

Signal Processing First

Lecture 15 Zeros of H(z) and the Frequency Domain

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LECTURE OBJECTIVES

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
 - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

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DESIGN PROBLEM

- Example:
 - Design a Lowpass FIR filter (Find b_k)
 - Reject completely 0.7π , 0.8π , and 0.9π
 - This is NULLING
 - Estimate the filter length needed to accomplish this task. How many b_k ?
- Z POLYNOMIALS provide the TOOLS

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Z-Transform DEFINITION

- POLYNOMIAL Representation of LTI SYSTEM:

$$H(z) = \sum_n h[n]z^{-n}$$

- EXAMPLE:

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

APPLIES to
Any SIGNAL

$$\begin{aligned} H(z) &= 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4} \\ &= 2 - 3z^{-2} + 2z^{-4} \\ &= 2 - 3(z^{-1})^2 + 2(z^{-1})^4 \end{aligned}$$

POLYNOMIAL in z^{-1}

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CONVOLUTION PROPERTY

- Convolution in the n-domain
 - SAME AS
- Multiplication in the z-domain

$$y[n] = h[n] * x[n] \leftrightarrow Y(z) = H(z)X(z)$$

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=0}^M h[k]x[n-k] \end{aligned}$$

FIR Filter

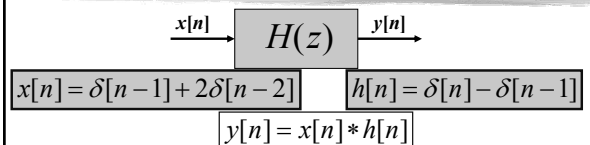
MULTIPLY
z-TRANSFORMS

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CONVOLUTION EXAMPLE

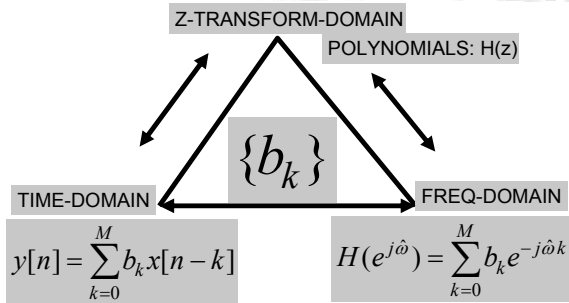


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THREE DOMAINS



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FREQUENCY RESPONSE ?

- Same Form:

$\hat{\omega}$ - Domain

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k (e^{j\hat{\omega}})^{-k}$$

$$z = e^{j\hat{\omega}}$$

z - Domain

$$H(z) = \sum_{k=0}^M b_k z^{-k}$$

SAME COEFFICIENTS

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ANOTHER ANALYSIS TOOL

- z-Transform POLYNOMIALS are EASY !
 - ROOTS, FACTORS, etc.
- ZEROS and POLES: where is $H(z) = 0$?**
- The z-domain is COMPLEX
 - $H(z)$ is a COMPLEX-VALUED function of a COMPLEX VARIABLE z .

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ZEROS of $H(z)$

- Find z , where $H(z)=0$

$$H(z) = 1 - \frac{1}{2}z^{-1}$$

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ZEROS of $H(z)$

- Find z , where $H(z)=0$
 - Interesting when z is ON the unit circle.

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

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PLOT ZEROS in z-DOMAIN

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POLES of H(z)

- Find z, where $H(z) \rightarrow \infty$
 - Not very interesting for the FIR case

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

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FREQ. RESPONSE from ZEROS

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- Relate H(z) to FREQUENCY RESPONSE
- EVALUATE H(z) on the UNIT CIRCLE
 - ANGLE is same as FREQUENCY

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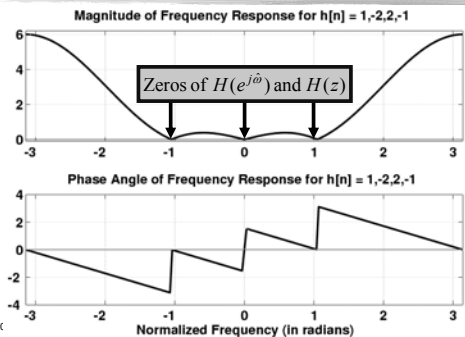
$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

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FIR Frequency Response

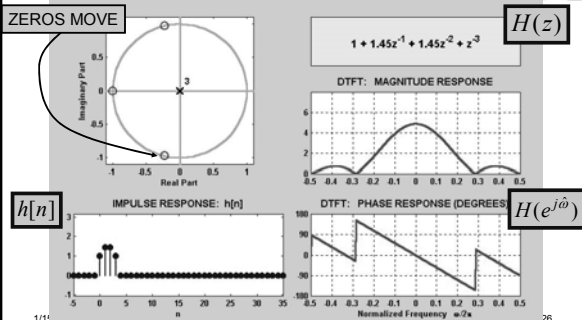


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3 DOMAINS MOVIE: FIR



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NULLING PROPERTY of H(z)

- When $H(z)=0$ on the unit circle.
 - Find inputs $x[n]$ that give zero output

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$x[n] = e^{j(\pi/3)n} \rightarrow H(z) \rightarrow y[n] = H(e^{j\pi/3}) = ?$$

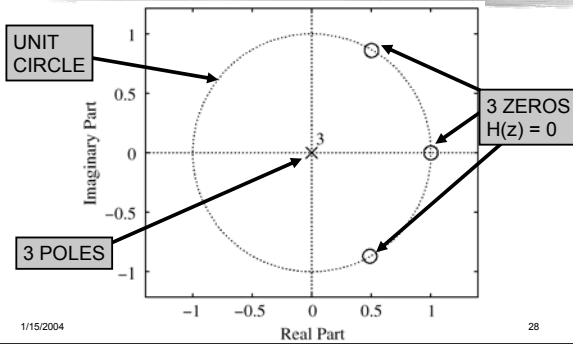
$$y[n] = H(e^{j(\pi/3)}) \cdot e^{j(\pi/3)n}$$

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PLOT ZEROS in z-DOMAIN



NULLING PROPERTY of $H(z)$

- Evaluate $H(z)$ at the input "frequency"

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

$$(1 - 2(\frac{1}{2} - j\frac{\sqrt{3}}{2}) + 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) - (-1))$$

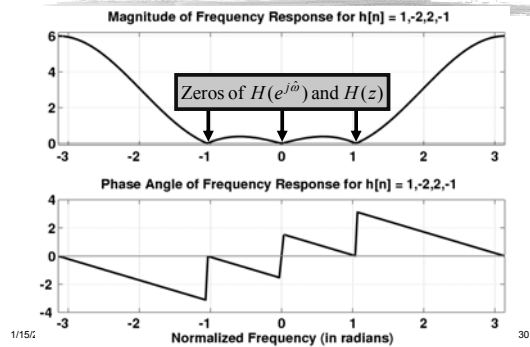
$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

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FIR Frequency Response



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PeZ Demo: Zero Placing

