Name Key

Rose-Hulman Institute of Technology
Electrical and Computer Engineering

EC 380 - Exam 3 — Tuesday, 11-Feb-2003

CLOSED BOOK. Work each problem in the space provided on its sheet. Be sure the work you present is clear I can understand what you have done. One 3" x 5" card and a calculator/computer are allowed. **NO MATLAB**. The filter tables are included with the exam. No other aids, animate or inanimate, are permitted. All problems have the same weight. Please do your own work. State answers in engineering form. **Box your answer, please, and don't forget units!**

Problem 1 – What order (n) digital Chebychev filter is needed for a sampling rate of 44.1 kHz, a -3dB cutoff at 6510 Hz, and a -60dB stopband at 11k Hz. Clearly label each step and the results of that step.

$f_{sam} = 44100$

$\hat{w}_p = \frac{2\pi 6510}{44100} = \frac{31\pi}{105} = 0.927$

$\hat{w}_s = \frac{2\pi 11000}{44100} = \frac{220\pi}{441} = 1.567$

Prewarp

$w_p = \tan\left(\frac{.927}{2}\right) \doteq \frac{1}{2}$

$w_s = \tan\left(\frac{1.567}{2}\right) \doteq 1$

$t = \frac{1}{\frac{1}{2}} = 2$

from graph

$\boxed{n = 6}$

Time doesn't permit you to design the above filter. Instead design a 2nd-order digital Butterworth filter with a sampling rate of 44.1 kHz, a -3dB cutoff at 6510 Hz. Write your final answer as a difference equation with y[n] on the left hand side. Clearly label each step and the results of that step.

From Table

$$H(s) = \frac{1}{\left(\frac{s}{2\pi f_c}\right)^2 + \sqrt{2}\,\frac{s}{2\pi f_c} + 1}$$

Let $f_c = \frac{\frac{1}{2}}{2\pi}$

$$H(s) = \frac{1}{4s^2 + 2\sqrt{2}\,s + 1}$$

$$H(z) = H(s)\Big|_{s=\frac{z-1}{z+1}} = \frac{.128 + .256z^{-1} + .128z^{-2}}{1 + \cancel{0.278}z^{-1} + 0.278z^{-2}}$$

$-0.766z^{-1}$

$y[n] = 0.766\,y[n-1] - 0.278\,x[n-2] +$

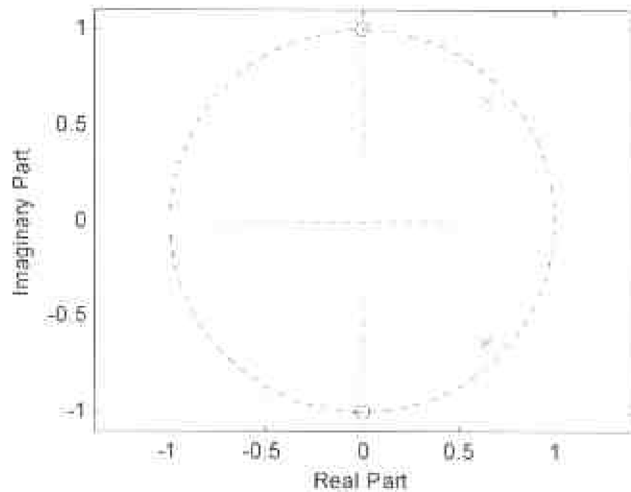
$0.128\,x[n] + 0.256\,x[n-1] + 0.128\,x[n-2]$

EC 380 Exam 3, 2/11/2003, page 2

Problem 2 - What are the poles and zeros for the following? Please express in polar form.

Poles:

$0.9e^{\pm j\frac{\pi}{4}}$



Zeros:

$e^{\pm j\frac{\pi}{2}}$

What is H(z)?

$$H(z) = \frac{\left(z - e^{j\frac{\pi}{2}}\right)\left(z - e^{-j\frac{\pi}{2}}\right)}{\left(z - 0.9e^{j\frac{\pi}{4}}\right)\left(z - 0.9e^{-j\frac{\pi}{4}}\right)}$$

$$H(z) = \frac{z^2 + 1}{z^2 - 0.9\left(e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}\right) + 0.81}$$

What is y[n]?

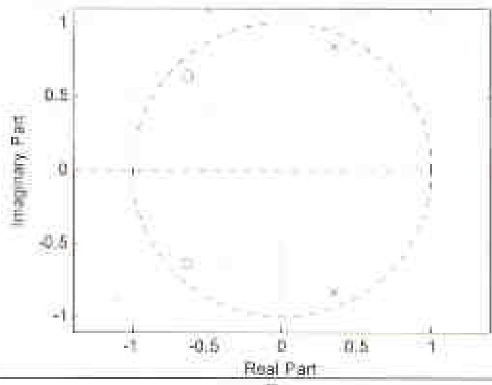
$$\frac{z^2+1}{z^2 - 1.27z + .81} = \frac{1 + z^{-2}}{1 - 1.27z^{-1} + .81z^{-2}}$$

$$y[n] = 1.27\,y[n-1] - 0.81\,y[n-2] + x[n] + x[n-2]$$

Problem 3 - Match the following systems and pole/zero plots to their corresponding frequency plots. Note that the number of entries is not equal, so match those which go together and appropriately mark any which have no match.

