

Test & Measurement

Application Note 150-1



Spectrum Analysis
**Amplitude
& Frequency
Modulation**



<http://www.hp.com/go/tmappnotes>

Contents

Spectrum Analysis AM and FM

Chapter 1 – Modulation Methods

Chapter 2 – Amplitude Modulation

Modulation Degree and Sideband Amplitude

Zero Span and Markers

The Fast Fourier Transform (FFT)

Special Forms of Amplitude Modulation

Single Sideband

Chapter 3 – Angular Modulation

Definitions

Bandwidth of FM Signals

FM Measurements with the Spectrum Analyzer

AM Plus FM (Incidental FM)

Appendix A – Mathematics of Modulation

Appendix B – Frequency Modulation in Broadcast Radio

Relevant Products and Training

Contacting Hewlett-Packard Worldwide


$$e = A \cos(\omega t + f)$$

1 Modulation Methods

Modulation is the act of translating some low-frequency or base-band signal (voice, music, data) to a higher frequency. Why do we modulate signals? There are at least two reasons: to allow the simultaneous transmission of two or more baseband signals by translating them to different frequencies, and to take advantage of the greater efficiency and smaller size of higher-frequency antennas.

In the modulation process, some characteristic of a high-frequency sinusoidal carrier is changed in direct proportion to the instantaneous amplitude of the baseband signal. The carrier itself can be described by the equation

$$e(t) = A \cdot \cos(\omega t + f)$$

where: A = peak amplitude of the carrier,

ω = angular frequency of the carrier in radians per second,

t = time, and

f = initial phase of the carrier at time $t = 0$.

In the expression above there are two properties of the carrier that can be changed, the amplitude (A) and the angular position (argument of the cosine function). Thus we have amplitude modulation and angular modulation. Angular modulation can be further characterized as either frequency modulation or phase modulation.

Milestones.

In 1864 James Clark Maxwell predicted the existence of electromagnetic waves that travel at the speed of light. In Germany, Heinrich Hertz proved Maxwell's theory and in 1888 was the first to create electromagnetic radio waves.



2

Amplitude Modulation Modulation Degree and Sideband Amplitude

Amplitude modulation of a sine or cosine carrier results in a variation of the carrier amplitude that is proportional to the amplitude of the modulating signal. In the time domain (amplitude versus time), the amplitude modulation of one sinusoidal carrier by another sinusoid resembles Figure 1A. The mathematical expression for this complex wave shows that it is the sum of three sinusoids of different frequencies.

One of these sinusoids has the same frequency and amplitude as the unmodulated carrier. The second sinusoid is at a frequency equal to the sum of the carrier frequency and the modulation frequency; this component is the upper sideband. The third sinusoid is at a frequency equal to the carrier frequency minus the modulation frequency; this component is the lower sideband. The two sideband components have equal amplitudes, which are proportional to the amplitude of the modulating signal. Figure 1B shows the carrier and sideband components of the amplitude-modulated wave of Figure 1A as they appear in the frequency domain (amplitude versus frequency).

Figure 1A.
Time domain display
of an amplitude-
modulated carrier.

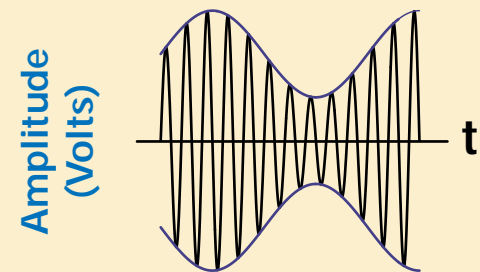
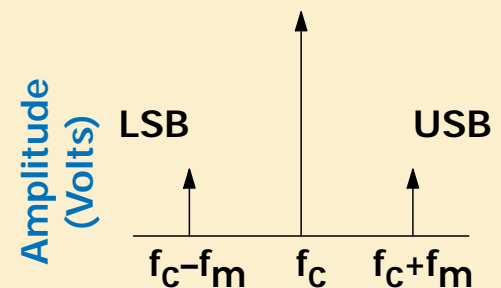


Figure 1B.
Frequency domain
(spectrum analyzer)
display of an amplitude-
modulated carrier.



2 Amplitude Modulation

Modulation Degree and Sideband Amplitude

A measure of the amount of modulation is m , the degree of modulation. This is usually expressed as a percentage called the percent modulation. In the time domain, the degree of modulation for sinusoidal modulation is calculated as follows, using variables shown in Figure 2:

$$m = \frac{E_{\max} - E_c}{E_c}$$

Since the modulation is symmetrical,

$$E_{\max} - E_c = E_c - E_{\min}$$

and

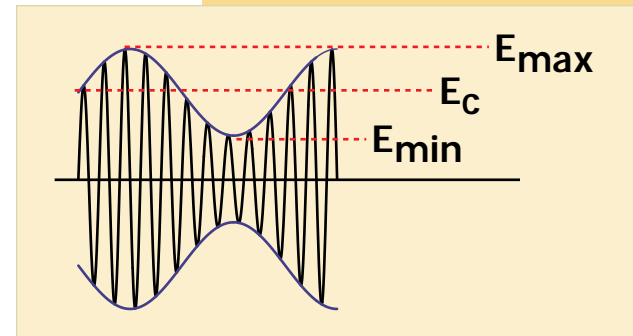
$$\frac{E_{\max} + E_{\min}}{2} = E_c.$$

From this it is easy to show that

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

for sinusoidal modulation.

Figure 2.
Calculation of
degree of amplitude
modulation from time
domain display.



2

Amplitude Modulation Modulation Degree and Sideband Amplitude

When all three components of the modulation signal are in phase, they add together linearly and form the maximum signal amplitude E_{\max} , as shown in Animation 1.

$$E_{\max} = E_c + E_{\text{USB}} + E_{\text{LSB}}$$

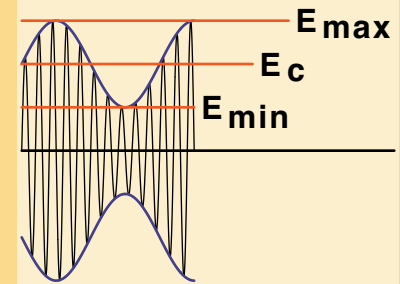
$$m = \frac{E_{\max} - E_c}{E_c} = \frac{E_{\text{USB}} + E_{\text{LSB}}}{E_c}$$

and, since $E_{\text{USB}} = E_{\text{LSB}} = E_{\text{SB}}$, then

$$m = \frac{2E_{\text{SB}}}{E_c}$$

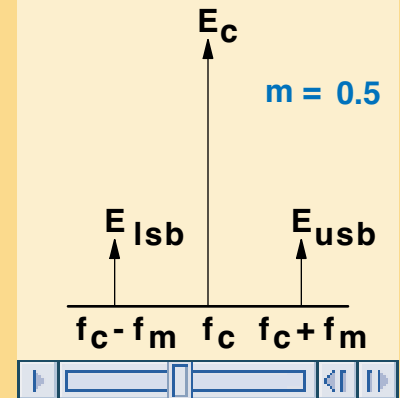
For 100% modulation ($m=1.0$), the amplitude of each sideband will be one-half of the carrier amplitude (voltage). Thus, each sideband will be 6 dB less than the carrier, or one-fourth the power of the carrier. Since the carrier component does not change with amplitude modulation, the total power in the 100% modulated wave is 50% higher than in the unmodulated carrier.

Animation 1.



Animation 1.

Calculation of degree of amplitude modulation displayed in both time and frequency domain. *Click over the chart to activate the animation.*



2

Amplitude Modulation Modulation Degree and Sideband Amplitude

Although it is easy to calculate the modulation percentage M from a linear presentation in the frequency or time domain ($M = m \cdot 100\%$), the logarithmic display on a spectrum analyzer offers some advantages, especially at low modulation percentages. The wide dynamic range of a spectrum analyzer (over 70dB) allows measurement of modulation percentages as low as 0.06%. This can easily be seen in Figure 3, where $M=2\%$; that is, where the sideband amplitudes are only 1% of the carrier amplitude.

Figure 3A shows a time domain display of an amplitude-modulated carrier with $M=2\%$. It is difficult to measure M on this display. Figure 3B shows the signal displayed logarithmically in the frequency domain. The sideband amplitudes can easily be measured in dB below the carrier and then converted into M . (The vertical scale is 10 dB per division.)

Figure 3A.

Time and frequency domain views of low level (2%) AM.

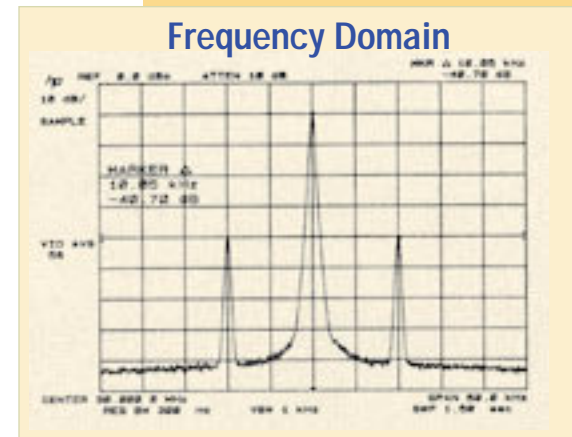
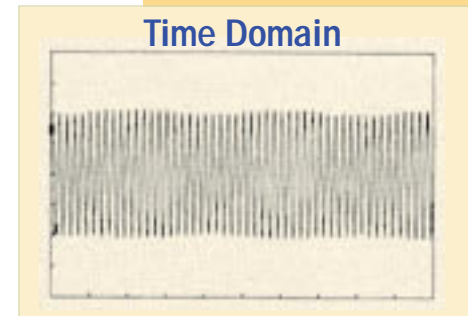


Figure 3B.

