Signal Processing First

Lecture 16
IIR Filters: Feedback and H(z)

ZZZZZ-Transform

LECTURE OBJECTIVES

- INFINITE IMPULSE RESPONSE FILTERS
  - Define **IIR** DIGITAL Filters
  - Have **FEEDBACK**: use PREVIOUS OUTPUTS

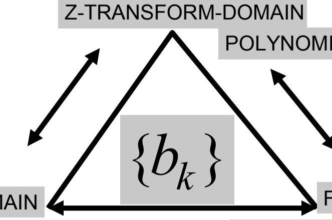
$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

  - Show how to compute the output y[n]
    - FIRST-ORDER CASE (N=1)
    - Z-transform: Impulse Response h[n] ←→ H(z)

THREE DOMAINS

Z-TRANSFORM-DOMAIN

POLYNOMIALS: H(z)



TIME-DOMAIN

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FREQ-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

Quick Review: Delay by n_d

$$y[n] = x[n-n_d]$$

IMPULSE RESPONSE

SYSTEM FUNCTION

FREQUENCY RESPONSE

LOGICAL THREAD

- FIND the IMPULSE RESPONSE, h[n]
  - INFINITELY LONG
  - **IIR** Filters

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n}$$

- EXPLOIT THREE DOMAINS:
  - Show Relationship for IIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

ONE FEEDBACK TERM

- ADD PREVIOUS OUTPUTS

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

PREVIOUS FEEDBACK

FIR PART of the FILTER

FEED-FORWARD

- CAUSALITY

FILTER COEFFICIENTS

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

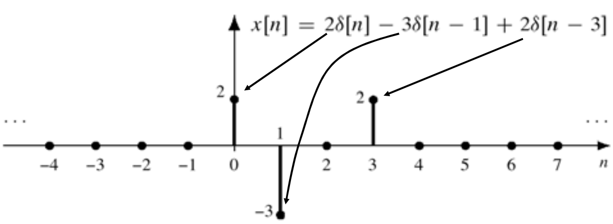
FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB
  - `yy = filter([3,-2],[1,-0.8],xx)`

COMPUTE OUTPUT

$$y[n] = 0.8y[n-1] + 5x[n]$$



COMPUTE y[n]

- FEEDBACK DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 5x[n]$$

AT REST CONDITION

- y[n] = 0, for n<0
- BECAUSE x[n] = 0, for n<0

INITIAL REST CONDITIONS

1. The input must be assumed to be zero prior to some starting time n_0, i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied*.
2. The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

COMPUTE y[0]

- THIS STARTS THE RECURSION:

With the initial rest assumption, $y[n] = 0$ for $n < 0$,

$$y[0] = 0.8y[-1] + 5(2) = 0.8(0) + 5(2) = 10$$

- SAME with MORE FEEDBACK TERMS

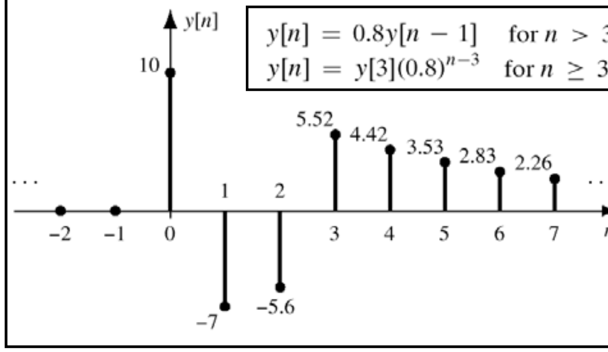
$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

COMPUTE MORE y[n]

- CONTINUE THE RECURSION:

PLOT y[n]

$$y[n] = 0.8y[n-1] \quad \text{for } n > 3$$
$$y[n] = y[3](0.8)^{n-3} \quad \text{for } n \geq 3$$



IMPULSE RESPONSE

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

IMPULSE RESPONSE

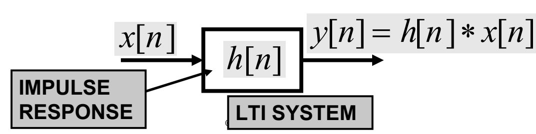
- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n]$$

- Find h[n]

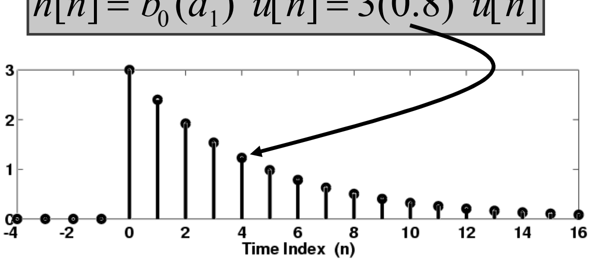
$$h[n] = 3(0.8)^n u[n]$$

- **CONVOLUTION** in TIME-DOMAIN



PLOT IMPULSE RESPONSE

$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$



Infinite-Length Signal: h[n]

- POLYNOMIAL Representation

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

APPLIES to Any SIGNAL

- SIMPLIFY the SUMMATION

$$H(z) =$$

Derivation of H(z)

- Recall Sum of Geometric Sequence:
- Yields a COMPACT FORM

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

$$H(z) =$$

H(z) = z-Transform{ h[n] }

- FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$h[n] = b_0 (a_1)^n u[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

H(z) = z-Transform{ h[n] }

- ANOTHER FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

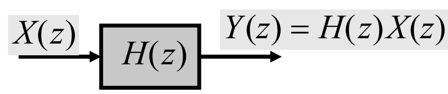
$$h[n] = b_0 (a_1)^n u[n] + b_1 (a_1)^{n-1} u[n-1]$$

z^{-1} is a shift

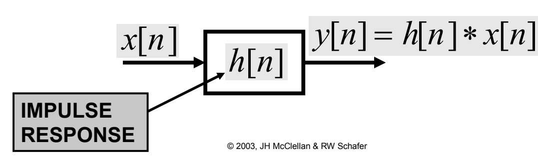
$$H(z) = \frac{b_0}{1 - a_1 z^{-1}} + \frac{b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

CONVOLUTION PROPERTY

- MULTIPLICATION of z-TRANSFORMS



- CONVOLUTION in TIME-DOMAIN



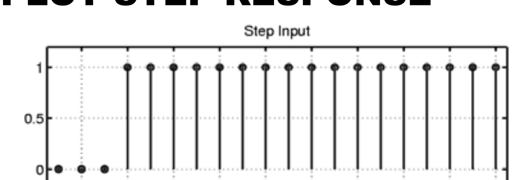
STEP RESPONSE: x[n]=u[n]

Given: $y[n] = a_1 y[n-1] + b_0 x[n]$

$u[n] = 1, \quad \text{for } n \geq 0$

Find: y[n]

PLOT STEP RESPONSE



$y[n] = 0.8 y[n-1] + 3u[n]$ $\quad$ $y[n] = 15\left(1 - 0.8^{n+1}\right)u[n]$