Name Key

Fall 1993 — EE 391 Introduction to Communication Systems — Mark A. Yoder

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Exam II

There are four questions, all of equal weight.
Read the questions carefully before answering them, as credit will not be given for the right answer to the wrong question.
Mark your answers clearly and be sure to give units.
The examination will last fifty minutes. You may use on 3" by 5" card of notes. There is a Fourier Transform table.

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1. *(25)* Given: $x(t)$ and $X(f)$ are Fourier Transform pairs. We know that if $x(t)$ is real then $X(f) = X^*(-f)$.

Suppose $x(t)$ is real and **odd**, show that $X(f)$ is imaginary and odd.

$x(t) = x^*(t)$ real

$x(t) = -x(-t)$ odd

Def.

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

take *

$$X^*(f) = \int_{-\infty}^{\infty} x^*(t) e^{j2\pi f t} dt$$

$$X^*(-f) = \int_{-\infty}^{\infty} x^*(t) e^{-j2\pi f t} dt = X(f)$$

$\alpha = -t \quad d\alpha = -dt$

$$X(f) = \int_{\infty}^{-\infty} x(-\alpha) e^{j2\pi f \alpha} \, d\alpha$$

$$= -\int_{-\infty}^{\infty} x(\alpha) e^{j2\pi f \alpha} d\alpha$$

$$-X(-f) = \int_{-\infty}^{\infty} x(\alpha) e^{-j2\pi f \alpha} d\alpha$$

$$= X(f)$$

$$\boxed{-X(-f) = X(f)} \text{ odd}$$

$x(t) \leftrightarrow X(f)$ scaling in $a = -1$

$x(-t) \leftrightarrow X(-f)$

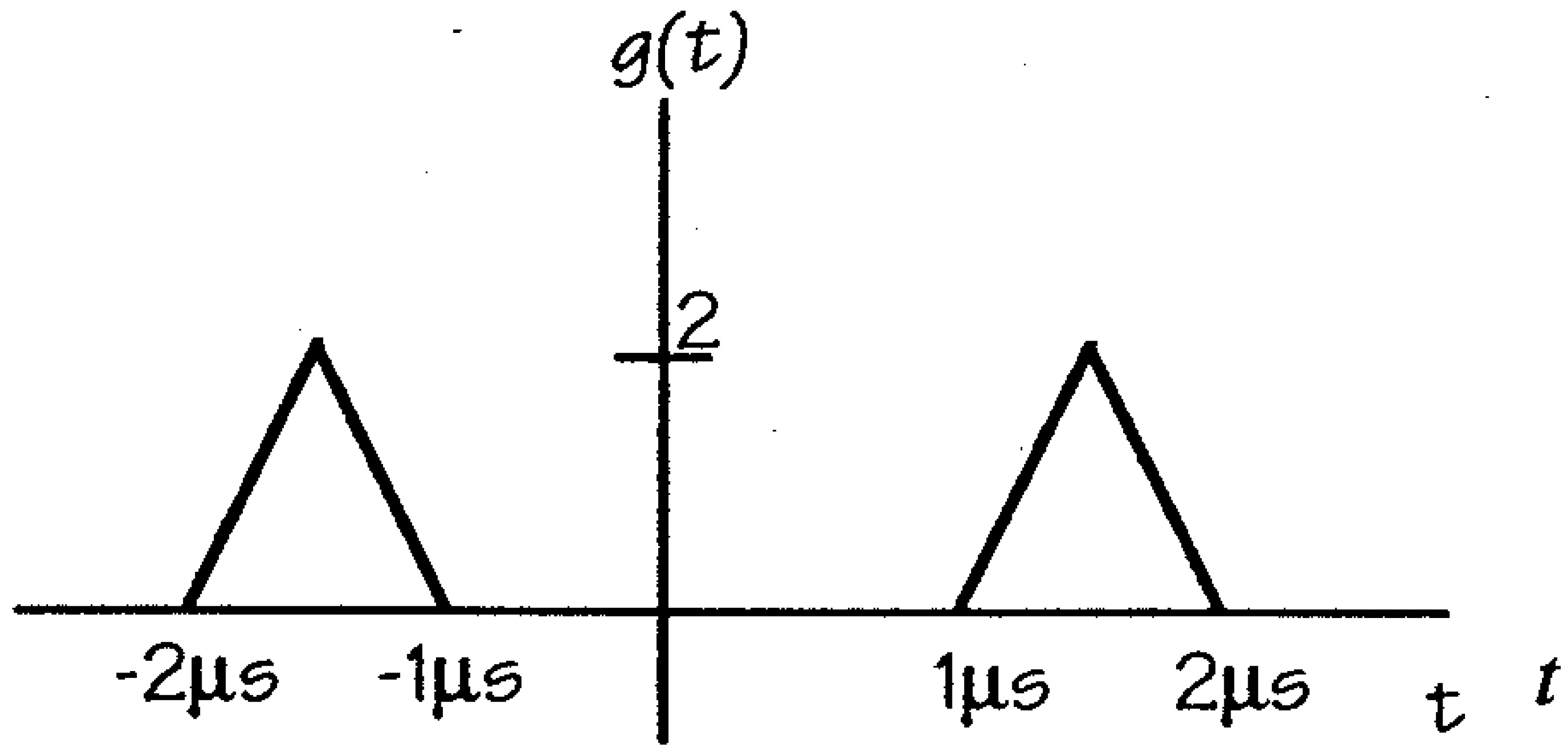
$x(t) = -x(-t)$

$X(f) = -X(-f)$

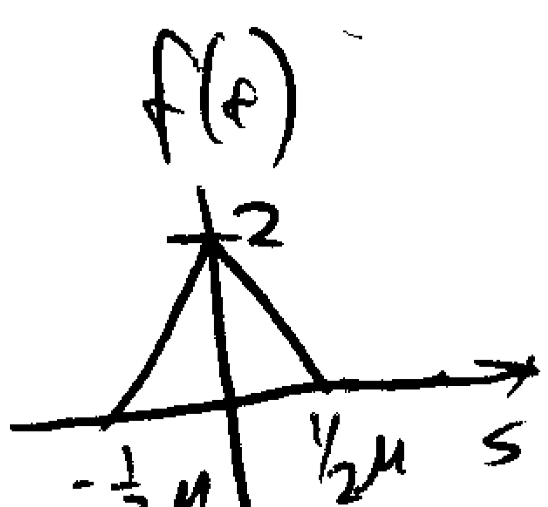
$-X(f) = X^*(-f)$

Imaginary

2. *(25)* Find the Fourier transform of the signal $g(t)$ as shown below:



Show all your work. You may use any transforms in the table on the last page, and any of the transform properties. For full credit do not resort to evaluation of the Fourier transform integral.



