

# Signal Processing First

## Lecture 10 FIR Filtering Intro

# LECTURE OBJECTIVES

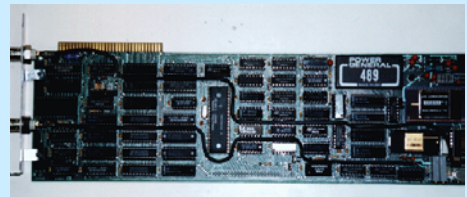
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output  $y[n]$  from the input signal,  $x[n]$

# DIGITAL FILTERING



- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- **DSP**: DIGITAL SIGNAL PROCESSING

# The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.

# Rockland Digital Filter, 1971

**Model 4136 PROGRAMMABLE DIGITAL FILTER**

Variable-Order Digital Filter for Realizing All Classical Designs

The Rockland Model 4136 Programmable Digital Filter consists of a special hardware digital filter section with 25 16-bit parallel four-stage 160-ns digital center filtering. Each of the four stages has fully-programmable coefficients which are stored internally in read-only memory.

Filter input and output word size is 16-bit parallel form at a maximum sampling rate of 80 kHz while internal operations are clocked only 20 kHz.

TRANSFER FUNCTION

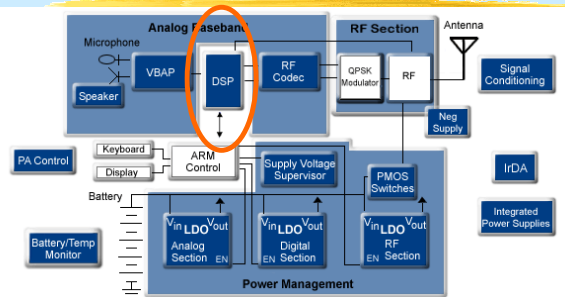
The transfer function from filter input to filter output in a variable-order filter is given by

$$H(z) = \frac{K(z^M + a_{M-1}z^{M-1} + \dots + a_1z + a_0)}{1 - b_1z^{-1} - b_2z^{-2} - \dots - b_{N-1}z^{-(N-1)} - b_Nz^{-N}}$$

where  $M, N, K, a_0, \dots, a_{M-1}, b_1, \dots, b_N$  are

For the price of a small house, you could have one of these.

# Digital Cell Phone (ca. 2000)



Free (?) with 2 year contract

## DISCRETE-TIME SYSTEM



- OPERATE on  $x[n]$  to get  $y[n]$
- WANT a **GENERAL CLASS** of SYSTEMS
  - ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - SYNTHESIZE** the SYSTEM

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## D-T SYSTEM EXAMPLES



- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING:  $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time  $n$  is the average of three consecutive input values"

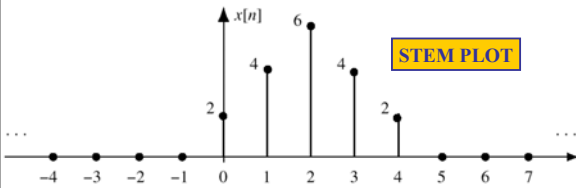
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## DISCRETE-TIME SIGNAL

- $x[n]$  is a LIST of NUMBERS
  - INDEXED by " $n$ "



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## 3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each " $n$ "

Make a TABLE

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$

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## PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter 123

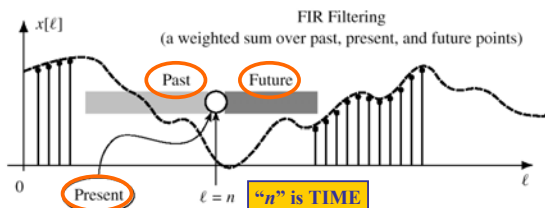


Figure 5.4 The running-average filter calculation at time index  $n$  uses values within a sliding window (shaded). Dark shading indicates the future ( $\ell > n$ ); light shading, the past ( $\ell < n$ ).

## ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of  $x[n]$ 
  - IMPORTANT IF " $n$ " represents REAL TIME
    - WHEN  $x[n]$  &  $y[n]$  ARE STREAMS

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

$n$	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

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## GENERAL FIR FILTER

- FILTER COEFFICIENTS  $\{b_k\}$

- DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- For example,  $b_k = \{3, -1, 2, 1\}$

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## GENERAL FIR FILTER

- FILTER COEFFICIENTS  $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- FILTER **ORDER** is  $M$
- FILTER **LENGTH** is  $L = M+1$ 
  - NUMBER of FILTER COEFFS is  $L$

