Signal Processing First

Lecture 12
Frequency Response of FIR Filters

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

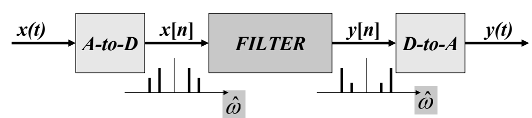
MAG PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t) continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

DIGITAL "FILTERING"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

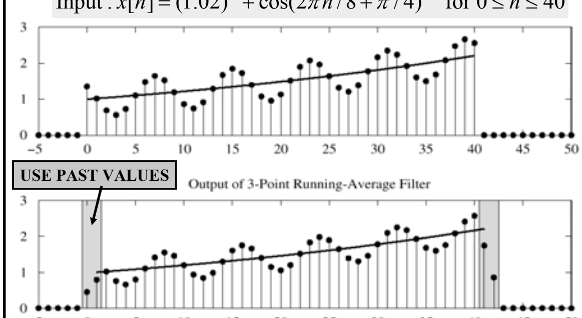
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

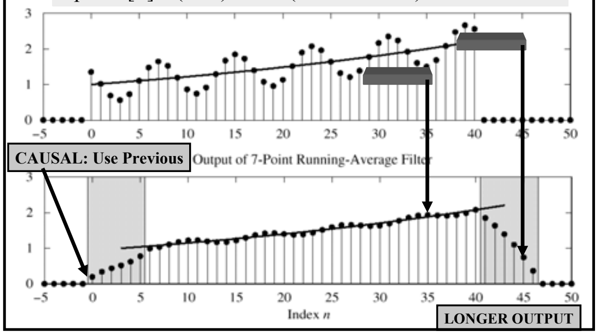
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \le n \le 40$



7-pt FIR EXAMPLE (AVG)

$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$



SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

FREQUENCY REPONSE

- At each frequency, we can DEFINE

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

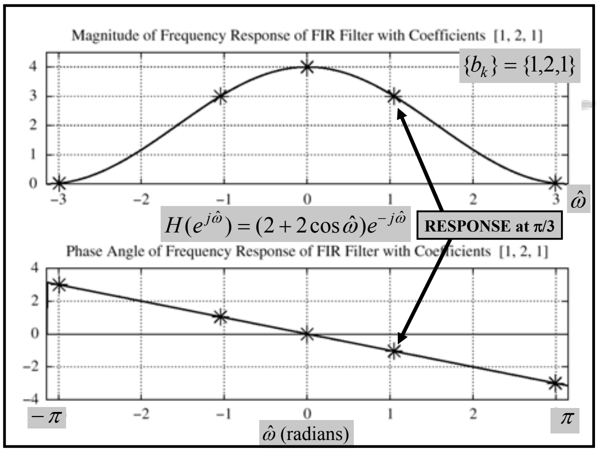
FREQUENCY RESPONSE

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

EXAMPLE 6.1

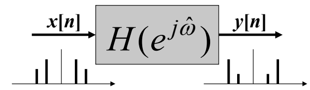
Given: $\{b_k\} = \{1,\ 2,\ 1\}$

Find: $H(e^{j\omega})$



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

MATLAB: FREQUENCY RESPONSE

- `HH = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - SP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **FREQUENCY RESPONSE**, or
  - IMPULSE RESPONSE h[n]
- **Sinusoid IN -----> Sinusoid OUT**
  - **At the SAME Frequency**

Time & Frequency Relation

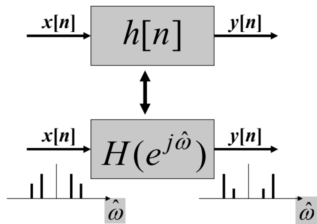
- Get Frequency Response from h[n]
  - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

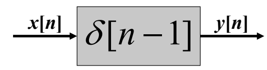
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$



FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n-1]$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?

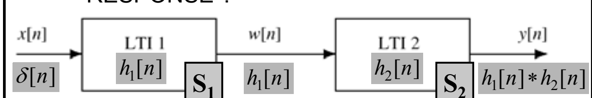
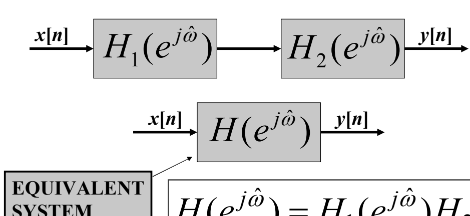


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- <u>MULTIPLY</u> the Frequency Responses



$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}})$$