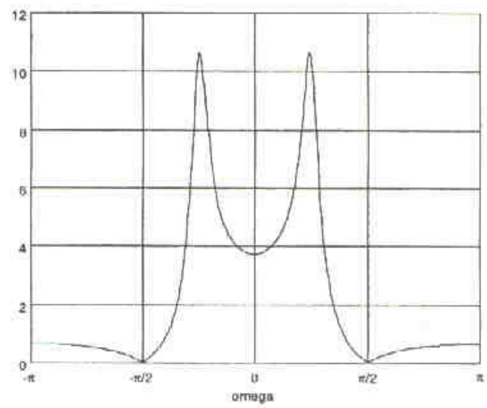
Name Key

Rose-Hulman Institute of Technology
Electrical and Computer Engineering

EC 380 - Final Exam — Monday, May 22, 2000

CLOSED BOOK. Work each problem in the space provided on its sheet. Be sure the work you present is clear so the grader can understand what you have done. One 8.5" x 11" sheet and a calculator are allowed. No other aids, animate or inanimate, are permitted. All problems have the same weight. Please do your own work. State answers in engineering form. **Box your answer, please, and don't forget units!**

Problem 1 – Congratulations, you have just landed a new job as a summer intern. Your first task is to design a 2nd-order filter whose magnitude of the frequency response is the following:



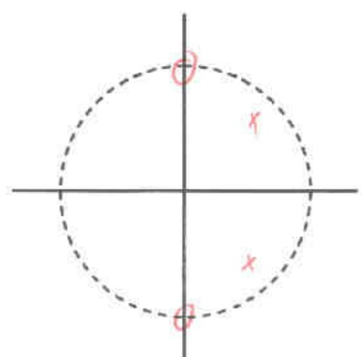
Estimate the *poles* for this filter and express them in polar form.

$z = 0.9e^{\pm j\pi/4}$

Estimate the *zeros* for this filter and express them in polar form.

$z = e^{\pm j\pi/2}$

Sketch the pole/zero plot. Use **x**'s to mark poles and **o**'s to mark zeros.



What is H(z) for this filter?

$$H(z) = \frac{(z-e^{j\pi/2})(z-e^{-j\pi/2})}{(z-0.9e^{+j\pi/4})(z-0.9e^{-j\pi/4})}$$

$$\frac{z^2+1}{z^2-1.27z+0.81} = \frac{1+z^{-2}}{1-1.27z^{-1}+0.81z^{-2}}$$

What is the difference equation for this filter?

y[n] = $1.27y[n-1] - 0.81y[n-2] + x[n] + x[n-2]$

Problem 2 - Given the system

$$y[n] = \sum_{k=-M/2}^{M/2} b_k x[n-k]$$

Explain why you think the system is or is not:

(Note: only half credit for correct yes or no answer. Circle one.)

a. Linear yes no why: FIR is LTI

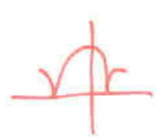
b. Time Invariant yes no why: "

c. Memoryless yes no why: y[n] depends on x[n-k]

d. Causal yes no why: y[0] depends on x[1] if M>1

e. BIBO Stable yes no why: FIR is always stable

f. Invertable yes no Under what conditions?



← certain frequencies can be zero'd out. depending on b_k

Problem 4 - Suppose that **S** is a linear, time-invariant system whose exact form is unknown. It needs to be tested by running some inputs into the system, and then observing the output signals. Suppose that the following input/output pair is the result of the test:

$$x_1[n] = \delta[n] \rightarrow y_1[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2]$$

What is the frequency response, $H(\hat{\omega})$, of the system? Express in the form $Ae^{-jB\hat{\omega}}(C\cos(D\hat{\omega}) + E)$

$$\mathcal{H}(\hat{\omega}) = 2 + e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(2e^{j\hat{\omega}} + 1 + 2e^{-j\hat{\omega}})$$

$$\boxed{= e^{-j\hat{\omega}}(4\cos(\hat{\omega}) + 1)}$$

$A = 1$
$B = 1$
$C = 4$
$D = 1$
$E = 1$

What is the output, $y_2[n]$, if the input is $x_2[n] = \cos(\pi/3n + \pi/2)$?

Express in the form $F\cos(Gn + H)$

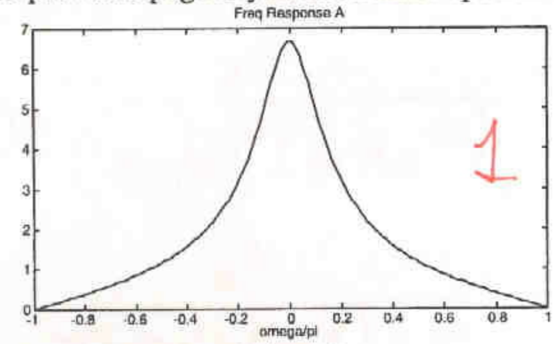
$$\mathcal{H}(\hat{\omega})\Big|_{\hat{\omega}=\frac{\pi}{3}} = e^{-j\frac{\pi}{3}}(4\underbrace{\cos(\tfrac{\pi}{3})}_{1/2} + 1)$$