$F(f)$

$\frac{1}{2}\times10^{-6}\,\text{sinc}\left(f\,\frac{1}{2}\times10^{-6}\right)$

$\tau\,\text{sinc}^2(f\tau)$

$2\left(1-\frac{|t|}{\tau}\right)$

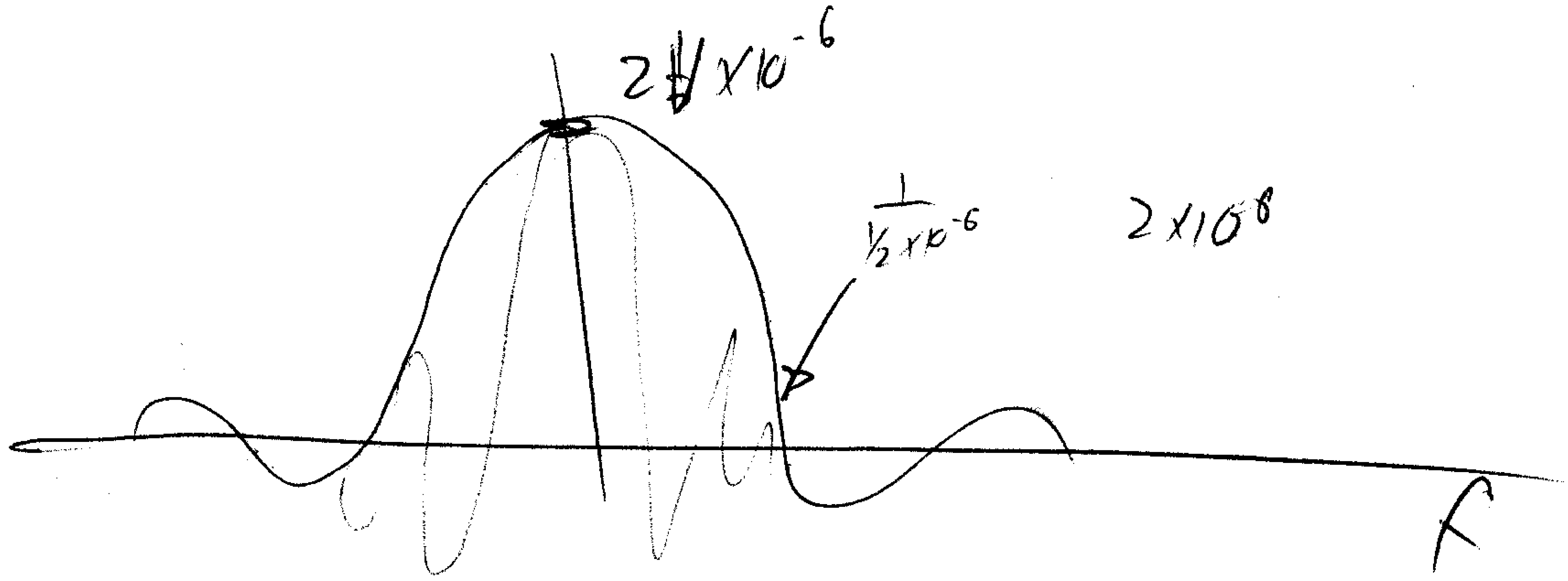
$\tau=\frac{1}{2}\mu s$

$$g(t)=2f(t)*\left[\delta(t+1.5\mu)+\delta(t-1.5\mu)\right]$$

$$G(f)=2\cdot F(f)\cdot 2\cos(2\pi\cdot 1.5\mu\, f)$$

$$=2\times10^{-6}\,\text{sinc}\left(\tfrac{1}{2}\times10^{-6}f\right)\cos\left(2\pi\, 1.5\times10^{-6}f\right)$$

Provide a neat, labeled sketch of the magnitude spectrum $|G(f)|$. Label all significant values and frequencies.



3. *(25)* A certain linear, time-invariant system has impulse response *h(t)* given by:

$$h(t) = 6000\, e^{-6000t}\, u(t)$$

a. Find *H(f)*

$$H(f) = \mathcal{F}\{h(t)\} = \frac{6000}{6000 + j2\pi f}$$

too many terms

b. The real-valued input signal *x(t)* is given by

$$x(t) = c_{-3}\, e^{-j2\pi 1500t} + c_{-2}\, e^{-j2\pi 1000t} + c_{-1}\, e^{-j2\pi 500t} + c_1\, e^{j2\pi 500t} + c_2\, e^{j2\pi 1000t} + c_3\, e^{j2\pi 1500t}$$

where $c_1 = 4 + j\,3$, $c_2 = -2 + j\,5$, and $c_3 = 2$

What is f_0? 500

c. Given the input signal, *x(t)*, is passed through the system with *h(t)* above, find the output signal *y(t)*; Express your answer as a Fourier series in exponential form. Express the coefficients of the output in *polar* form in degrees.

k	c_k	$H(k500)$	d_k
-3	2	$.64e^{+j46}$	$1.07e^{+j57^\circ}$
-2	$-2-j5$	$.88e^{+j27^\circ}$	
-1	$4-j3$	1	0
0	0		$4.4e^{j\cancel{64}^\circ\, 9.\cancel{45}\, 2.0}$
1	$4+j3$	$.48e^{-j27^\circ}$	$3.7e^{j\cancel{158}^\circ\, 65}$
2	$-2+j5$	$.\cancel{89}e^{-j46^\circ}$	$1.07e^{-j57^\circ}$
3	2	$.53e^{-j57^\circ}$	

4. *(25)* A couple weeks ago two groups of student (*a* and*b*) performed LabX which directed them to look at the Fourier Transform of a *rect* centered about *t=0*. The plots below and on the next page show the results. (Note: the plot have been scaled to correspond to the continuous Fourier Transform used in class, not the `fft` in Matlab). Use your knowledge of the continuous Fourier Transform to support you answers to the questions below.

a. Here are group *a*'s results. What is the height and width of the *rect*? Why does the angle look the way it does? Is it centered about *t=0*? If not, what is the shift in seconds?

