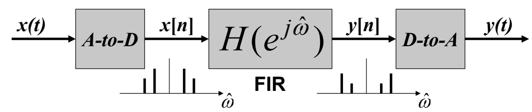
Signal Processing First

Lecture 13
Digital Filtering
of Analog Signals

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

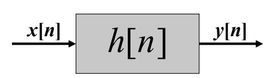
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$

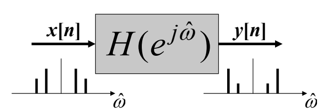
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



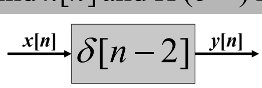
$b_k = \{0, 0, 1\}$

$h[n] = \delta[n-2]$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



$H(e^{j\hat{\omega}})$

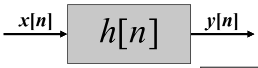
GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-n_d]$

$h[n] = \delta[n-n_d]$

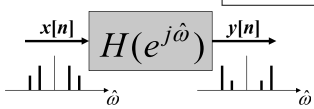
FREQ DOMAIN --> TIME ??

- START with $H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$h[n] = ?$

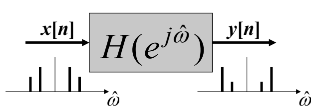
$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$



FREQ DOMAIN --> TIME

EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

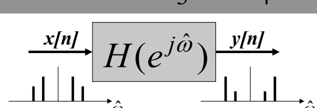
$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6\,e^{-j\pi/12}e^{j(\pi/3)n}$$

EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT (ans-1)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

EX: COSINE INPUT (ans-2)

$$\text{Find } y[n] \text{ when } x[n] = 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

SINUSOID thru FIR

- IF $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

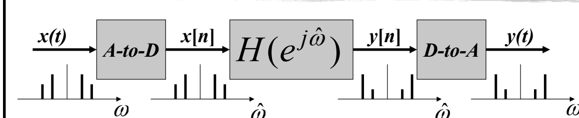
$$x[n] = A\cos(\hat{\omega}_1 n + \phi)$$

$$\Rightarrow y[n] = A\left|H(e^{j\hat{\omega}_1})\right|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

EX: COSINE INPUT (ans-3)

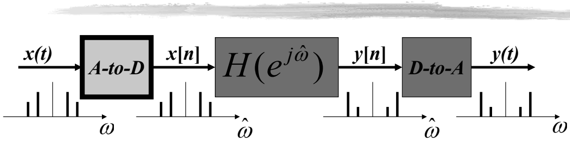
$$\text{Find } y[n] \text{ when } x[n] = 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

DIGITAL "FILTERING"



- ω SPECTRUM of x(t) (SUM of SINUSOIDS)
- $\hat{\omega}$ SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- SPECTRUM y[n] (FIR Gain or Nulls)
- ω Then, OUTPUT y(t) = SUM of SINUSOIDS

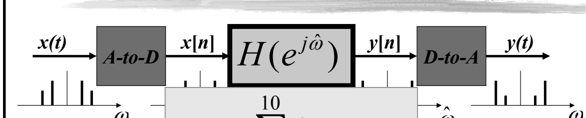
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} \tfrac{1}{11} x[n-k]$$

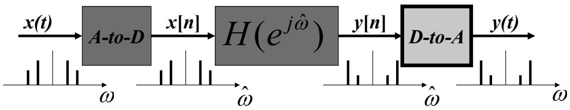
250 Hz

25 Hz

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}} \quad ?$$

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

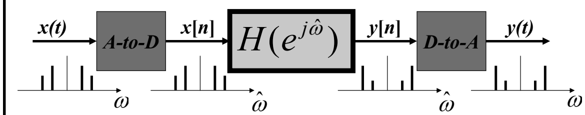
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES

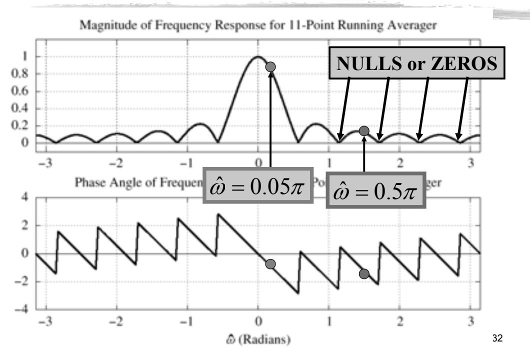


250 Hz	0.5π	$H(e^{j0.5\pi})$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(e^{j0.05\pi})$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

