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Rose-Hulman Institute of Technology
Electrical and Computer Engineering

EC 380 - Exam 2 Solution Friday, January 24th, 2003

CLOSED BOOK. Work each problem in the space provided on its sheet. Be sure the work you present is clear so I can understand what you have done. One 3" x 5" card and a calculator/computer are allowed. No other aids, animate or inanimate, are permitted. Please do your own work. State answers in engineering form. **Box your answer, please, and don't forget units!**

Problem 1 – [25 points] A filter is described by the following difference equation:

$$y[n] = x[n] + 2x[n-1] - 2x[n-2] - x[n-3].$$

a. Find $H(e^{j\hat{\omega}})$. Express it in the magnitude/angle form we've used in class. Express complex values in polar form.

$$h[n] = \delta[n] + 2\delta[n-1] - 2\delta[n-2] - \delta[n-3]$$

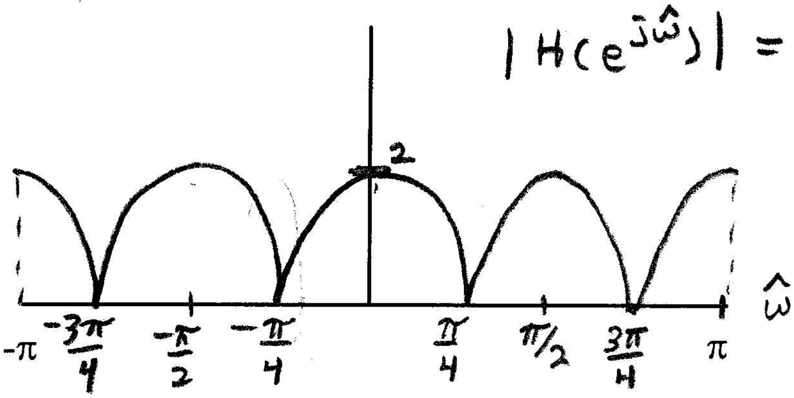
$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{3} h[k]e^{-j\hat{\omega}k} = 1 + 2e^{-j\hat{\omega}} - 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$= e^{-j\frac{3}{2}\hat{\omega}}\left[e^{+j\frac{3}{2}\hat{\omega}} + 2e^{+j\hat{\omega}/2} - 2e^{-j\hat{\omega}/2} - e^{-j\frac{3}{2}\hat{\omega}}\right]$$

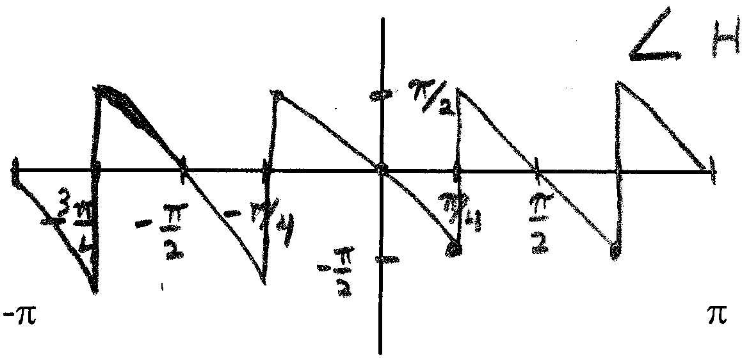
$$= e^{-j\frac{3}{2}\hat{\omega}}\underbrace{e^{j\pi/2}}_{+j}\left[2\sin(\tfrac{3}{2}\hat{\omega}) + 4\sin(\tfrac{1}{2}\hat{\omega})\right] = e^{j[\pi/2 - \frac{3}{2}\hat{\omega}]}\cdot\left[2\sin(\tfrac{3}{2}\hat{\omega}) + 4\sin(\tfrac{1}{2}\hat{\omega})\right]$$

b. Suppose $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}\cos(2\hat{\omega})$ Sketch $|H(e^{j\hat{\omega}})|$ and $\angle H(e^{j\hat{\omega}})$. Be sure to label all important frequencies and amplitudes.

$$|H(e^{j\hat{\omega}})| = |2\cos(2\hat{\omega})|$$



$$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega} \pm \pi \left(\begin{matrix}\text{where} \\ \cos 2\hat{\omega} < 0\end{matrix}\right)$$



Problem 2 – [25 points] The signal

$$x[n] = 5\cos\left(\frac{3}{4}\pi n\right) + 2\cos\left(\frac{1}{6}\pi n\right) + \delta[n-2]$$

We need $H(e^{j\,3/4\pi})$

need $H(e^{j\pi/6})$

need $h[n-2]$

is passed through the filter, y[n] = x[n] – x[n-2] + x[n-4].

Find the output y[n]. Express it in the same form as the input.