2 Amplitude Modulation

Modulation Degree and Sideband Amplitude

The relationship between m and the logarithmic display can be expressed as

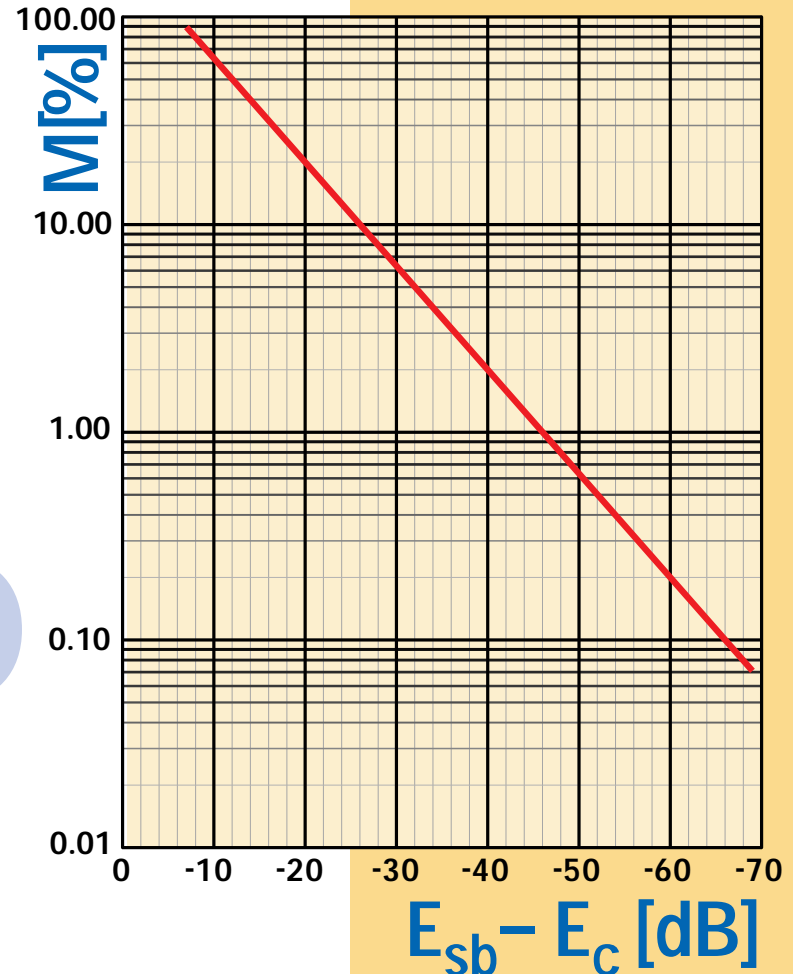
$$E_{SB} \text{ (dB)} - E_c \text{ (dB)} = 20 \log \frac{m}{2}$$

or

$$E_{SB} \text{ (dB)} - E_c \text{ (dB)} + 6\text{dB} = 20 \log m$$

Figure 4 shows the modulation percentage M as a function of the difference in dB between a carrier and either sideband.

Figure 4.
Modulation percentage M vs. sideband level (log display).



2

Amplitude Modulation Modulation Degree and Sideband Amplitude

Figures 5 and 6 show typical displays of a carrier modulated by a sine wave at different modulation levels in the time and frequency domains.

Figure 5A shows an amplitude-modulated carrier in the time domain. The percent modulation is

$$M = (6-2)/(6+2) = 4/8 = 50\%.$$

(Scope calibration is 0.1 msec/division, 50mV/division.)

Figure 5B shows the same waveform measured in the frequency domain. Since the carrier and sidebands differ by 12 dB, $M=50\%$. Frequency span is 10 kHz/division, centered at 60 MHz, and the log reference level is +10 dBm. You can also measure 2nd and 3rd harmonic distortion on this waveform. Second harmonic sidebands at $f_c \pm 2f_m$ are 55 dB down.

Figure 5A.

An amplitude-modulated carrier in the time domain.

Time Domain



Frequency Domain

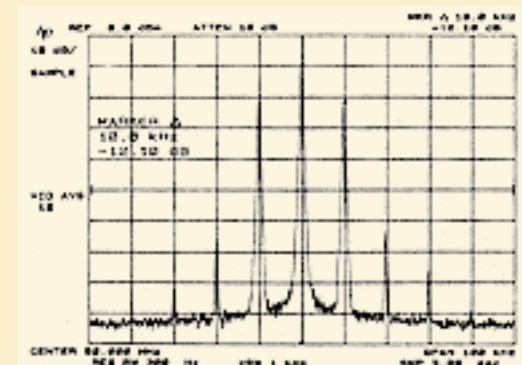


Figure 5B.

The same waveform measured in the frequency domain.

2

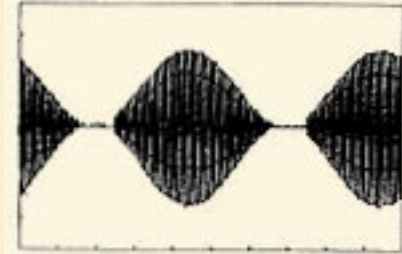
Amplitude Modulation Modulation Degree and Sideband Amplitude

Figure 6A shows an overmodulated ($M > 100\%$) 60 MHz signal in the time domain; $f_m = 2$ kHz (0.2 msec/Div, 50 mV/Div). The carrier is cut off at the modulation minima.

Figure 6B is the frequency domain display of the signal. Note that the first sideband pair is only 6 dB lower than the carrier. Also, the occupied bandwidth is much greater because of severe distortion of the modulating signal. (50 kHz span, 10 dB/Div, BW 100 Hz.)

Figure 6A.
An over-modulated
60MHz signal.

Time Domain



Frequency Domain

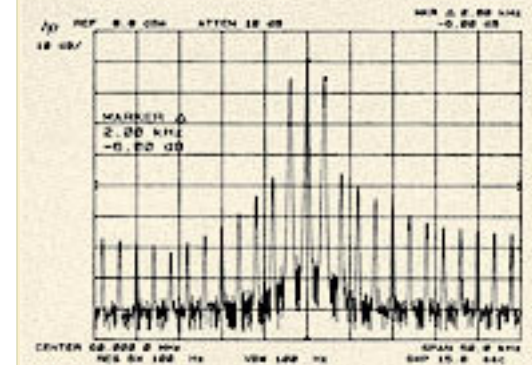


Figure 6B.
The frequency domain
display of the signal.

2

Amplitude Modulation Modulation Degree and Sideband Amplitude

AM Signal Animations The effects of varying two of the adjustable parameters in amplitude modulation, the degree of modulation m and the modulation frequency f_m , are demonstrated in Animations 2 and 3 in the time domain.

The amplitude of the envelope of the modulated signal varies linearly with m . This is demonstrated in Animation 2. The signal becomes fully modulated at $m=1$. Values of $m > 1$ produce overmodulated signals which cannot be recovered well in most detection systems.

Increasing f_m causes the frequency of the envelope signal to increase, as shown in Animation 3. For most amplitude modulation applications, f_m is taken to be much smaller than the carrier frequency, f_c .

Animation 3.

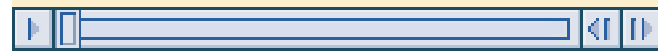
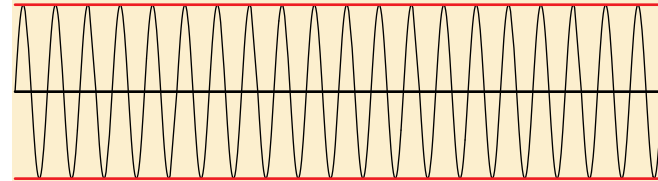
Effect of varying f_m from $0.06f_c$ to $0.2f_c$ on an AM signal. Live interactive AM and FM signal models that allow you to explore and experience modulation theory are available at the HP World Wide Website.

Animation 2.

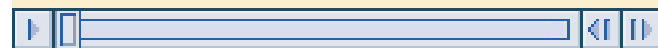
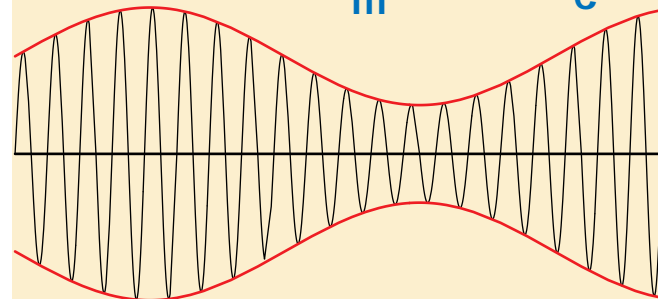
Effect of varying m from 0 to 1 on an AM signal.

Click over to activate.

$m = 0$



$f_m = 0.06 f_c$



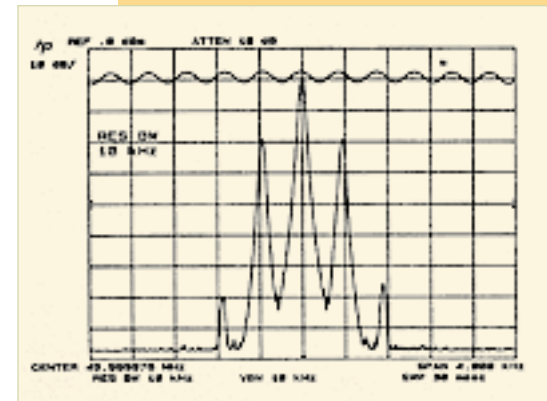
2 Amplitude Modulation

Zero Span and Markers

So far, the assumption has been that the spectrum analyzer has a resolution bandwidth narrow enough to resolve the spectral components of the modulated signal. But we may want to view low-frequency modulation with an analyzer that does not have sufficient resolution. For example, a common modulation test tone is 400 Hz. What can we do if our analyzer has a minimum resolution bandwidth of 1 kHz?

