Problem 7 - For an N point DFT answer the following questions. You may assume that X[k] is the DFT of x[n]:

a. If x[n] is real, which two values of X[k] are always real?

$X(0)$ + $X(\frac{N}{2})$

b. The following sequences, x1[n] = [1, 2, 3, 4], x2[n] = [1, 4, 3, 2], x3[n] = [-1, -2, -3, -4], have the same **magnitude** DFT as |X1[k]|. Why does x2[n] have the same magnitude DFT?.

3 4 1 2

$x2[n]$ is a shifted version of $x1[n]$

shift in n causes a phase change in $\hat{w}$, but not magnitude change

c. Why does x3[n] have the same magnitude DFT?.

$x3[n]$ is $x1[n]$ time -1 or times $e^{j\pi}$

The angle is changed but not the magnitude

d. If x[n] is real, X[k] is equal to X*[-k]. Prove it.

If $x[n]$ is real $x[n] = x^*[n]$

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\frac{\pi n k}{N}}$$

take conjugate of both sides

$$X^*(k) = \sum_{n=0}^{N-1} x^*[n] e^{j2\frac{\pi n k}{N}}$$

$x^*[n] = x[n]$, ~~for~~ let $k = -k$

$$X^*(-k) = \sum_{n=0}^{N-1} x[n] e^{-j2\frac{\pi n k}{N}} = X(k)$$

$$\therefore X^*(-k) = X(k)$$