$h[n] = \delta[n] - \delta[n-2] + \delta[n-4]$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{4} h[n] e^{-j\hat{\omega}k} = 1 - e^{-j\hat{\omega}2} + e^{-j\hat{\omega}4}$$

$$= e^{-j\hat{\omega}2}\left[e^{j\hat{\omega}2} - 1 + e^{-j\hat{\omega}2}\right] = e^{-j\hat{\omega}2}\left[2\cos(2\hat{\omega}) - 1\right]$$

$$H(e^{j\,3/4\pi}) = e^{-j\,3/2\pi}\left[\underbrace{2\cos(3/2\,\pi)}_{=0} - 1\right] = e^{-j\,3/2\pi}\cdot \underbrace{e^{+j\pi}}_{-1} = e^{-j\pi/2}$$

$$H(e^{j\pi/6}) = e^{-j\pi/3}\left[2\cos(\pi/3) - 1\right] = 0$$

$$\boxed{y[n] = 5\cos\left(\frac{3}{4}\pi n - \frac{\pi}{2}\right) + h[n-2]}$$

$$h[n-2] = \delta[n-2] - \delta[n-4] + \delta[n-6]$$

$$\therefore \boxed{y[n] = 5\cos\left(\frac{3}{4}\pi n - \frac{\pi}{2}\right) + \delta[n-2] - \delta[n-4] + \delta[n-6]}$$

Problem 3 – [25 points] For the system:

$$y[n] = x[n] - x[n-L] \qquad Y(z) = X(z) - z^{-L} X(z)$$

a. Give the z-transform, $H(z)$, for this filter as a ratio of polynomials.

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-L} = \frac{z^{L} - 1}{z^{L}}$$

b. List and sketch the poles and zeros for L=2. Use **x**'s to mark poles and **o**'s to mark zeros.

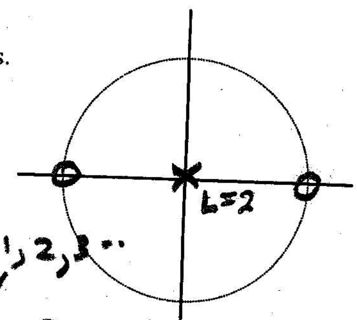
$$H(z) = \frac{z^2 - 1}{z^2}$$

zeros at $z^2 = 1$

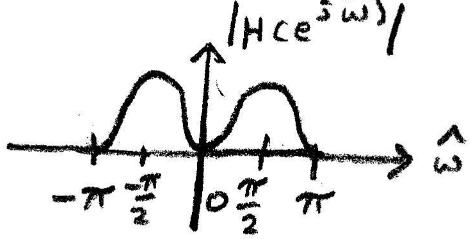
$z^2 = e^{j2\pi N}$ $\quad N = 0, 1, 2, 3 \ldots$

$z = e^{j\pi N}$ $\quad \pi$

only 2 roots (rest repeat)



c. Sketch the magnitude of the frequency response based on your pole/zero plot.



zeros at $\hat{\omega} = 0, \pi$

d. Where is/are the peak(s) on your plot (give a value for $\hat{\omega}$)? What is the value at the peak?

The peak lies in between zeros. The zeros are at $\hat{\omega} = 0, \pi$

Peak at $\Rightarrow$ $\hat{\omega} = \pm \pi/2$

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}2} = e^{-j\hat{\omega}}\underbrace{[e^{+j\hat{\omega}} - e^{-j\hat{\omega}}]}_{j2\sin(\hat{\omega})}$$

$$H(e^{j\hat{\omega}}) = e^{j(\pi/2 - \hat{\omega})}[2\sin(\hat{\omega})]$$

$j2\sin(\hat{\omega}) = (e^{j\pi/2})2\sin(\hat{\omega})$

$$|H(e^{\pm j\pi/2})| = \boxed{2}$$

e. **Problem 4** – [25 points] Find the impulse response, h[n], for the IIR filter below. Simplify h[n] so it doesn't contain δ[n]'s or other h[n]'s.

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$h[n] = a_1 h[n-1] + b_0 \delta[n] + b_1 \delta[n-1]$$

No credit unless you show and explain your work.

$$H(z) = z^{-1} a_1 H(z) + b_0 + b_1 z^{-1}$$

$$H(z)\left[1 - a_1 z^{-1}\right] = b_0 + b_1 z^{-1}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0}{1 - a_1 z^{-1}} + \overset{\text{(delay by 1)}}{z^{-1}} \left(\frac{b_1}{1 - a_1 z^{-1}}\right)$$

Use the z-Transform pair derived in class

$$\frac{1}{1-a_1 z^{-1}} \Leftrightarrow (a_1)^n u[n]$$

$$h[n] = b_0 (a_1)^n u[n] + b_1 (a_1)^{n-1} u[n-1]$$