One possibility, if the percent modulation is high enough, is to use the analyzer as a fixed-tuned receiver, demodulate the signal using the envelope detector of the analyzer, view the modulation signal in the time domain, and make measurements as we would on an oscilloscope. To do so, we would first tune the carrier to the center of the spectrum analyzer display, then set the resolution bandwidth wide enough to encompass the modulation sidebands without attenuation, as shown in Figure 7, making sure that the video bandwidth is also wide enough. (The ripple in the upper trace of Figure 7 is caused by the phasing of the various spectral components, but the mean of the trace is certainly flat).

Figure 7.
Resolution bandwidth is set wide enough to encompass the modulation sidebands without attenuation.



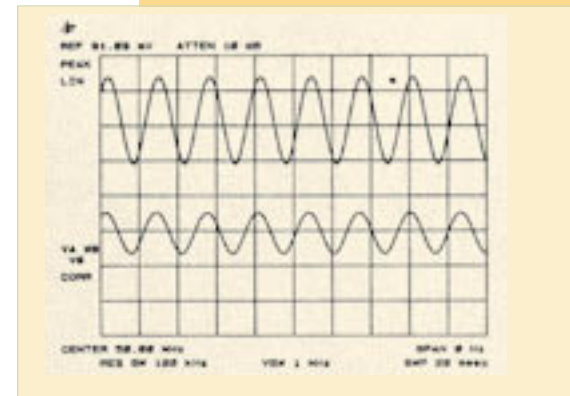
2 Amplitude Modulation

Zero Span and Markers

Next we select the zero-span function to fix-tune the analyzer, adjust the reference level to move the peak of the signal near the top of the screen, select the linear display mode, select video triggering and adjust trigger level, and adjust the sweep time to show several cycles of the demodulated signal. See Figure 8. Now we can determine the degree of modulation using the expression

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

Figure 8.
Moving the signal up and down on the screen changes the number of divisions between E_{\max} and E_{\min} .



2 Amplitude Modulation

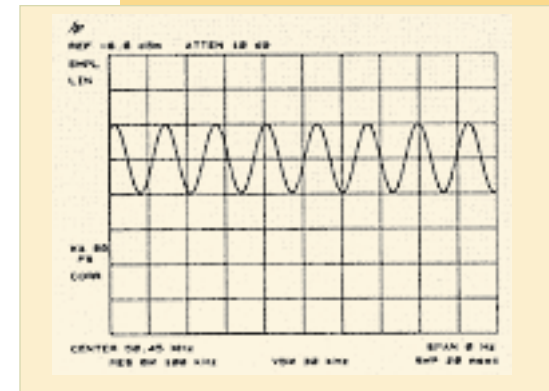
Zero Span and Markers

This is a relative measurement, and as we adjust the reference level to move the signal up and down on the screen, the number of divisions between E_{\max} and E_{\min} changes, as shown in Figure 8, but the ratio remains constant. We may be able to find a convenient location on the CRT; that is, if we can place the maxima and minima on graticule lines, the arithmetic is often easy, as in Figure 9. Here we have E_{\max} of six divisions and E_{\min} of four divisions, so

$$m = \frac{(6 - 4)}{(6 + 4)} = 0.2, \text{ or } 20\% \text{ AM}$$

The frequency of the signal can be determined from the calibrated sweep time of the analyzer (20 msec in this case). One cycle of the signal is about 1.3 divisions wide, yielding a modulation frequency of about 385 Hz.

Figure 9.
Placing the maxima and minima on graticule lines makes the measurement easier.



2 Amplitude Modulation

Zero Span and Markers

Many spectrum analyzers with digital displays also have markers and delta markers. These can make the measurements much easier. For example, in Figure 10 we have used the delta markers to find the ratio E_{\min} / E_{\max} . By modifying the expression for m , we can use the ratio directly:

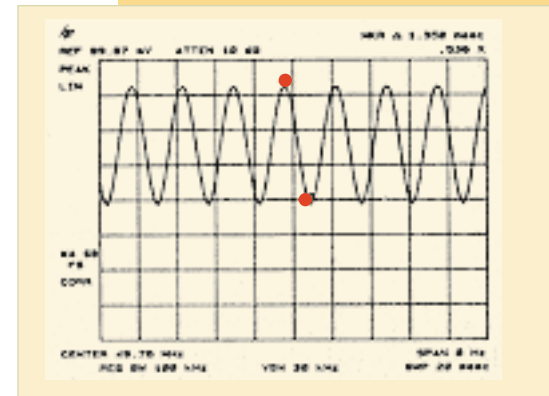
$$m = (1 - E_{\min}/E_{\max}) / (1 + E_{\min}/E_{\max}).$$

Since we are using linear units, the analyzer displays the delta value as a decimal fraction, exactly what we need for our expression. Figure 10 shows the ratio as 0.536, giving us

$$m = (1 - 0.536) / (1 + 0.536) = 0.302, \text{ or } 30.2\% \text{ AM.}$$

This percent AM would have been awkward to measure on an analyzer without markers, because there is no place on the display where the maxima and minima are both on graticule lines. The technique of using markers works well down to modulation levels that are quite low.

Figure 10.
Delta markers
can be used to
find the ratio
 E_{\min}/E_{\max} .



2 Amplitude Modulation

Zero Span and Markers

The percent AM (0.86%), computed from the 0.983 ratio in Figure 11A, agrees with the value determined from the carrier/sideband ratio of -47.28 dB shown in Figure 11B.

Note that the delta marker readout also shows the time difference between the markers. This is true of most analyzers in zero span. By setting the markers for one or more full periods, (Figure 12), we can take the reciprocal and get the frequency; in this case, $1/(2.6 \text{ msec})$, or 385 Hz.

Figures 11A and 11B.

Using the markers to measure percent AM works well even at low modulation levels. The percent AM computed from ratio in 11A agrees with values determined from carrier/sideband ratio in 11B.

Figure 12.

Time difference indicated by delta marker readout can be used to calculate frequency by taking the reciprocal.

Figure 11A.

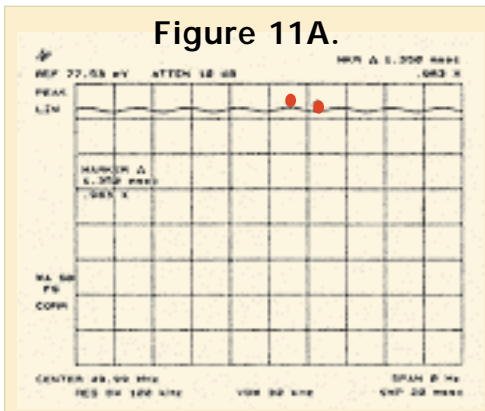


Figure 11B.

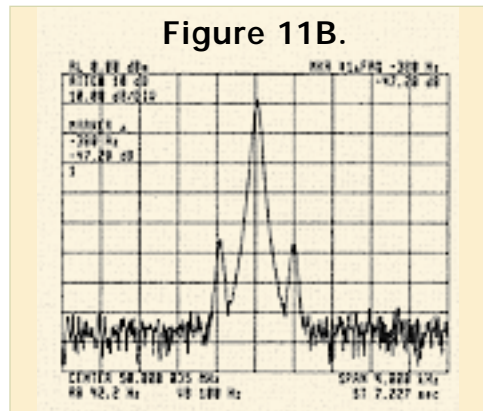
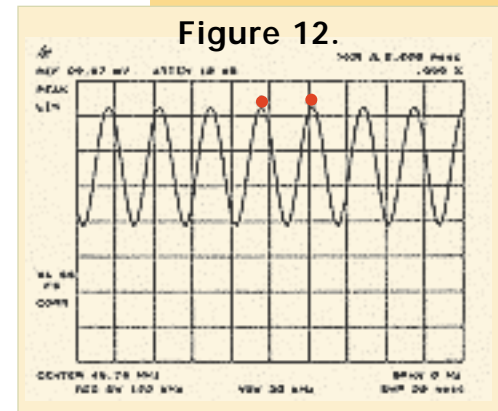


Figure 12.



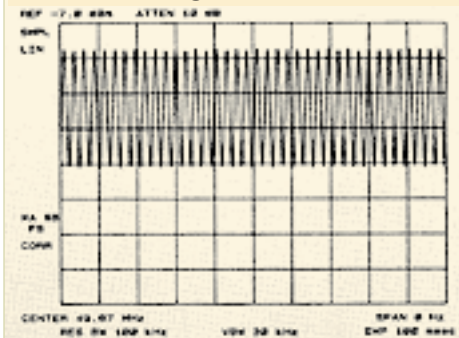
2 Amplitude Modulation

The Fast Fourier Transform (FFT)

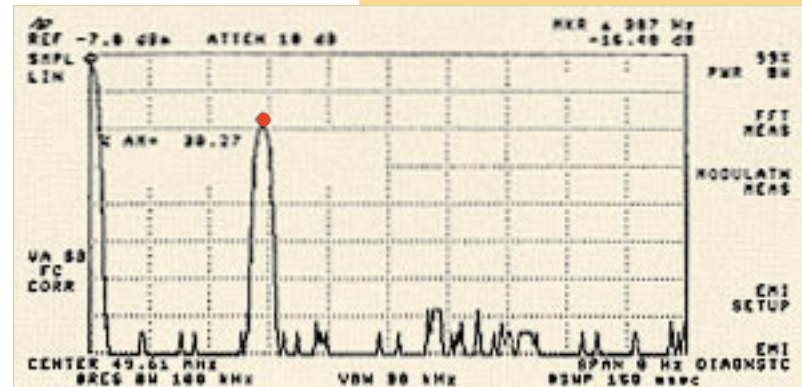
There is an even easier way to make the measurements above if the analyzer has the ability to perform an FFT on the demodulated signal. On the HP 8590 and HP 8560 portable spectrum analyzers, FFT is available on a soft key. On the HP 8566B, 8567A, 8568B and 71000 series spectrum analyzers, FFT is available in ROM, and we can write a DLP (down-loadable program) to take advantage of it.