$A\,\mathrm{rect}\left(\frac{t}{T}\right) = AT\,\mathrm{sinc}(fT)$

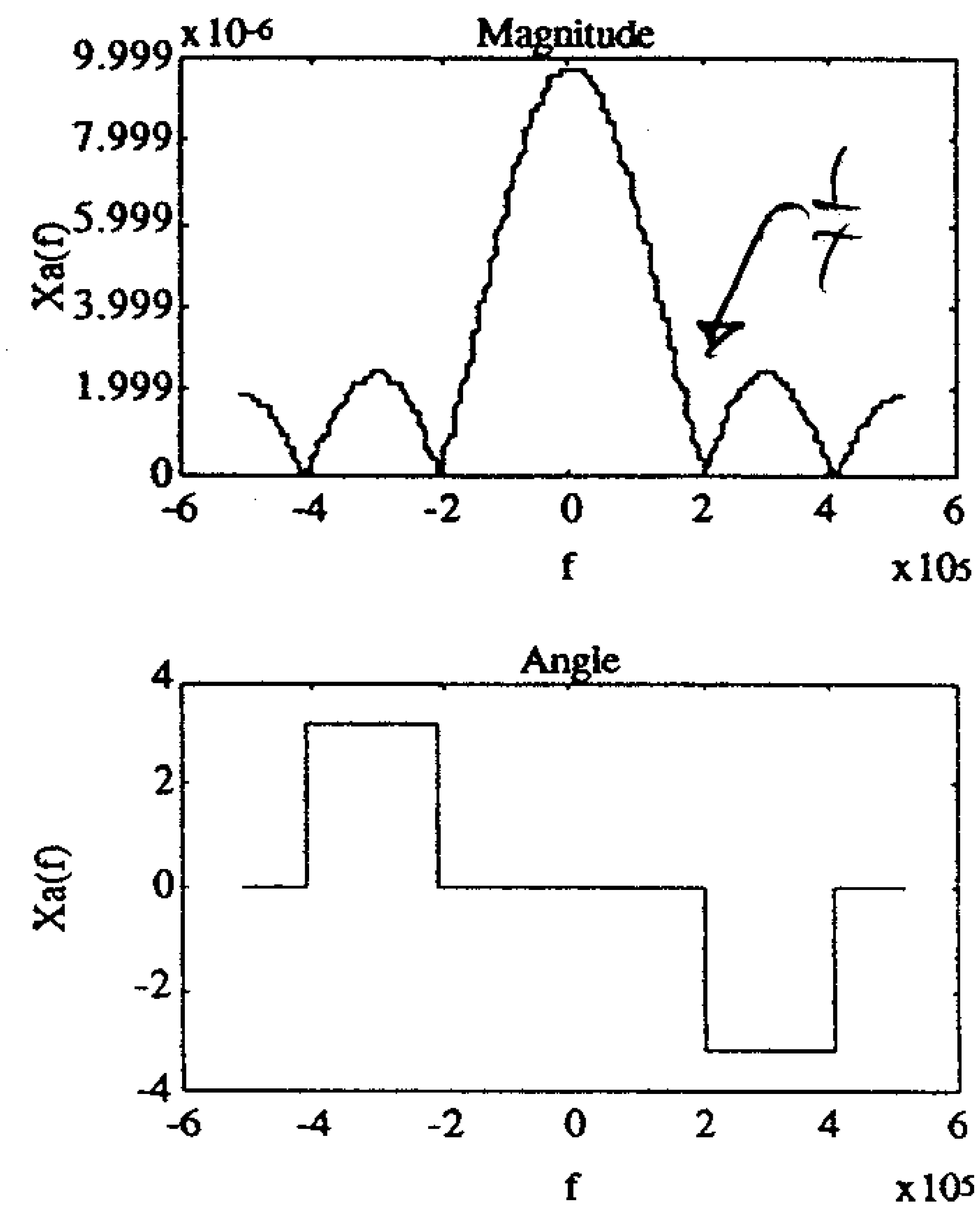
$\frac{1}{T} = 2\times10^{5} \quad T = \frac{1}{2}\times10^{-5} = \boxed{5\mu s}$

