ECE380 Homework 6. Filter Implementation Structures

1) Consider the following 4^{th}-order digital filter

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^4 - 2z^2 + 1}{z^4 - z^3 - 0.31z^2 + 0.81z - 0.405}$$

   a) Draw its ***Direct Form #1*** structure using the symbols introduced in class.
   b) Draw its ***Direct Form #2*** structure.
   c) Draw its implementation as a ***cascade of two 2^{nd}-order Direct Form #2 structures***. Because there are several ways to decompose H(z) into two second-order sections, please choose the solution where the 2^{nd}-order structure that connects to the input has two zeros at z = -1 and a pair of complex conjugate poles at z = 0.5 + j0.5 and z = 0.5 – j0.5.
   d) Draw its implementation as a ***<u>parallel realization of two 2nd-order Direct Form #2</u>*** structures. See Appendix A of this handout for a discussion of the H(z) partial fraction expansion method.

2) From the Direct Form #2 structure of Problem 1(b), now draw its **Transposed Direct Form #2 structure**, using the steps listed in Section 8-3.2.3 of our "SP First" textbook. Because these rules seem magical, and it may be hard to believe that the same H(z) is still implemented by this structure, write equations for the intermediate variable V(z) and the output variable Y(z) that comes from this resulting structure, and then substitute the first equation into the second to eliminate the intermediate variable V(z), and finally solve for H(z). You should be able to show that the H(z) corresponding to the Transposed Direct Form #2 structure does indeed match the original H(z) stated at the beginning of Problem 1!

Appendix A. <u>Partial Fraction Expansion for Parallel Filter Realization</u>

Because all poles are different, you should be able to expand H(z) into a simple partial fraction expansion of first-order terms of the form:

$$H(z) = C_o + \sum_{k=1}^{N} \frac{C_k z}{z - p_k}$$

Note that H(z) consists of the sum of a constant and a number of first-order terms. This partial fraction expansion differs from the more common one, because of the "z" in the numerator of each fractional term. We require the fractional terms in this expansion to have a "z" in their numerator so that 1^{st}-order IIR filter is obtainable from each term after we divide through by z to get the term into the form $C_k/(1-p_k z^{-1})$. Note this type of expansion is also needed when performing inverse Z transformation, since a term of the form $C_k/(1-p_k z^{-1})$ readily inverse-Z transforms into $C_k a^n u(n)$. If two poles that are complex conjugates of each other, say $p_2 = p_1^*$ are present, then, in order for the coefficients of H(z) to be real-valued we must require that $C_2 = C_1^*$.

Under these conditions, these two first-order partial fraction terms with complex conjugate poles and complex conjugate coefficients will combine into one second-order term with real coefficients, as shown below:

$$\frac{C_1 z}{z-p_1}+\frac{C_1^* z}{z-p_1^*}=\frac{C_1 z(z-p_1^*)+C_1^* z(z-p_1)}{(z-p_1)(z-p_1^*)}=\frac{(C_1^*+C_1)z^2-(C_1 p_1^*+C_1^* p_1)z}{z^2-(p_1+p_1^*)z+|p_1|^2}$$

$$=\frac{2\operatorname{Re}(C_1)z^2-(|C_1||p_1|e^{\angle C_1-\angle p_1}+|C_1||p_1|e^{\angle p_1-\angle C_1})z}{z^2-2\operatorname{Re}(p_1)z+|p_1|^2}=\frac{2\operatorname{Re}(C_1)z^2-2(|C_1||p_1|\cos(\angle C_1-\angle p)z}{z^2-2\operatorname{Re}(p_1)z+|p_1|^2}$$

$$=\frac{az^2+bz}{z^2+cz+d}$$

Also, note that any two first-order terms containing real poles p_1 and p_2 (and therefore possessing real-valued coefficients C_1 and C_2) may similarly be combined into a second-order term of exactly the same form:

$$\frac{C_1 z}{z-p_1}+\frac{C_2 z}{z-p_2}=\frac{(C_1+C_2)z^2-(C_1 p_2+C_2 p_1)z}{z^2-z(p_1+p_2)+p_1 p_2}=\frac{az^2+bz}{z^2+cz+d}$$

The most convenient way to obtain the partial fraction expansion of H(z) in the form required above (with z in the numerator of the terms) is to partial fraction expand H(z)/z using MAPLE, then multiply the result by z to get the z in the numerators of each term.

The following example shows how to use MAPLE to obtain the partial fraction expansion of

$$H(z)=\frac{z^2-z+1}{z^2+z+2}$$

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> Hofz:=(z^2-z+1)/(z^2+z+2);
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$$Hofz := \frac{z^2-z+1}{z^2+z+2}$$

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> Hofzoverz:=Hofz/z;
>
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$$Hofzoverz := \frac{z^2-z+1}{(z^2+z+2)\,z}$$

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> Hparfracoverz:=convert(Hofzoverz,parfrac,z);
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$$Hparfracoverz := \frac{1}{2}\frac{1}{z} + \frac{\frac{1}{2}(-3+z)}{z^2+z+2}$$

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> Hparfrac:=expand(z*Hparfracoverz);
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$$Hparfrac := \frac{1}{2} - \frac{3}{2}\frac{z}{z^2+z+2} + \frac{\frac{1}{2}z^2}{z^2+z+2}$$

The resulting partial fraction expansion is $H(z) = \frac{1}{2} + \frac{0.5z^2 - 1.5z}{z^2+z+2}$

Note that because in this example H(z) had a pair of complex conjugate poles, MAPLE automatically combined the pair of first-order complex conjugate terms into the corresponding second-order form. However, if the example had possessed real poles, MAPLE would have given us two first-order terms, and we would have had to manually combine them into the second-order form before implementation as a parallel combination of second-order filters could begin.