In any case, we demodulate the signal as above, except we adjust the sweep time to display many, rather than a few, cycles, as shown in Figure 13. Then, calling the FFT routine yields a frequency-domain display of just the modulation signal, as shown in Figure 14. The carrier is displayed at the left edge of the CRT, and a single-sided spectrum is displayed across the screen. The delta markers can be used on this display, showing the modulation sideband offset by 387 Hz (the modulating frequency) and down by 16.4 dB (representing 30% AM). The HP 8590 series and the HP 8592A compute and display percent AM when the softkey MODULATN MEAS is pressed, as in Figure 14.

Figure 13.



Figures 13 and 14.
Using the FFT yields
a frequency-domain
display of just the
modulation signal.



2 Amplitude Modulation

The Fast Fourier Transform (FFT)

FFT capability is particularly useful for measuring distortion. Figure 15 shows our demodulated signal at 60% AM level. It is impossible to determine the modulation distortion from this display. The FFT display in Figure 16, on the other hand, indicates about 0.5% second-harmonic distortion.

The maximum modulating frequency for which the FFT can be used on a spectrum analyzer is a function of the rate at which the data are sampled (digitized); that is, it's directly proportional to the number of data points across the CRT and inversely proportional to the sweep time.

Figure 15.
The modulation distortion of our signal cannot be read from this display.

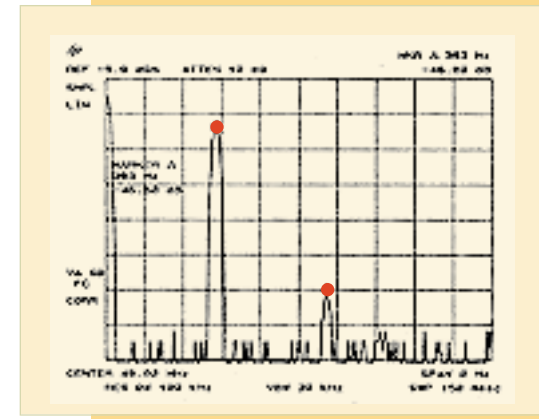
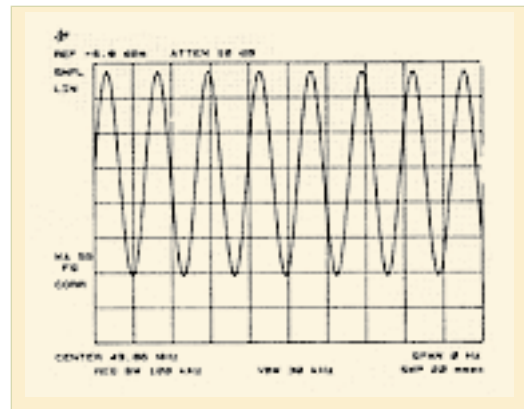


Figure 16.
An FFT display indicates the modulation distortion; in this case, about 0.5% second-harmonic distortion.

2 Amplitude Modulation

The Fast Fourier Transform (FFT)

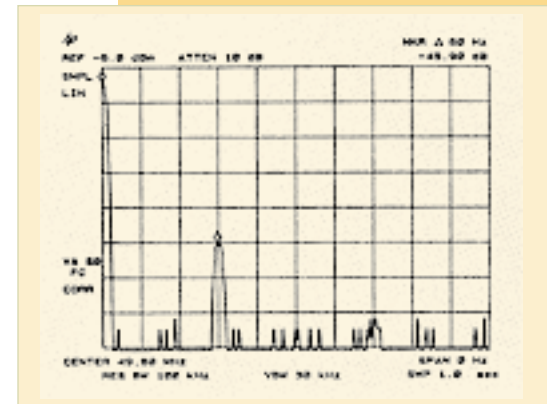
For the HP 8566B, 8567A, and 8568B, the maximum frequency (right edge of the CRT) is 25 kHz; for the HP 8590 and HP 8560 portable series, it is 10 kHz; for the HP 71000 series, it's 10 kHz.

Note that lower frequencies can be measured: very low frequencies, in fact. With a 20-second sweep, the HP 8566B, 8567A, and 8568B can measure down to 0.5 Hz.

Figure 17 shows a measurement of power-line hum (60 Hz in this case) on the HP 8590 series using a 1-second sweep time.

Figure 17.

A 60 Hz power-line hum measurement uses a 1-second sweep time.



2 Amplitude Modulation

The Fast Fourier Transform (FFT)

An example of a down-loadable program (DLP) that uses the FFT on the HP 8566B, 8567A, and 8568B is given in the programming note “Amplitude Modulation Measurements Using the Fast Fourier Transform,” (HP literature number 5954-2754). Figure 18 shows the instructions for, and the results of using, the DLP. A similar DLP could be written for the HP 71000 series of spectrum analyzers.

Setting an analyzer to zero span allows us not only to observe a demodulated signal on the CRT and measure it, but to listen to it as well. Most analyzers, if not all, have a video output that allows us access to the demodulated signal. This output generally drives a headset directly. If we want to use an external speaker, we probably would need to add an amplifier to drive it.

Figures 18A and 18B.
The DLP instructions for the FFT shown in 18A produce the result shown in 18B.

Figure 18A.

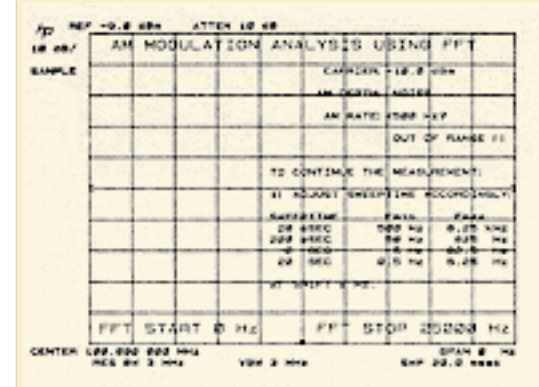
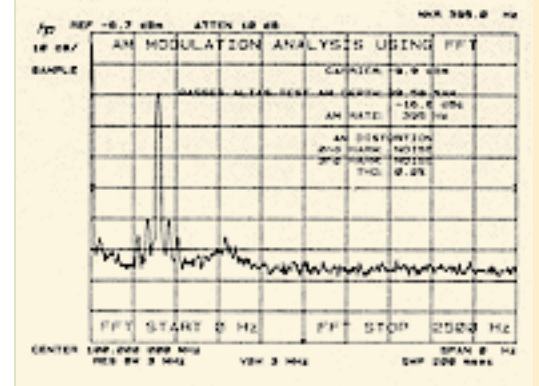


Figure 18B.



2 Amplitude Modulation

The Fast Fourier Transform (FFT)

Many analyzers include an AM demodulator and an internal speaker so that we can listen to signals without external hardware. As a convenience, some analyzers provide a marker pause function so we need not even be in zero span to hear the signals.

To use this feature, we set the frequency span to cover the desired range (that is, the AM broadcast band), set the active marker on the signal of interest, set the length of the pause (dwell time), and activate the AM demodulator. The analyzer then sweeps to the marker and pauses for the set time, allowing us to listen to the signal for that interval, before completing the sweep. If the marker is the active function, we can move it, and thus listen to any other signal on the display.

Milestones.

In 1895 Guglielmo Marconi of Italy made the first radio for communicating with ships at sea. In 1901 Marconi sent the first signal across the Atlantic.



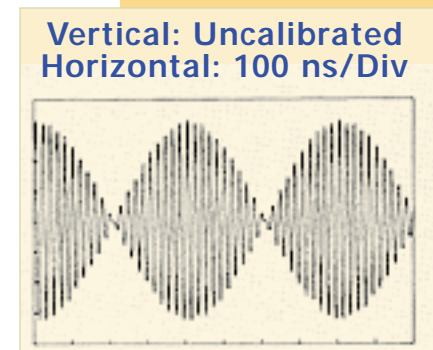
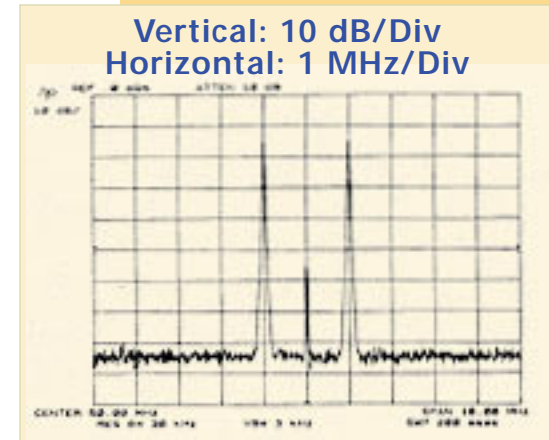
2

Amplitude Modulation Special Forms of Amplitude Modulation

We know that changing the degree of modulation of a particular carrier does not change the amplitude of the carrier component itself. Instead, the amplitude of the sidebands changes, thus altering the amplitude of the composite wave. Since the amplitude of the carrier component remains constant, all the transmitted information is contained in the sidebands. This means that the considerable power transmitted in the carrier is essentially wasted. For improved power efficiency, the carrier component may be suppressed (usually by the use of a balanced modulator circuit), so that the transmitted wave consists only of the upper and lower sidebands. This type of modulation is called double sideband-suppressed carrier, or DSB-SC. The carrier must be reinserted at the receiver, however, to recover this modulation. In the time and frequency domains, DSB-SC modulation appears as shown in Figure 19.