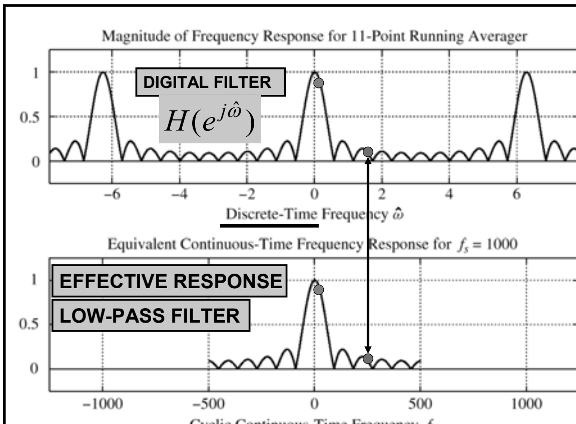
At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

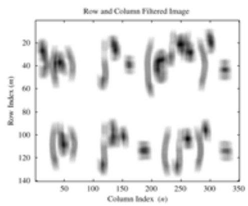
$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

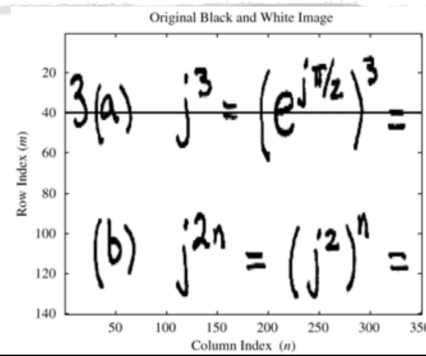


FILTER TYPES

- LOW-PASS FILTER (**LPF**)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (**BPF**)

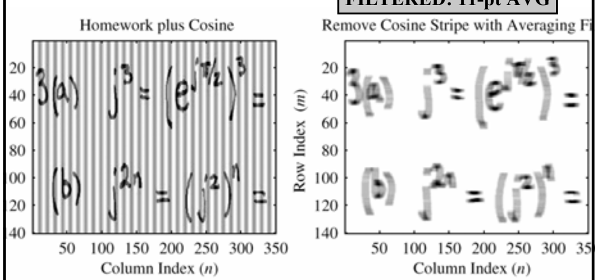


B & W IMAGE

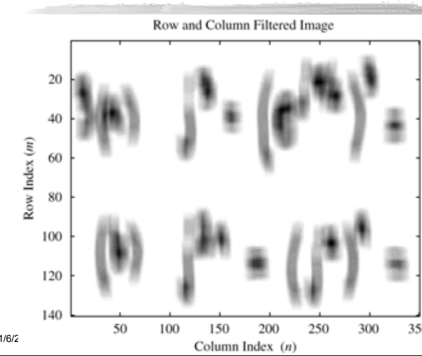


B&W IMAGE with COSINE

FILTERED: 11-pt AVG



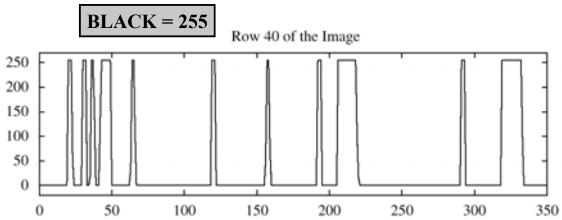
FILTERED B&W IMAGE



LPF:
BLUR

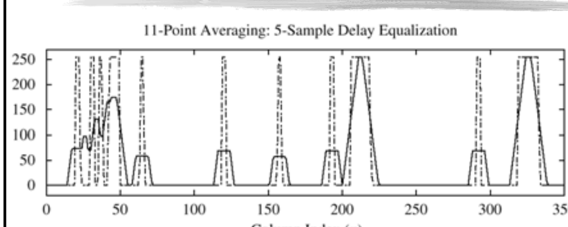
ROW of B&W IMAGE

BLACK = 255



WHITE = 0

FILTERED ROW of IMAGE



ADJUSTED DELAY by 5 samples