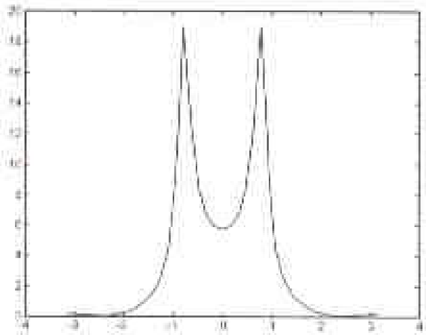
DO NOT USE PeZ.

A



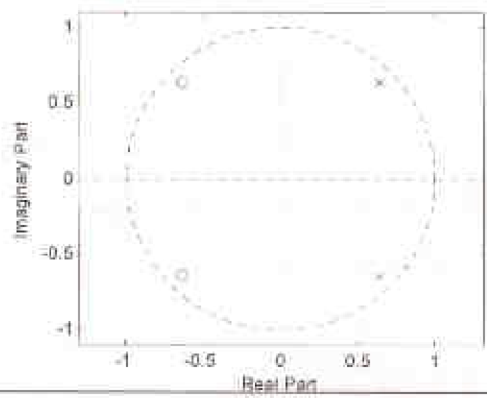
3

1



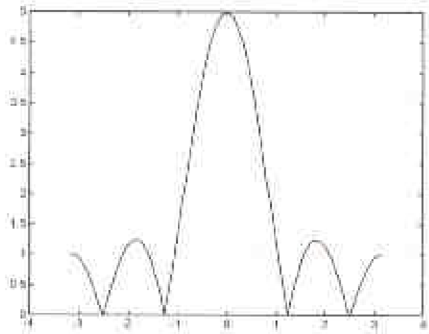
B

B



1

2



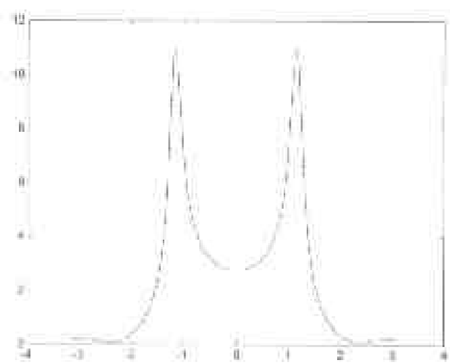
E

C: y[n] = x[n] + x[n-1] + x[n-2]

D: y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] 4

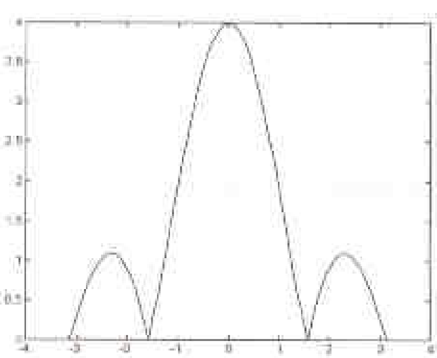
E: y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] 2

3



A

4



D

Problem 4 - No automated DFT's on this problem. (i.e. no Maple, MATLAB, or HP `fft` command, etc.)

A. Write the DFT equation of X[4] expanded into first four terms, assuming N = 8. You may write your answer in terms of x[] but you must simplify the coefficients into rectangular form (no $\sum$ or e^j may appear in your answer).

Note: $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}}$

$X[4] = x[0] + x[1]e^{-j\frac{2\pi \cdot 1 \cdot 4}{8}} + x[2]e^{-j\frac{2\pi \cdot 2 \cdot 4}{8}} + x[3]e^{-j\frac{2\pi \cdot 3 \cdot 4}{8}} + \ldots$

$= x[0] - x[1] + x[2] - x[3] + x[4] - x[5] + x[6] - x[7]$

B. If x(t) is a discrete function, what property(s) can you predict that the Fourier Transform X(~~f~~ ω) will have? Why?

X(ω) will be periodic

Sample in time-domain by mult by ||||||

convolve in freq-domain ~~b~~ with ||||| make freq periodic.

C. x[n] = [4, 3, 2, 1] Find X[2], which is the 2nd element of the DFT of x[n].

$X[2] = x[0] + x[1]e^{-j\frac{2\pi \cdot 1 \cdot 2}{4}} + x[2]e^{-j\frac{2\pi \cdot 2 \cdot 2}{4}} + x[3]e^{-j\frac{2\pi \cdot 3 \cdot 2}{4}}$

$= x[0] - x[1] + x[2] - x[3]$

$= 4 - 3 + 2 - 1 = \boxed{2}$

D. X[k] = [5, 2+j2, 3, __] If x[n] is real, what is X[3]?

$\boxed{2-j2}$

What is X[5]?

$X[5] = X[~~N~~ 5-N] = X[1] = \boxed{2+j2}$

E. In the output of a 16 point DFT, which value of X[k] corresponds to the digital frequency $\hat{\omega} = \frac{\pi}{2}$ (give a value of k)?

$\hat{\omega} = \frac{2\pi k}{N}$

$\frac{\pi}{2} = \frac{2\pi k}{16}$

$\boxed{k = 4}$