Note the suppression of the carrier (>40 dB).
(Inputs: $f_c = 50$ MHz at 5 mV, $f_m = 1$ MHz at 140 mV.)

Figure 19. Frequency and time domain presentations of balanced modulator output.



2 Amplitude Modulation

Single Sideband

In communications, an important type of amplitude modulation is single sideband with suppressed carrier (SSB). Either the upper or lower sideband can be transmitted, written as SSB-USB or SSB-LSB (or the SSB prefix may be omitted). Since each sideband is displaced from the carrier by the same frequency, and since the two sidebands have equal amplitudes, it follows that any information contained in one must also be in the other. Eliminating one of the sidebands cuts the power requirement in half and, more importantly, halves the transmission bandwidth (frequency spectrum width).

SSB has been used extensively throughout telephone systems to combine many separate messages into a composite signal (baseband) by frequency multiplexing. This method allows the combination of up to several thousand 4-kHz-wide channels containing voice, routing signals, and pilot carriers. The composite signal can then be either sent directly via coaxial lines or used to modulate microwave line transmitters.

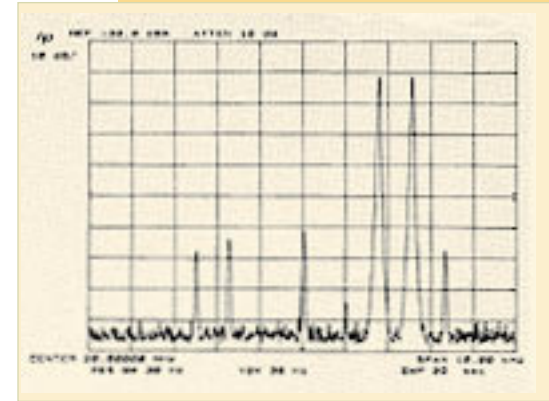


Figure 20A.

A SSB generator, modulated with two sine-wave signals of 1800 and 2600 Hz. The 20MHz carrier (display center) is suppressed 50 dB; lower sideband signals and inter-modulation products are more than 50 dB down.

Log ref is -30 dBm.
Span is 1 kHz/Div
with 10 dB/Div and
30 Hz bandwidth.

2 Amplitude Modulation

Single Sideband

The SSB signal is commonly generated at a fixed frequency by filtering or by phasing techniques. This necessitates mixing and amplification in order to get the desired transmitting frequency and output power. These latter stages, following the SSB generation, must be extremely linear to avoid signal distortion, which would result in unwanted in-band and out-of-band intermodulation products. Such distortion products can introduce severe interference in adjacent channels.

Thus, intermodulation measurements are a vital requirement for designing, manufacturing, and maintaining multi-channel communication networks. The most commonly used measurement is called the “two-tone test.” Two sine-wave signals in the audio frequency range (300-3100 Hz), each with low harmonic content and a few hundred Hertz apart, are used to modulate the SSB generator. The output of the system is then examined for intermodulation products with the aid of a selective receiver. The spectrum analyzer displays all intermodulation products simultaneously, thereby substantially decreasing measurement and alignment time.

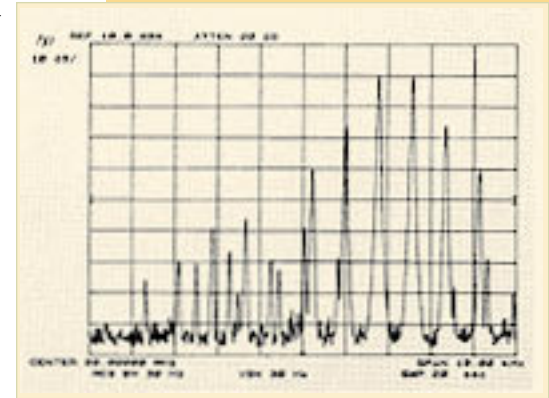


Figure 20B.

The same signal after passing through an amplifier with 1 dB compression. Intermodulation products are penetrating into the suppressed sideband. Log ref is +10 dBm. Span is 1 kHz/Div, with 10 dB/Div and 30 Hz bandwidth.

Figure 20 shows an intermodulation test of an SSB transmitter.

3 Angular Modulation

Definitions

In Chapter 1 we described a carrier as

$$e(t) = A \cdot \cos (wt + f)$$

and, in addition, stated that angular modulation can be characterized as either frequency or phase modulation. In either case, we think of a constant carrier plus or minus some incremental change.

Frequency Modulation

The instantaneous frequency deviation of the modulated carrier with respect to the frequency of the unmodulated carrier is directly proportional to the instantaneous amplitude of the modulating signal.

Phase Modulation

The instantaneous phase deviation of the modulated carrier with respect to the phase of the unmodulated carrier is directly proportional to the instantaneous amplitude of the modulating signal.

Milestone.

The Superheterodyne radio circuit, invented by E.H. Armstrong in 1918, did much to improve radio receivers and circuits. In 1933 Armstrong invented Frequency Modulation, known today as FM.

84.3 FM

3 Angular Modulation

Definitions

For angular modulation, there is no specific limit to the degree of modulation, so there is no equivalent of 100% AM.

Modulation index is expressed as:

$$m = \Delta f_p / f_m = \Delta \phi_p = \Delta f_p / f_m = \Delta \phi_p$$

where

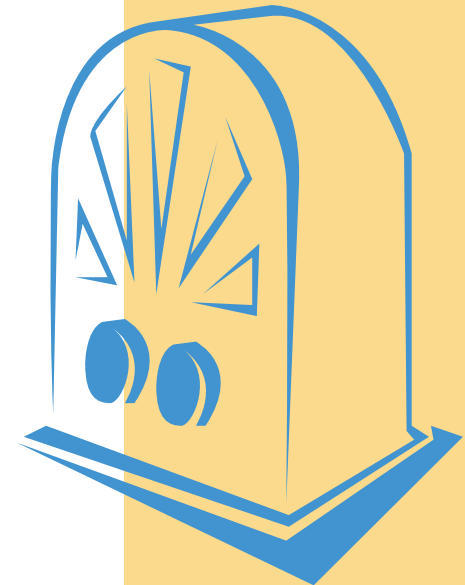
m = modulation index,

Δf_p = peak frequency deviation,

f_m = frequency of the modulating signal, and

$\Delta \phi_p$ = peak phase deviation in radians.

This expression tells us that the angular modulation index is really an indication of peak phase deviation, even in the FM case. Also, note that the definitions for frequency and phase modulation do not include the modulating frequency. In each case, the modulated property of the carrier, frequency or phase, deviates in proportion to the instantaneous amplitude of the modulating signal, regardless of the rate at which the amplitude changes.



3 Angular Modulation

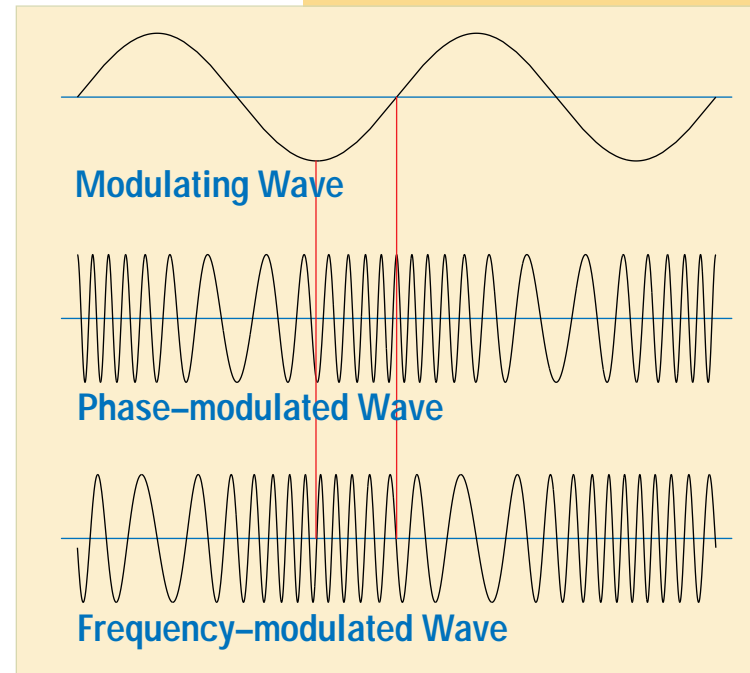
Definitions

However, the frequency of the modulating signal is important in FM. In the expression for the modulating index, it is the ratio of peak frequency deviation to modulation frequency that equates to phase. Comparing this basic equation with the two definitions of modulation, we find

(1) A carrier sine wave modulated with a single sine wave of constant frequency and amplitude will have the same resultant signal properties (that is, the same spectral display) for frequency and phase modulation. A distinction in this case can be made only by direct comparison of the signal with the modulating wave, as shown in Figure 21.

(2) Phase modulation can generally be converted into frequency modulation by choosing the frequency response of the modulator so that its output voltage will be proportional to $1/f_m$ (integration of the modulating signal). The reverse is also true if the modulator output voltage is proportional to f_m (differentiation of the modulating signal).

Figure 21.
Phase and frequency modulation of a sine-wave carrier by a sine-wave signal.



3 Angular Modulation

Definitions

Because phase modulation can be applied at the amplifier stage of a transmitter, a very stable crystal-controlled oscillator can be used. Thus, “indirect FM” is commonly used in VHF and UHF communication stations where highly stable carrier frequencies are required.

We can see that the amplitude of the modulated signal always remains constant, regardless of modulation frequency and amplitude. The modulating signal adds no power to the carrier in angular modulation as it does with amplitude modulation.

