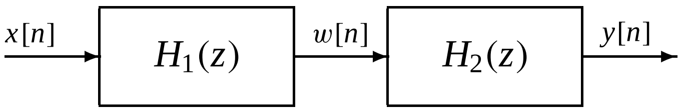
PROBLEM:

An *inverse filter* is an LTI system which, when cascaded with another LTI system, "undoes" the effects of the other LTI system. You saw an approximate inverse filter in one of the Laboratory experiments. The reason that you were only able to do an approximation then was that we had not yet studied IIR systems.



(a) Suppose that $H_1(z)$ describes a given LTI system. Determine $H_2(z)$ so that the output is $y[n] = x[n]$; i.e., so that the second system compensates exactly for the effects of the first system.

(b) If $H_1(z)$ represents an FIR system, what can you say about the second system; i.e., is it FIR or IIR? Explain.

(c) Suppose that the first system is defined by the difference equation

$$w[n] = \sum_{k=0}^{9} \alpha^k x[n-k] \qquad \text{where } 0 < \alpha < 1.$$

Show that $H_1(z)$ can be expressed the following ratio of polynomials in the variable z^{-1}:

$$H_1(z) = \frac{1 - \alpha^{10} z^{-10}}{1 - \alpha z^{-1}}.$$

Plot the poles and zeros of $H_1(z)$ in the complex z-plane. *Hint: You will need to find the values of* z *that satisfy the equation* $1 - \alpha^{10} z^{-10} = 0$. *This is done just as in the case of the moving average filter. You should find that the zeros are not on the unit circle, but on a circle of a different radius.*

(d) Now, for the system $H_1(z)$ of part (c), determine the inverse system function $H_2(z)$ and plot its poles and zeros in the complex z-plane. What happens to the poles and zeros of $H_1(z)$ and $H_2(z)$ when we form the product $H_1(z)H_2(z)$?

(e) From $H_2(z)$ obtained in part (d), determine the difference equation that relates the output $y[n]$ to $w[n]$, the input to the second system.

(f) The system of part (e) could be implemented in Matlab by the statement

```
yy=filter(b,a,ww);
```

What should `b` and `a` be in order to implement the inverse system for the example of part (c)?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION