ECE380 HW # 8 Some Simple DFT Problems

1. Using the definition of the DFT (not the MATLAB or MAPLE FFT functions), calculate the DFT of the sequence {2, 1, -1, -2}. Show your calculations for each point. Check your results using the MATLAB FFT function. Show the MATLAB command you used and the result (which should match your hand results, of course!)

2. Now compute the 4-point IDFT of the result of Problem 1, using the definition of the IDFT (not the MATLAB or MAPLE IFFT functions). Once again, show your calculations for each point. Also, show that your result is indeed equal to {2, 1, -1, -2}. Check yourself with the MATLAB IFFT function. Show the MATLAB command you used and the result.

3. Zero padding is sometimes used to "pad out" a set of data points. Here let us investigate the effects of zero-padding the 4-point sequence of Problem 1 out to an 8-point sequence before the DFT is computed. In other words, calculate the DFT (working by hand) of {2, 1, -1, -2, 0, 0, 0, 0}. Once again show your work. Check your results via MATLAB. How do your results compare with the results of Problem 1? (Hint: zero padding should perform "frequency interpolation". Now the frequency step is fs/8 instead of fs/4.

4. Let us examine the relationship between the FFT and the Z-transform that was discussed in class. Begin by computing the Z-transform of the sequence {2,1,-1,-2,0, 0, 0, …} where the leftmost element in this sequence corresponds to n = 0. Now replace z by exp(j*omega*T) and let omegat*T = 0, Pi/2, Pi, and 3Pi/2. Note that we are merely evaluating the Z transform at 4 equally-spaced points around the unit circle in the z-plane. Show that these values yield the DFT of the sequence {2,1-1,-2} that was obtained in Problem 1.

5. A 100 Hz sine wave is sampled at a rate of fs = 200 samples per second, yielding 2 samples per cycle {1, -1, 1, -1, 1, -1, 1, -1} . Use the MATLAB FFT function to find the FFT of this sequence. What analog frequency does each point in the FFT represent? Explain why the peak magnitude of the FFT occurs where it does.

6. A 50 Hz sine wave is sampled at a rate of fs = 200 samples per second, yielding 4 samples per cycle {1, 0, -1, 0, 1, 0, -1, 0}. Use the MATLAB FFT function to find the FFT of this sequence. What analog frequency does each point in the FFT represent? Explain why the peak magnitude of the FFT occurs where it does.

7. Use the MATLAB FFT to verify the results presented in class for the DFT of the unit sample function and the DFT of a constant. Take the FFT of {1, 0, 0, 0, 0, 0, 0, 0} and also {1,1,1,1,1,1,1,1}. Compare with the general results derived in the class notes.

8. Use the MATLAB FFT to take the DFT of the N=8 point sequence [1, 2, 3, 4, 5, 6, 7, 8]. The resulting N complex FFT values may be numbered X_0 up to X_7. Upon examining this result, propose a simple formula that predicts the first FFT point, X_0. Also, find a simple formula that relates $X_{\frac{N}{2}+p}$ to $X_{\frac{N}{2}-p}$ for p = 1… $\frac{N}{2}-1$. Take the FFT's of several other sequences to make sure that this rule applies to each of the other sequences as well. Using this relationship, only the first half of an FFT need be calculated; the rest may be found from this symmetry formula!