Mathematical treatment shows that, in contrast to amplitude modulation, angular modulation of a sine-wave carrier with a single sine wave yields an infinite number of sidebands spaced by the modulation frequency, f_m . In other words, AM is a linear process, whereas FM is a nonlinear process. For distortion-free detection of the modulating signal, all sidebands must be transmitted. The spectral components (including the carrier component) change their amplitudes when the modulation index m is varied. The sum of the squares of these components always yields a composite signal with an average power that remains constant and equal to the average power of the unmodulated carrier wave.



Milestones.

There are more than 10,000 AM and FM radio stations in the United States, more than anywhere else in the world.

It is estimated that 30,000 radio stations operate outside the United States.

3 Angular Modulation

Definitions

The curves of Figure 22 show the relation (Bessel function) between the carrier and sideband amplitudes of the modulated wave as a function of the modulation index m .

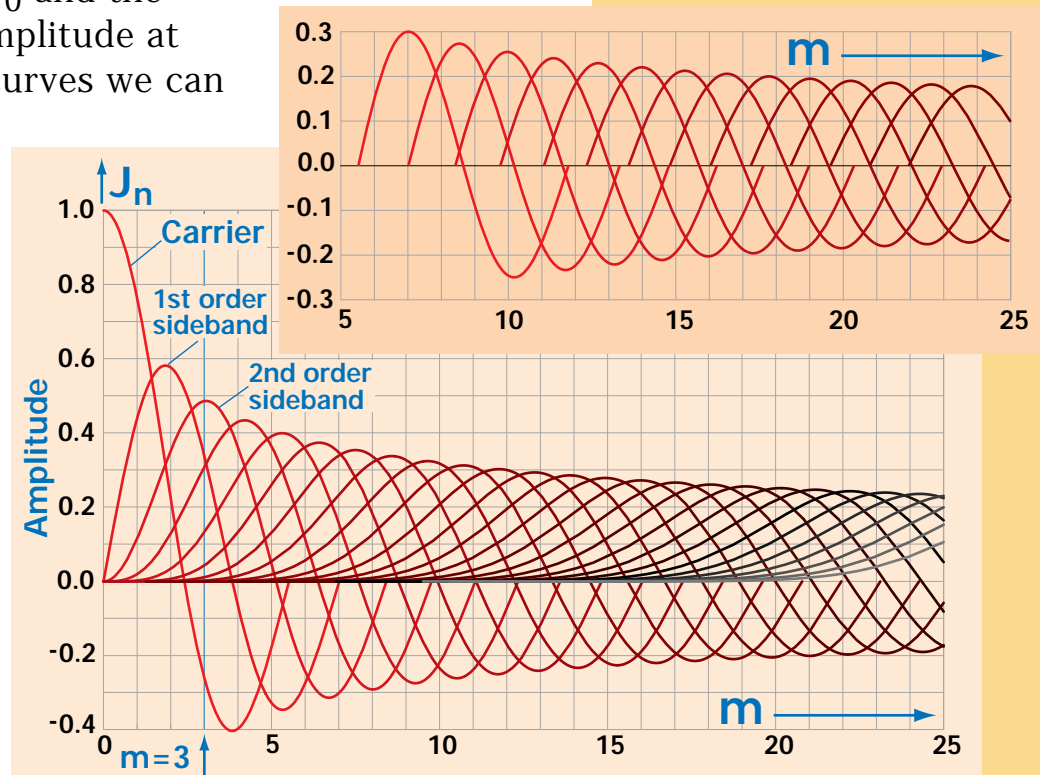
Note that the carrier component J_0 and the various sidebands J_n go to zero amplitude at specific values of m . From these curves we can determine the amplitudes of the carrier and the sideband components in relation to the unmodulated carrier.

For example, we find for a modulation index of $m=3$ the following amplitudes:

Carrier	$J_0 = 0.27$
First order sideband	$J_1 = 0.33$
Second order sideband	$J_2 = 0.48$
Third order sideband	$J_3 = 0.33$

etc.

Figure 22.
Carrier and sideband
amplitudes for angle-
modulated signals.



3 Angular Modulation

Definitions

The sign of the values we get from the curves is of no significance, since the spectrum analyzer displays only the absolute amplitudes.

The exact values for the modulation index corresponding to each of the carrier zeros are listed in Table 1.

Order of Carrier Zero	Modulation Index
1	2.40
2	5.52
3	8.65
4	11.79
5	14.93
6	18.07
$n(n > 6)$	$18.07 + p(n - 6)$

Table 1.

Jo

3 Angular Modulation

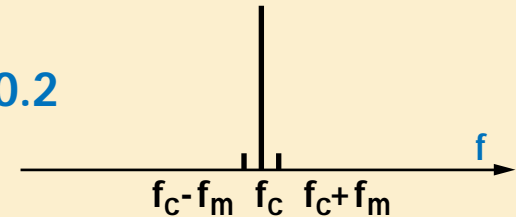
Bandwidth of FM Signals

In practice, the spectrum of an FM signal is not infinite. The sideband amplitudes become negligibly small beyond a certain frequency offset from the carrier, depending on the magnitude of m . We can determine the bandwidth required for low distortion transmission by counting the number of significant sidebands. (Significant sidebands usually means all those sidebands that have a voltage at least 1 percent (-40 dB) of the voltage of the unmodulated carrier.)

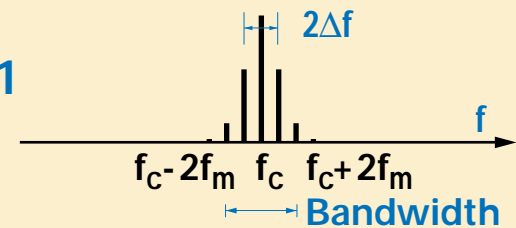
We will now investigate the spectral behavior of an FM signal for different values of m . In Figure 23 we see the spectra of a signal for $m=0.2, 1, 5,$ and 10 . The sinusoidal modulating signal has the constant frequency f_m , so the m is proportional to its amplitude.

Figure 23.
Amplitude-frequency spectrum of an FM signal (with sinusoidal modulating signal, f_m , fixed and amplitude varying).

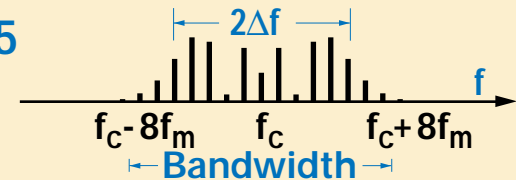
$m = 0.2$



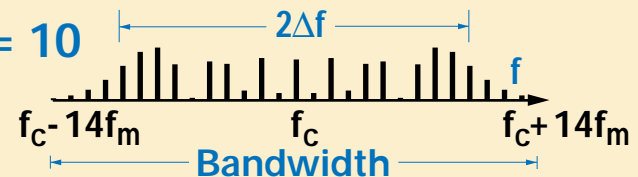
$m = 1$



$m = 5$



$m = 10$



3 Angular Modulation

Bandwidth of FM Signals

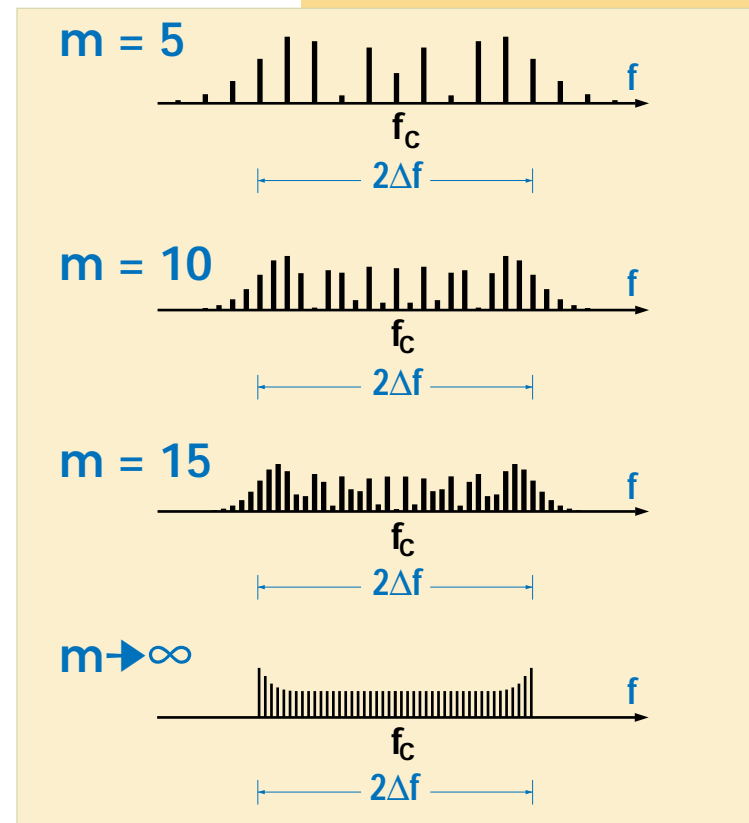
In Figure 24 the amplitude of the modulating signal is held constant and m is varied by changing the modulating frequency. Note that the individual spectral components are shown for $m=5, 10,$ and 15 . For $m \ll 1$, the components are not resolved, but the envelope is correct.

Two important facts emerge from Figures 23 & 24:

(1) For very low modulation indices (m less than 0.2), we get only one significant pair of sidebands. The required transmission bandwidth in this case is twice f_m , as for AM.

(2) For very high modulation indices (m more than 100), the transmission bandwidth is twice Df_p . For values of m between these margins, we have to count the significant sidebands to determine the transmission bandwidth.

Figure 24.
Amplitude-frequency spectrum
of an FM signal (with amplitude
of Df fixed and f_m decreasing).



