## Problem Set 9

1. Consider a filter having frequency response  $H(\omega)$ . The "3 dB" frequency is that frequency for which the magnitude of the response has dropped by 3 dB. Show using hand analysis that the "3 dB" frequency is expressed by the following relationship

$$\left|H\left(\omega_{3cB}\right)\right| = \frac{1}{\sqrt{2}} \left|H_{PB}\right|$$

- 2. Consider a filter with a perfectly flat passband ( $H_{PB} = 1$ ) and a group delay which varies linearly with frequency, having a slope of one. Use a computer tool such as MATLAB to perform the following:
  - A. Plot the magnitude and phase of the frequency response of the filter..
  - B. Find the output of the filter given a square wave input of peak-to-peak amplitude 1, DC offset 0.5, and period 10  $\mu$ s.
  - C. Plot the input wave and the output wave, and compare their shapes. How is the output wave impacted by the specified filter characteristics?
- 3. The frequency response of a particular class of Butterworth filters is given by

$$\left|H\left(\omega\right)\right|^{2} = \frac{1}{1 + 0.015 \left(\frac{\omega}{\omega_{p}}\right)^{2n}}$$

where  $\omega_n$  is the passband edge.

- A. Suppose the stopband edge is the frequency  $\omega_s$  at which the attenuation reaches 35 dB. For this class of filters find and plot the transition ratio  $t = \frac{\omega_s}{\omega_p}$  as a function of the order *n*, as *n* varies from one to ten.
- B. Find the 3 dB frequency of the Butterworth filter when n = 5.



4. A Chebyshev low-pass filter is required that will meet the specifications shown below:

Find values for the parameters  $\omega_p$ ,  $\varepsilon^2$ , and *n* so that  $|H(\omega)|^2$  meets the specifications.

This problem set is due at the start of class on Tuesday, November 12.