(z)$$
$$x[n-1] \leftrightarrow z^{-1} X(z)$$

Z-Transform of IIR Filter

- DERIVE the SYSTEM FUNCTION H(z)
  - Use **DELAY** PROPERTY

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

EASIER with DELAY PROPERTY

Time delay of n_0 samples multiplies the z-transform by z^{-n_0}

$$x[n-n_0] \iff z^{-n_0} X(z)$$

SYSTEM FUNCTION of IIR

- NOTE the FILTER COEFFICIENTS

SYSTEM FUNCTION

- Given: DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

- **READ** the FILTER COEFFS:

POLES & ZEROS

- Find the Poles and Zeros
- ROOTS of Numerator & Denominator

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

EXAMPLE: Poles & Zeros

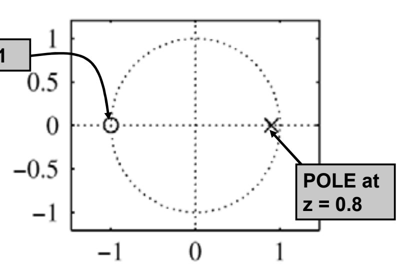
- VALUE of H(z) at POLES is **INFINITE**

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$$

$$H(z) = \frac{2 + 2(-1)}{1 - 0.8(-1)} = 0$$ **ZERO at z= -1**

$$H(z) = \frac{2 + 2(\frac{5}{4})^{-1}}{1 - 0.8(\frac{5}{4})^{-1}} = \frac{\frac{9}{2}}{0} \to \infty$$ **POLE at z=0.8**

POLE-ZERO PLOT



$$\frac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$$

FREQUENCY RESPONSE

- SYSTEM FUNCTION: H(z)
- H(z) has DENOMINATOR
- FREQUENCY RESPONSE of IIR
  - We have H(z)

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

FREQUENCY RESPONSE

- EVALUATE on the UNIT CIRCLE

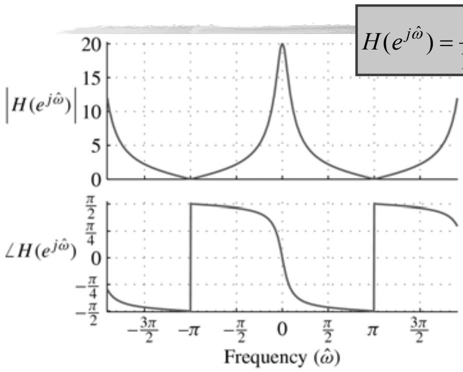
$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$$

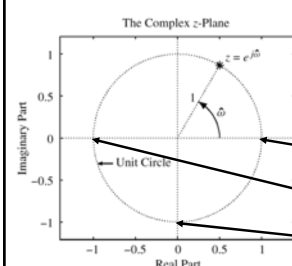
Frequency Response Plot

$$H(e^{j\hat{\omega}}) = \frac{2 + 2e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$$



UNIT CIRCLE

- MAPPING BETWEEN z and $\hat{\omega}$



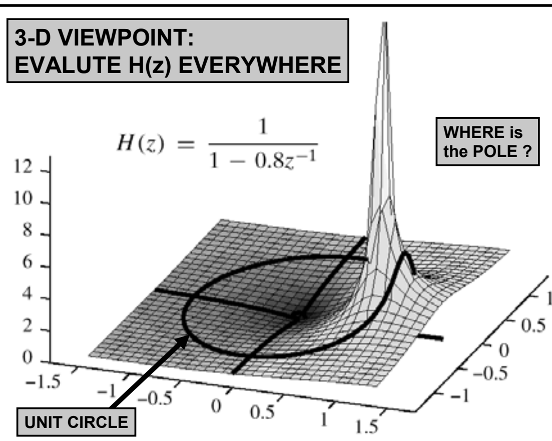
$$z = e^{j\hat{\omega}}$$

$z = 1$	$\leftrightarrow$	$\hat{\omega} = 0$
$z = -1$	$\leftrightarrow$	$\hat{\omega} = \pm\pi$
$z = \pm j$	$\leftrightarrow$	$\hat{\omega} = \pm\frac{1}{2}\pi$

3-D VIEWPOINT: EVALUTE H(z) EVERYWHERE

$$H(z) = \frac{1}{1 - 0.8z^{-1}}$$

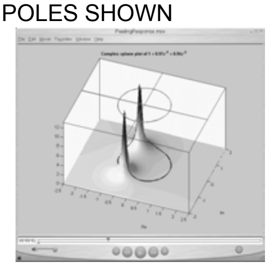
WHERE is the POLE ?



UNIT CIRCLE

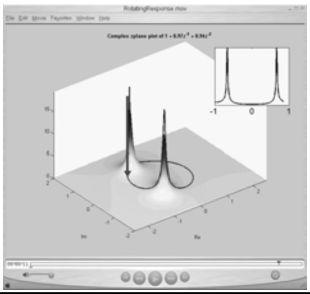
MOVIE for H(z) in 3-D

- POLES to H(z) to Frequency Reponse
  - TWO POLES SHOWN

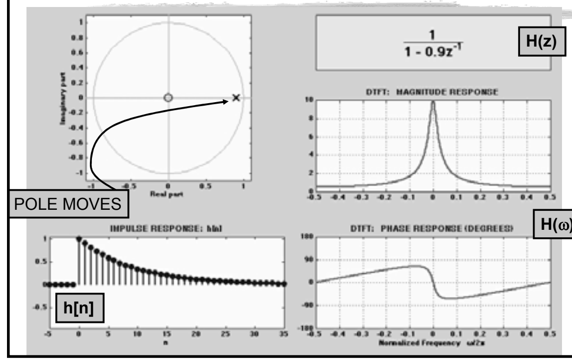


Frequency Response from H(z)

Walking around the Unit Circle



3 DOMAINS MOVIE: IIR



PeZ Demo: Pole-Zero Placing