PROBLEM: 4.1

Factor the following polynomial and plot its roots in the complex plane.

$$P(z) = 1 + \tfrac{1}{2}z^{-1} + \tfrac{1}{2}z^{-2} + z^{-3}$$

In MATLAB see the functions called `roots` and `zplane`.

PROBLEM: 4.2

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] = \sum_{k=0}^{3} x[n-k]$$

(a) What is the impulse response, $h[n]$, of this system?

(b) Determine the system function $H(z)$ for this system.

(c) Plot the zeros of $H(z)$ in the complex z-plane. *Hint: Remember the N-th roots of unity.*

PROBLEM: 4.3

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 - z^{-1})(1 - e^{j\pi/2}z^{-1})(1 - e^{-j\pi/2}z^{-1})(1 - 0.5e^{j\pi/3}z^{-1})(1 - 0.5e^{-j\pi/3}z^{-1})$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. Make sure that all the filter coefficients $\{b_k\}$ in your difference equation are purely real.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = \delta[n-2] + 2\delta[n-4] - \delta[n-5].$$

(d) Plot the poles and zeros of $H(z)$ in the z-plane.

(e) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of this system.

(f) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of frequency $\hat{\omega}$ will the output signal be zero for all n (i.e., $y[n] = 0$) ? Find all possible frequencies in the range $-\pi \le \hat{\omega} \le \pi$. *Hint: Take a look at the locations of the zeros of $H(z)$ as plotted in part (d).*

PROBLEM: 4.4

We now have four ways of describing an LTI system: the difference equation with filter coefficients $\{b_k\}$; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the system function $H(z)$.

(a) $y[n] = 3(x[n] - x[n-3])$.

(b) $h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$.

(c) $H(e^{j\hat{\omega}}) = [2j\sin(2\hat{\omega})]e^{-j3\hat{\omega}}$.

(d) $h[n] = \delta[n] + \delta[n-3]$.