# Signal Processing First

## Lecture 10
## FIR Filtering Intro

# LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output y[n] from the input signal, x[n]

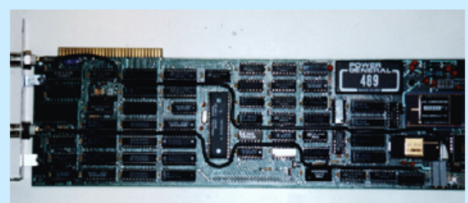
  

# DIGITAL FILTERING


- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

# The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.

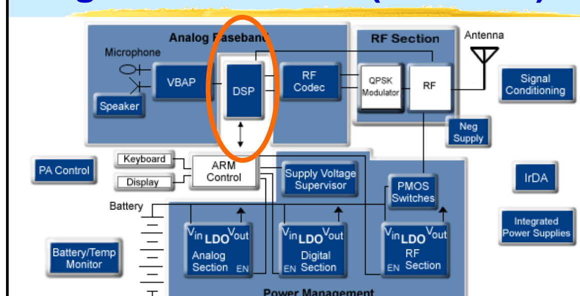
  

# Rockland Digital Filter, 1971


For the price of a small house, you could have one of these.

# Digital Cell Phone (ca. 2000)




Free (?) with 2 year contract

# DISCRETE-TIME SYSTEM


- OPERATE on x[n] to get y[n]
- WANT a **GENERAL** CLASS of SYSTEMS
  - **ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM



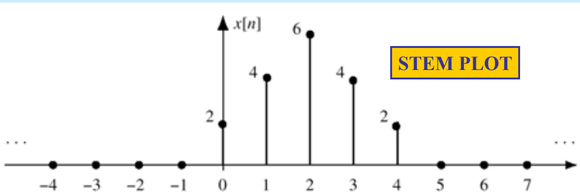
# D-T SYSTEM EXAMPLES


- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"



# DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by "$n$"






# 3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "$n$"

**Make a TABLE**

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$



# PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter **123**

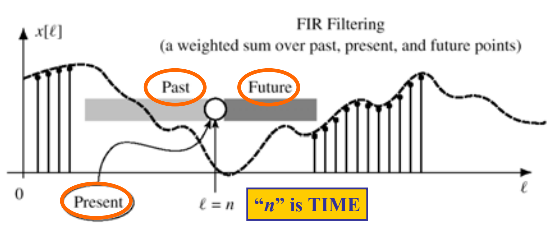



**Figure 5.4** The running-average filter calculation at time index $n$ uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

# ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

| $n$ | $n < -2$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | $7$ | $n > 7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | **6** | **4** | **2** | 0 | 0 | 0 | 0 |
| $y[n]$ | 0 | 0 | 0 | $\frac{2}{3}$ | 2 | 4 | $\frac{14}{3}$ | **4** | 2 | $\frac{2}{3}$ | 0 | 0 |

# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $b_k = \{3, -1, 2, 1\}$





# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **LENGTH** is L = M+1
  - NUMBER of FILTER COEFFS is L





# GENERAL FIR FILTER

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$





# FILTERED STOCK SIGNAL


*50-pt Averager*



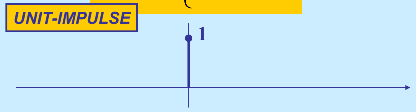

# FILTERED STOCK SIGNAL


# SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE (LATER)**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

**UNIT-IMPULSE**



## UNIT IMPULSE SIGNAL δ[n]

| $n$ | ... | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta[n]$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta[n-3]$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

δ[n] is NON-ZERO When its argument is equal to ZERO

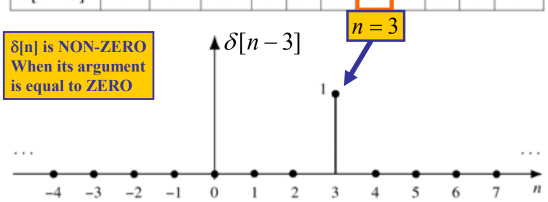
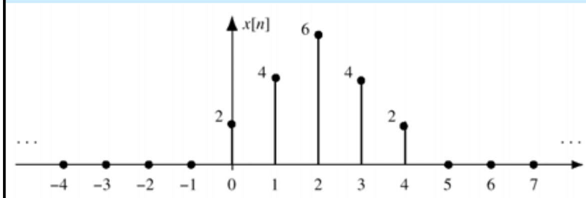



**Figure 5.7** Shifted impulse sequence, $\delta[n-3]$.

## MATH FORMULA for x[n]

- Use SHIFTED IMPULSES to write x[n]






## 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = δ[n]

$$x[n] = \delta[n]$$
$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- What is the "IMPULSE RESPONSE"?



## 4-pt Avg Impulse Response

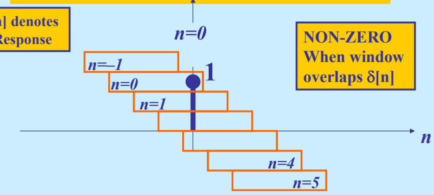
$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

δ[n] "READS OUT" the FILTER COEFFICIENTS

$$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

"h" in $h[n]$ denotes Impulse Response

NON-ZERO When window overlaps δ[n]






## FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

| $n$ | $n<0$ | 0 | 1 | 2 | 3 | ... | $M$ | $M+1$ | $n>M+1$ |
|---|---|---|---|---|---|---|---|---|---|
| $x[n]=\delta[n]$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y[n]=h[n]$ | 0 | $b_0$ | $b_1$ | $b_2$ | $b_3$ | ... | $b_M$ | 0 | 0 |

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

CONVOLUTION



## FILTERING EXAMPLE

- 7-point AVERAGER

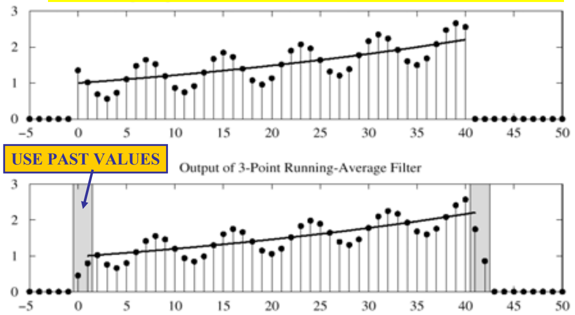
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

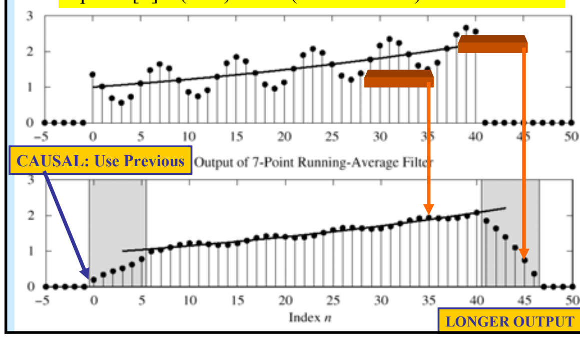
## 3-pt AVG EXAMPLE

$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$




## 7-pt FIR EXAMPLE (AVG)

$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$




## FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$