**PROBLEM: 3.1**

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^2. \tag{1}$$

(a) Determine whether or not the system defined by (1) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any squared powers of cosine functions in your answers.

(c) For the system of Equation (1), determine the output $y_2[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.75\pi(n-1)).$$

(d) This system produces output contain frequencies that are not present in the input signal. Explain how this system might cause "aliasing" of sinusoidal components of the input.

**PROBLEM: 3.2**

A linear time-invariant system is described by the difference equation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) Find the frequency response $H(\hat{\omega})$, and then express it as a mathematical formula, in polar form (magnitude and phase).

(b) $H(\hat{\omega})$ is a periodic function of $\hat{\omega}$; determine the period.

(c) Plot the magnitude and phase of $H(\hat{\omega})$ as a function of $\hat{\omega}$ for $-\pi < \hat{\omega} < 3\pi$. Do this by hand and then check with the MATLAB function `freqz`.

(d) Find all frequencies, $\hat{\omega}$, for which the output response to the input $e^{j\hat{\omega}n}$ is zero.

(e) When the input to the system is $x[n] = \sin(\pi n/13)$ determine the functional form for the output signal $y[n]$.

**PROBLEM:** 3.3

We have shown that an LTI system can be represented in several equivalent ways. In each part below, you are given one representation of an LTI system and you are to provide the other representations requested. (Frequency response formulas can be given in any convenient form. You do **NOT** have to simplify them.)

(a) Frequency response:

Impulse response:

Difference equation: $y[n] = x[n] + 2x[n-1] + x[n-2]$

(b) Frequency response: $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(2\cos(\hat{\omega}))$

Impulse response:

Difference equation:

(c) Frequency response:

Impulse response:

MATLAB Implementation: `y = conv([0,1,0,-1],x)`

**PROBLEM: 3.4**

Suppose that $\mathcal{S}$ is a linear, time-invariant system whose exact form is unknown. It needs to be tested by running some inputs into the system, and then observing the output signals. Suppose that the following input/output pairs are the result of the tests:

$$\begin{aligned} x[n] = \delta[n] \quad &\longrightarrow \quad y[n] = \delta[n] - \delta[n-3] \\ x[n] = \cos(2\pi n/3) \quad &\longrightarrow \quad y[n] = 0 \\ x[n] = \cos(\pi n/3 + \pi/2) \quad &\longrightarrow \quad y[n] = 2\cos(\pi n/3 + \pi/2) \end{aligned}$$

(a) Make a plot of the signal: $x[n] = 3\delta[n] - 2\delta[n-2] + \delta[n-3]$.

(b) What is the output of the system when the input is $x[n] = 3\delta[n] - 2\delta[n-2] + \delta[n-3]$.

(c) Determine the output when the input is $x[n] = \cos(\pi(n-3)/3)$.

(d) Is the following statement true or false: "$H(\pi/2) = 0$." EXPLAIN

**PROBLEM: 3.5**

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1: \quad y_1[n] = x_1[n] + x_1[n-2]$$

$$\mathcal{S}_2: \quad y_2[n] = 7x_2[n-5] + 7x_2[n-6]$$

$$\mathcal{S}_3: \quad \mathcal{H}_3(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j2\hat{\omega}}$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the difference equation for $\mathcal{S}_3$.

(b) Determine the frequency response of the first two systems: $\mathcal{H}_i(\hat{\omega})$ for $i = 1, 2$.

(c) Determine the frequency response of the overall cascaded system.

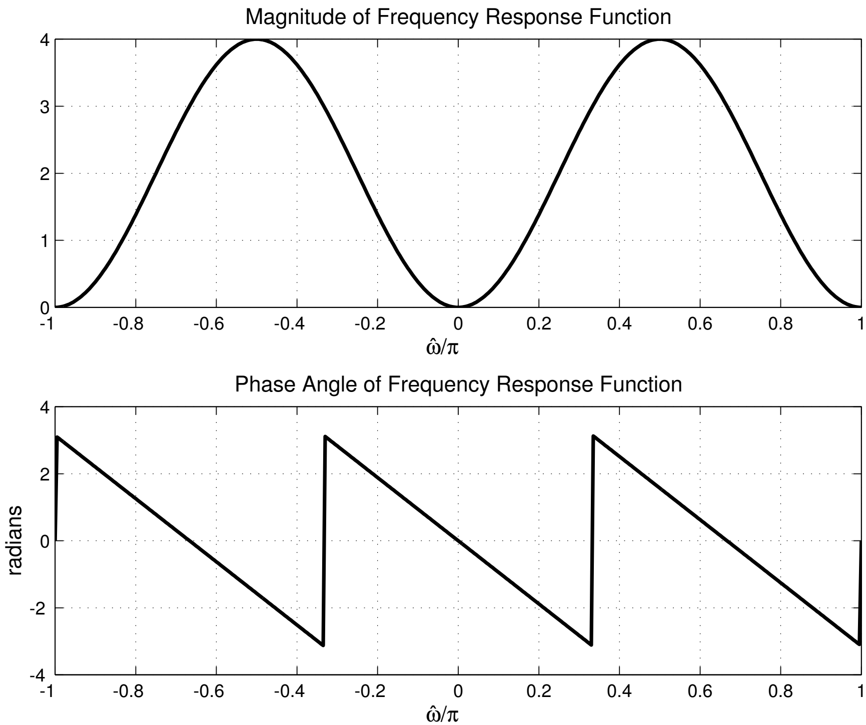
(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

**PROBLEM: 3.6**

A linear time-invariant system is defined by the system function

$$H(z) = -z^{-1} + 2z^{-3} - z^{-5}$$

The magnitude and phase of the frequency response of this system are plotted in the following figure. **Note that the frequency scale is $\hat{\omega}/\pi$.**




(a) This filter is a *lowpass* *bandpass* *highpass* filter. (Circle one.)

(b) Use the above graph to determine (as accurately as you can) the output $y[n]$ of this system when the input is

$$x[n] = 10 + 10\cos(0.5\pi n).$$

**Mark the points on the graph that you used in your solution.**

(c) Determine an expression for the frequency response, $H(e^{j\hat{\omega}})$. Write your answer in the form $H(e^{j\hat{\omega}}) = A(\hat{\omega})e^{-j\hat{\omega}n_0}$, where $A(\hat{\omega})$ is real and $n_0$ is an integer.