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## GENERAL FIR FILTER

- SLIDE a WINDOW across  $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

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## FILTERED STOCK SIGNAL



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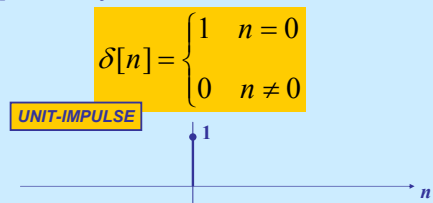
50-pt Averager

## FILTERED STOCK SIGNAL



## SPECIAL INPUT SIGNALS

- $x[n] = \text{SINUSOID}$  **FREQUENCY RESPONSE (LATER)**
- $x[n]$  has only one **NON-ZERO VALUE**



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## UNIT IMPULSE SIGNAL $\delta[n]$

$n$	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

$\delta[n]$  is NON-ZERO  
When its argument  
is equal to ZERO

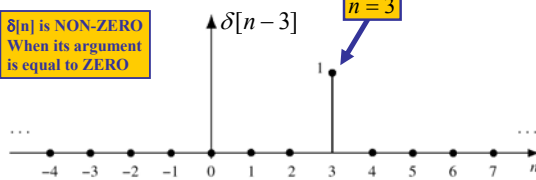
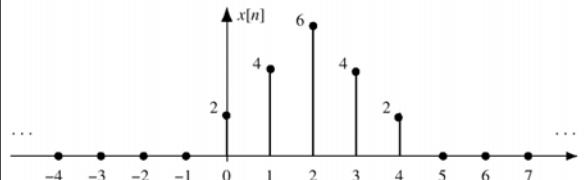


Figure 5.7 Shifted impulse sequence,  $\delta[n-3]$ .

## MATH FORMULA for $x[n]$

- Use SHIFTED IMPULSES to write  $x[n]$



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## 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL =  $\delta[n]$

$$x[n] = \delta[n]$$

$$y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$$

- What is the "IMPULSE RESPONSE"?

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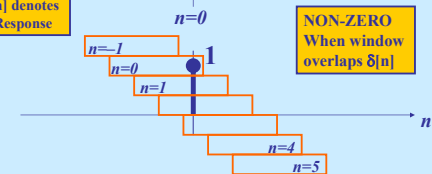
## 4-pt Avg Impulse Response

$$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

$\delta[n]$  "READS OUT" the FILTER COEFFICIENTS

$$h[n] = \{\dots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \dots\}$$

"h" in  $h[n]$  denotes  
Impulse Response



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## FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

$n$	$n < 0$	0	1	2	3	...	$M$	$M+1$	$n > M+1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	$b_0$	$b_1$	$b_2$	$b_3$	...	$b_M$	0	0

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = \sum_{k=0}^M h[k] x[n-k]$$

CONVOLUTION

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## FILTERING EXAMPLE

- 7-point AVERAGER

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right) x[n-k]$$

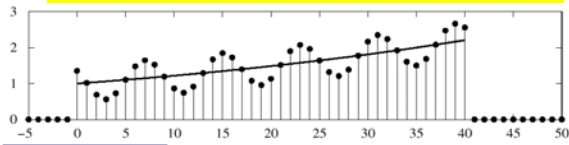
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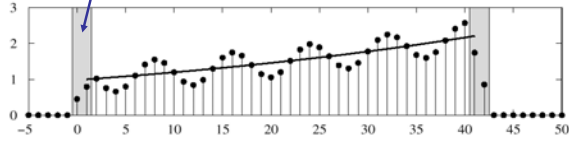
## 3-pt AVG EXAMPLE

Input :  $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$  for  $0 \leq n \leq 40$



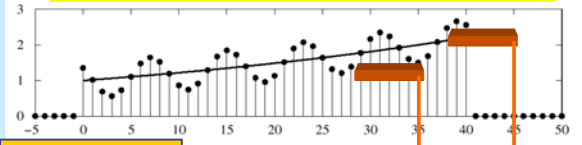
USE PAST VALUES

Output of 3-Point Running-Average Filter



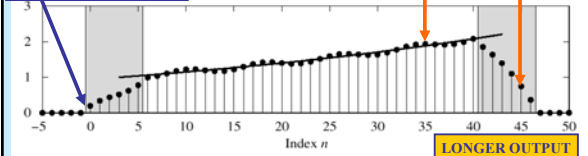
## 7-pt FIR EXAMPLE (AVG)

Input :  $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$  for  $0 \leq n \leq 40$



CAUSAL: Use Previous

Output of 7-Point Running-Average Filter



LONGER OUTPUT

## FILTERING EXAMPLE

### 7-point AVERAGER

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right)x[n-k]$$

- Removes cosine
  - By making its amplitude (A) smaller

### 3-point AVERAGER

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right)x[n-k]$$

- Changes A slightly