$AT = 10^{-5}$

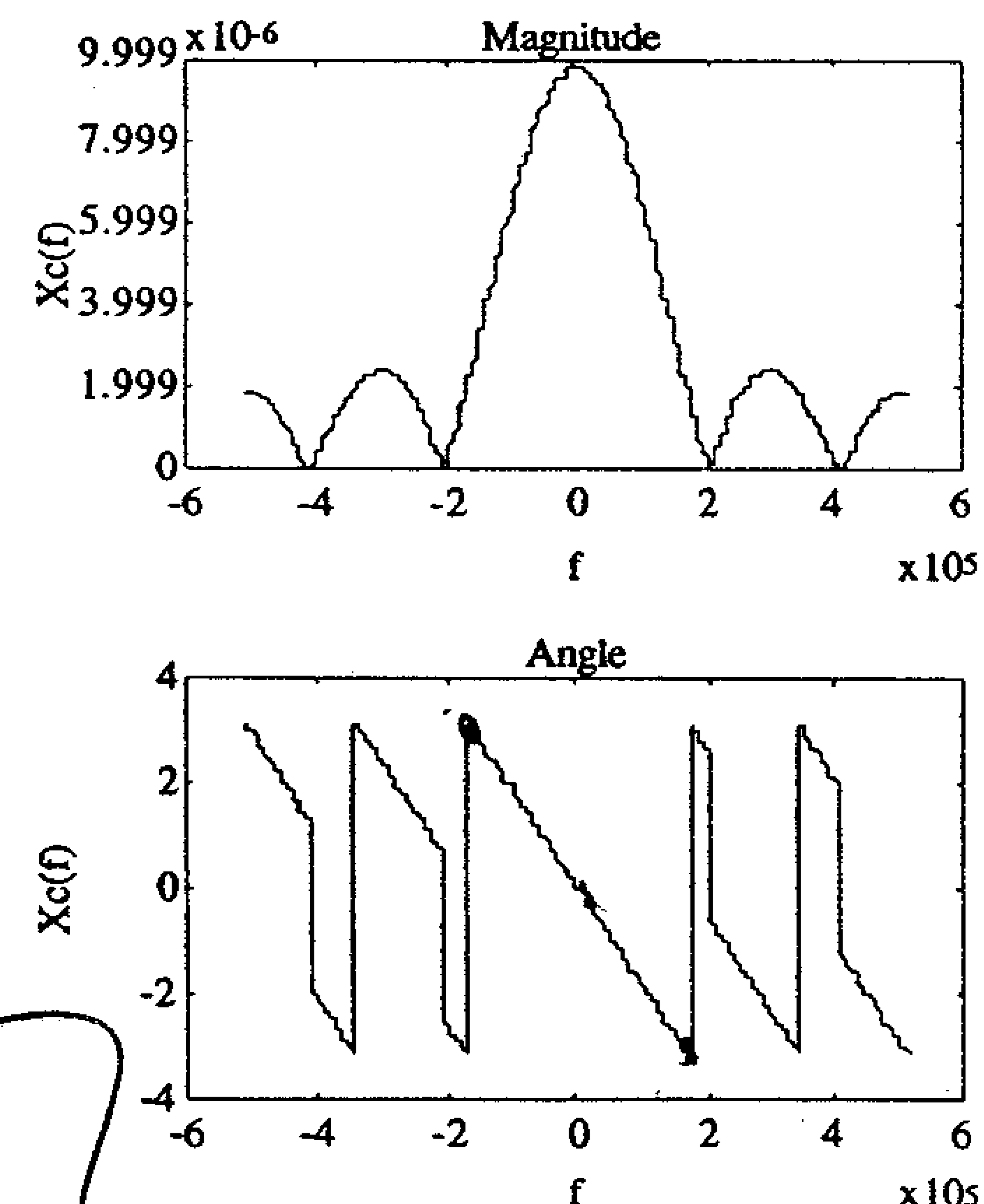
$A = \frac{10^{-5}}{2\times10^{-5}} = \boxed{2}$

$\angle = 0$

no shift



b. Here are group *b*'s reults. What is the height and width of their *rect*? Why does the angle look the way it does? Is it centered about *t=0*? If not, what is the shift in seconds?



$\angle = \frac{\pi}{4\times10^{5}} f = -f\,10^{-5}$

$t_0 = 10^{-5}$

$e^{-j2\pi(-1)f}$

shift $= \boxed{10\mu s}$

Which group did the lab right? Why?