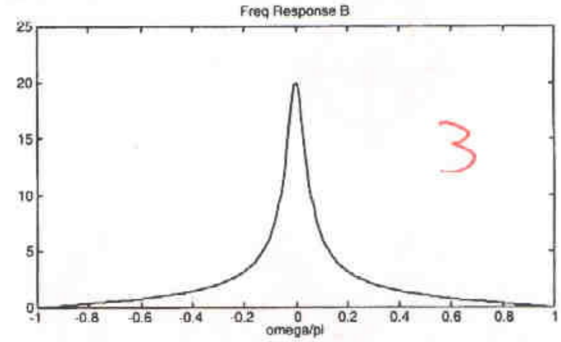
$$= 3e^{-j\pi/3}$$

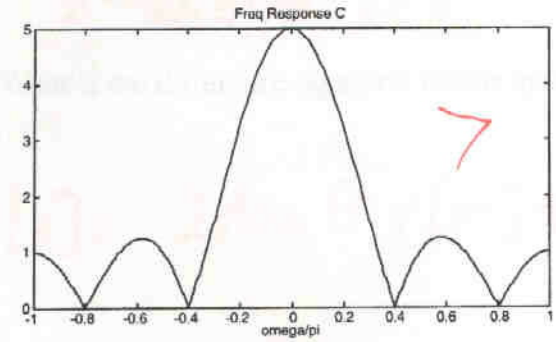
$$y_2[n] = 3\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{2} - \tfrac{\pi}{3})$$

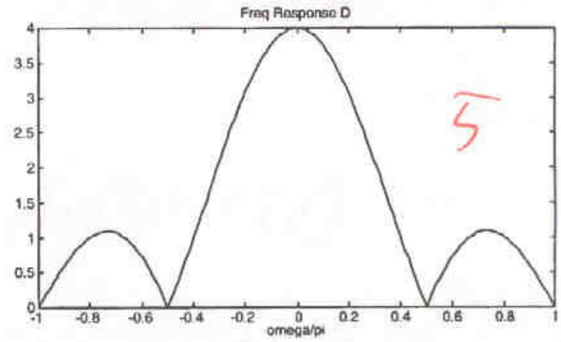
$$\boxed{= 3\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{6})}$$

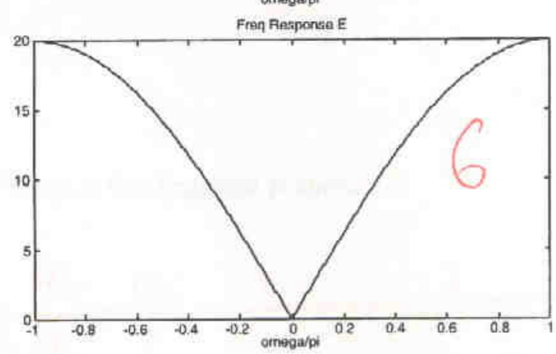
Problem 6 - For each of the frequency plots determine which one of the following systems matches the frequency response. Write on the plot the number of the system to which it corresponds. Use the back of the *previous* page if you need more space.

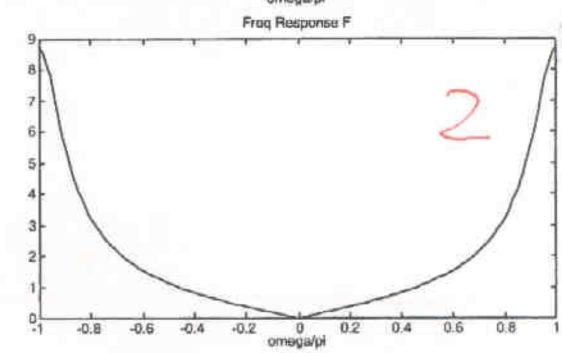












A S_1: $y[n] = 0.7y[n-1] + x[n] + x[n-1]$ $\quad \frac{1+z^{-1}}{1-0.7z^{-1}} = \frac{z+1}{z-0.7}$

F S_2: $y[n] = -0.7y[n-1] + x[n] - x[n-1]$ $\quad \frac{1-z^{-1}}{1+0.7z^{-1}} = \frac{z-1}{z+0.7}$

B S_3: $H(z) = \dfrac{1+z^{-1}}{1-0.9z^{-1}} = \dfrac{z+1}{z-0.9}$

S_4: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5}$

D S_5: $y[n] = \sum_{k=0}^{3} x[n-k]$

E S_6: $H(z) = 10 - 10z^{-1}$ $\quad \frac{10z-10}{z}$

C S_7: $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$

Problem 8 - For an N point DFT answer the following questions. You may assume that X[k] is the DFT of x[n]:

a,b,c. Given x[n] has N=8 and is real, fill in the missing values of X(k). Place an X on those that don't have enough information of fill in.

$4e^{j\pi/2}$, $2-j2$

X(k) = { X, 2+j2, $4e^{j\pi/2}$, $2+j2$, 7, 2-j2, $4e^{-j\pi/2}$, $2-j2$ }

d. Prove that if x[n] is multiplied by alternating 1's and –1's (i.e. [1, -1, 1, -1, …]]), X(k) (the DFT of x[n]) is circularly shifted by N/2 points.

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}} \quad \text{def. of DFT}$$

Find

$$X\left(k+\tfrac{N}{2}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi n(k+\frac{N}{2})}{N}}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j2\frac{\pi nk}{N}} e^{-j\frac{2\pi n\frac{N}{2}}{N}}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j\pi n} e^{-j\frac{2\pi nk}{N}}$$

same as multiplying x[n] by alternating 1's + -1's.