PROBLEM: 2.1

Let $x[n]$ be the complex exponential

$$x[n] = 11e^{j(0.3\pi n + 0.5\pi)}$$

If we define a new signal $y[n]$ to be the output of the difference equation:

$$y[n] = 2x[n] + 4x[n-1] + 2x[n-2]$$

it is possible to express $y[n]$ in the form

$$y[n] = Ae^{j(\omega_0 n + \phi)}$$

Determine the numerical values of A, ϕ and ω_0.

PROBLEM: 2.2

Evaluate the "running" average:

$$y[n] = \frac{1}{L} \sum_{k=0}^{L-1} x[n-k]$$

for two different input signals. In both cases, compute the numerical values of $y[n]$ over the range $-5 \leq n \leq 10$, assuming that $L = 4$. Then derive a general formula for $y[n]$ that will apply for any length L and for the index range $n \geq 0$.

(a) A signal that turns on at $n = 0$. This is called the *unit step* signal, and is usually denoted by $u[n]$. Make a plot of $u[n]$ before working out the answer for $y[n]$.

$$x[n] = u[n] = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

(b) A signal that starts increasing linearly at $n = 0$. This is called the *unit ramp* signal, and is usually denoted by $r[n]$. Make a plot of $r[n]$ before working out the answer for $y[n]$.

$$x[n] = r[n] = \begin{cases} 0 & \text{for } n < 0 \\ n & \text{for } n \geq 0 \end{cases}$$

(c) Use MATLAB to create a plot of the output for both over the range $0 \leq n \leq 15$. Let the length of the averaging window be $L = 7$.

PROBLEM: 2.3

For a particular linear time-invariant system, when the input is

$$x_1[n] = 4u[n] = \begin{cases} 0 & n < 0 \\ 4 & n \geq 0 \end{cases}$$

the corresponding output is

$$y_1[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4u[n-3] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ 2 & n = 1 \\ 3 & n = 2 \\ 4 & n \geq 3 \end{cases}$$

(a) Using the concepts of linearity and time-invariance, determine the impulse response of the system.

(b) The system is an FIR filter—determine the filter coefficients and the length of the filter.

(c) State a general procedure for deriving the impulse response of a LTI system from a measurement of its step response, i.e., if $s[n]$ is the step response of a LTI system, what simple operations can be done to $s[n]$ to produce the impulse response $h[n]$.

(d) Using the concepts of linearity and time-invariance, determine the output signal when the input signal is $x_2[n] = 7u[n-1] - 7u[n-4]$. Give your answer as a formula expressing $y_2[n]$ in terms of known sequences or as an equation for each value of $y_2[n]$ for $-\infty < n < \infty$.

PROBLEM: 2.4

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

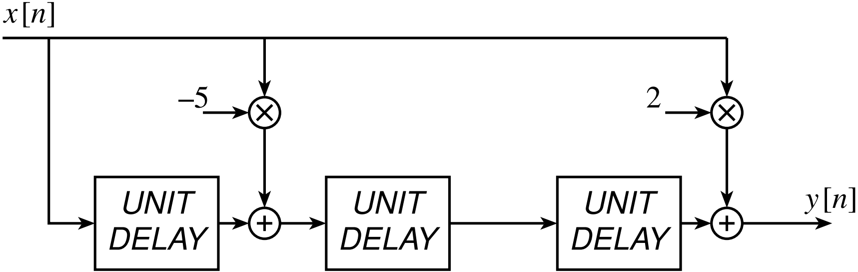
(a) $y[n] = 3x[n-1] + x[n] + 3x[n+1]$

(b) $y[n] = x[n]\cos(.3\pi n)$

(c) $y[n] = |x[-n]|$

PROBLEM: 2.5

The following signal flow graph structure defines a linear time-invariant system:



Write a simple formula for the the difference equation defined by the signal flow graph.