3 Angular Modulation

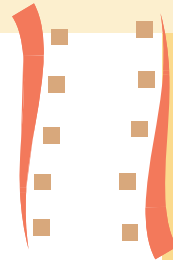
Bandwidth of FM Signals

FM Signal Animation

The main effects of varying the modulation index m , on a frequency modulated signal are visualized in Animation 4. The main points to observe are:

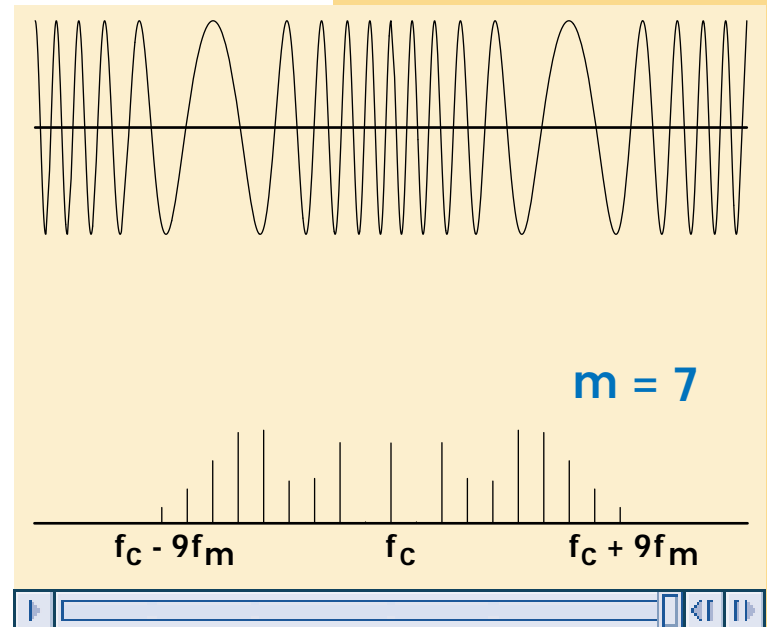
- As m increases, the time domain shows increasing variation in the instantaneous frequency.
- As m increases, higher-order sidebands in the frequency domain become more and more significant.
- At special values of m , various sideband amplitudes become zero.

Live interactive AM and FM signal models that allow you to explore and experience modulation theory are available at the HP World WideWebsite.



Animation 4.

How a frequency modulated signal varies in the time and frequency domain as m is changed from 0 to 7.



3 Angular Modulation

Bandwidth of FM Signals

Figures 25 and 26 show the analyzer displays of two FM signals, one with $m=0.2$, the other with $m=95$.

Figure 27 shows the bandwidth requirements for a low-distortion transmission in relation to m .

For voice communication, a higher degree of distortion can be tolerated; that is, we can ignore all sidebands with less than 10% of the carrier voltage (-20 dB). We can calculate the necessary bandwidth B using the approximation

$$B = 2\Delta f_{\text{peak}} + 2f_m \text{ or } B = 2f_m (1 + m)$$

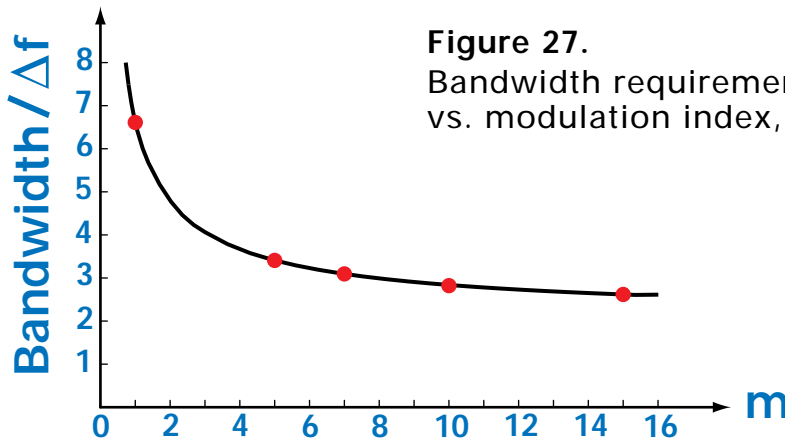
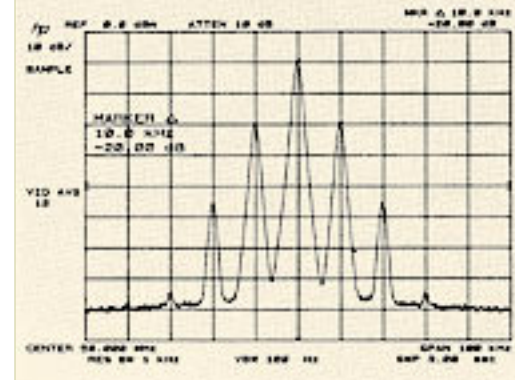


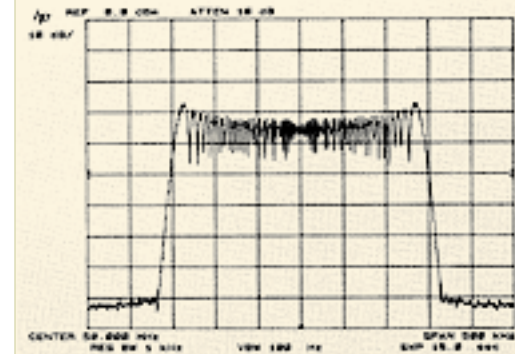
Figure 27.
Bandwidth requirements
vs. modulation index, m .

Figure 25, 26. Sinusoidal
modulating signals.

$f_m = 10 \text{ KHz and } m = 0.2$



$f_m = 1.5 \text{ KHz and } m = 95$



3 Angular Modulation

Bandwidth of FM Signals

So far, our discussion of FM sidebands and bandwidth has been based on having a single sine wave as the modulating signal. Extending the discussion to complex and more realistic modulating signals is difficult. We can, however, look at an example of single-tone modulation for some useful information.

An FM broadcast station has a maximum frequency deviation (determined by the maximum amplitude of the modulating signal) of $Df_{\text{peak}} = 75$ kHz. The highest modulation frequency, f_m , is 15 kHz. This yields a modulation index of $m = 5$, and the resulting signal has eight significant sideband pairs. Thus the required bandwidth can be calculated as $2 \times 8 \times 15$ kHz = 240 kHz. For modulation frequencies below 15 kHz (with the same amplitude assumed), the modulation index increases above 5, with the bandwidth eventually approaching $2 Df_{\text{peak}} = 150$ kHz for very low modulation frequencies.

We can, therefore, calculate the required transmission bandwidth using the highest modulation frequency and the maximum frequency deviation Df_{peak} .

Δf_{peak}

3

Angular Modulation

FM Measurements with the Spectrum Analyzer

The spectrum analyzer is a very useful tool for measuring Df_{peak} and m , and for making fast and accurate adjustments of FM transmitters. It is also frequently used for calibrating frequency deviation meters.

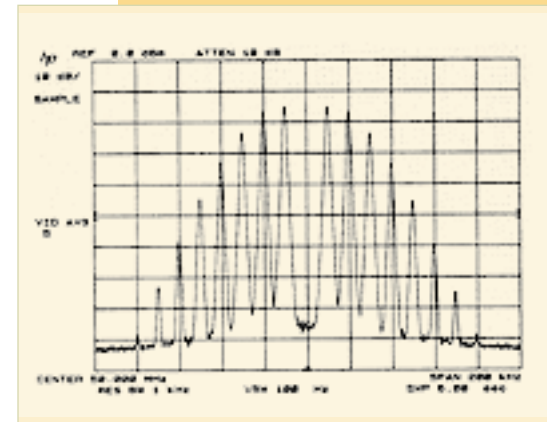
A signal generator or transmitter is adjusted to a precise frequency deviation with the aid of a spectrum analyzer, using one of the carrier zeros and selecting the appropriate modulating frequency. In Figure 28, a modulation frequency of 10 kHz and a modulation index of 2.4 (first carrier null) necessitate a carrier peak frequency deviation of exactly 24 kHz. Since we can accurately set the modulation frequency using the spectrum analyzer or, if need be, a frequency counter, and since the modulation index is also known accurately, the frequency deviation thus generated will be equally accurate.

Figure 28.

This is the spectrum of an FM signal at 50 MHz. The deviation has been adjusted for the first carrier null.

The f_m is 10 kHz;
therefore,

$$Df_{\text{peak}} = 2.4 \times 10 \text{ kHz} \\ = 24 \text{ kHz at } 200 \text{ kHz,} \\ 10 \text{ dB/Div, } B=1 \text{ kHz.}$$



3

Angular Modulation

FM Measurements with the Spectrum Analyzer



Spectrum Analysis 150-1
Amplitude and
Frequency Modulation

Table 2 gives the modulation frequency for common values of deviation for the various orders of carrier zeros.

Table 2.

Order of Carrier Zero	Modulation Index	Commonly Used Values of FM Peak Deviation										
		7.5 kHz	10 kHz	15 kHz	25 kHz	30 kHz	50 kHz	75 kHz	100 kHz	150 kHz	250 kHz	300 kHz
1	2.40	3.12	4.16	6.25	10.42	12.50	20.83	31.25	41.67	62.50	104.17	125.00
2	5.52	1.36	1.18	2.72	4.53	5.43	9.06	13.59	18.12	27.17	45.29	54.35
3	8.65	0.87	1.16	1.73	2.89	3.47	5.78	8.67	11.56	27.34	28.90	34.68
4	11.79	0.66	0.85	1.27	2.12	2.54	4.24	6.36	8.48	12.72	21.20	25.45
5	14.93	0.50	0.67	1.00	1.67	2.01	3.35	5.02	6.70	10.05	16.74	20.09
6	18.07	0.42	0.55	0.83	1.38	1.66	2.77	4.15	5.53	8.30	13.84	16.60

3

Angular Modulation

FM Measurements with the Spectrum Analyzer



Spectrum Analysis 150-1
Amplitude and
Frequency Modulation

The procedure for setting up a known deviation is:

(1) Select the column with the required deviation: for example, 250 kHz.

(2) Select an order of carrier zero that gives a frequency in the table commensurate with the normal modulation bandwidth of the generator to be tested. For example, if 250 kHz was chosen to test an audio modulation circuit, it will be necessary to go to the fifth carrier zero to get a modulating frequency within the audio pass-band of the generator (here, 16.74 kHz).

(3) Set the modulating frequency to 16.74 kHz, and monitor the output spectrum of the generator on the spectrum analyzer. Adjust the amplitude of the audio modulating signal until the carrier amplitude has gone through four zeros; stop when the carrier is at its fifth minimum. With a modulating frequency of 16.74 kHz and the spectrum at its fifth zero, the setup provides a unique 250 kHz deviation. The modulation meter can then be calibrated. Make a quick check by moving to the adjacent carrier zero and resetting the modulating frequency and amplitude (in this case, resetting to 13.84 kHz at the sixth carrier zero).

3

Angular Modulation

FM Measurements with the Spectrum Analyzer



Spectrum Analysis 150-1
Amplitude and
Frequency Modulation

Other intermediate deviations and modulation indexes can be set using different orders of sideband zeros, but these are influenced by incidental amplitude modulation. Since we know that amplitude modulation does not cause the carrier to change, but instead puts all the modulation power into the sidebands, incidental AM will not affect the carrier zero method described on the previous page.

If it is not possible or desirable to alter the modulation frequency to get a carrier or sideband null, there are other ways to obtain usable information about frequency deviation and modulation index. One method is to calculate m by using the amplitude information of five adjacent frequency components in the FM signal. These five measurements are used in a recursion formula for Bessel functions to form three calculated values of a modulation index. Averaging them yields a value for m that takes practical measurement errors into consideration. Because of the number of calculations necessary, this method is feasible only by using a computer. A somewhat easier method consists of the following two measurements.



Milestones.

In 1931 a stereophonic sound system was invented by A.D. Blumlein. It was not until 1982 that A.M. radio stations in the United States began broadcasting in stereo.

3

Angular Modulation

FM Measurements with the Spectrum Analyzer

First, the sideband spacing of the modulated carrier is measured by using a sufficiently small IF bandwidth (BW), to give the modulation frequency f_m . Second, the peak frequency deviation Df_{peak} is measured by selecting a convenient scan width and an IF bandwidth wide enough to cover all significant sidebands. The modulation index m can then be calculated easily.

Note that Figure 29 illustrates the peak-to-peak deviation.

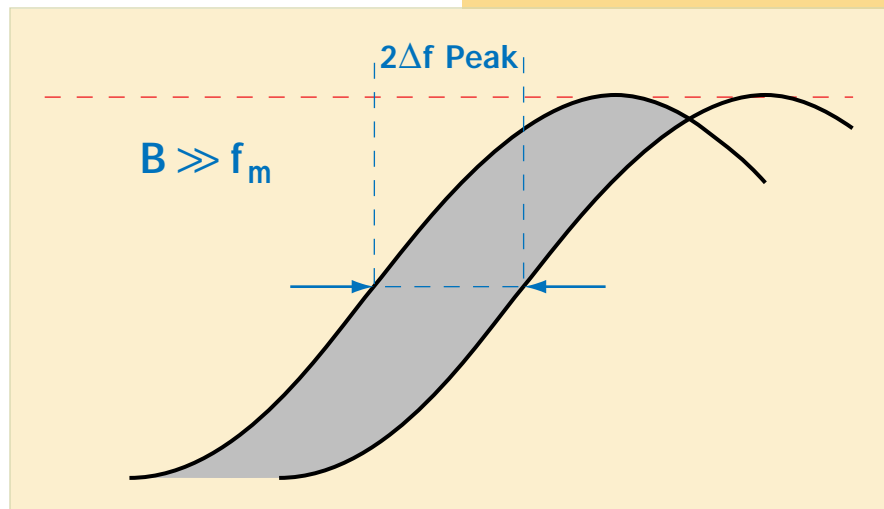
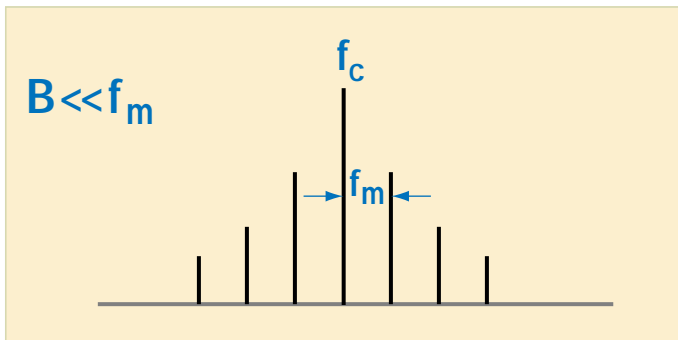


Figure 29.
Measurement of
 f_m and Df_{peak} .

3

Angular Modulation

FM Measurements with the Spectrum Analyzer

The two-step type of measurement is shown in Figure 30.

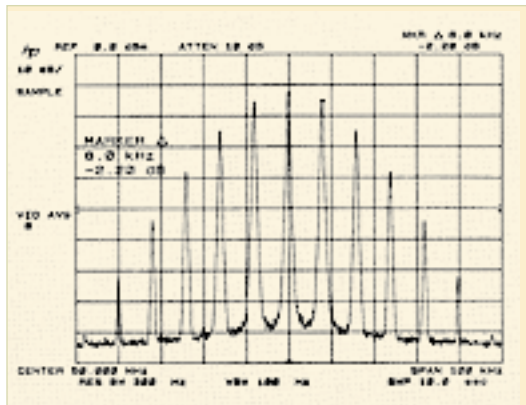


Figure 30A.
A frequency-modulated carrier. Sideband spacing is measured to 8 kHz. 100 kHz span, 10 dB/Div, B=300 Hz.

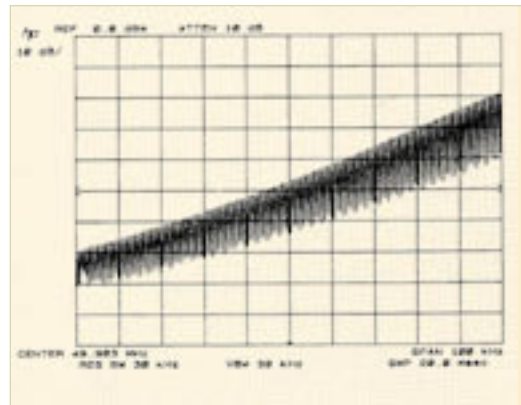


Figure 30B.
The peak-to-peak frequency deviation of the same signal is measured to 20 kHz. 100 Hz span, 10 dB/Div, B=30 kHz.

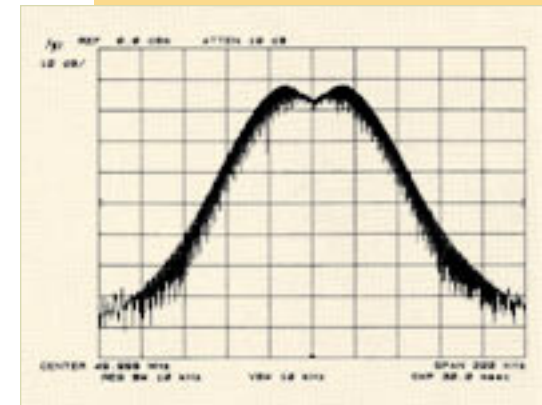


Figure 30C.
Insufficient Bandwidth: B=10 kHz.

3

Angular Modulation

FM Measurements with the Spectrum Analyzer

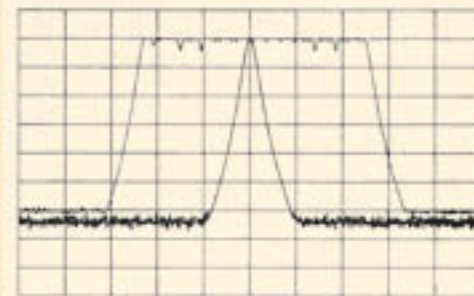
The spectrum analyzer can also be used to monitor FM transmitters (for example, at broadcast or communication stations) for occupied bandwidth. Here the statistical nature of the modulation must be considered. The signal must be monitored for a period of time sufficiently long to ensure that the peak frequency deviations occur. The MAX-HOLD capability available on spectrum analyzers with digitized traces is used to capture the signal, with those peak deviations.

To better keep track of what is happening, you can often take advantage of the fact that most analyzers of this type have two or more trace memories. That is, select the MAX-HOLD mode for one trace while the other trace is live. See Figure 31.

Figure 31.

The MAX-HOLD mode is used to measure the peak frequency

MAX HOLD



3

Angular Modulation

FM Measurements with the Spectrum Analyzer



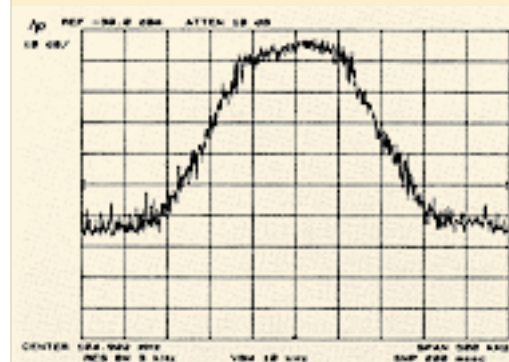
Spectrum Analysis 150-1
Amplitude and
Frequency Modulation

Figure 32 shows an FM broadcast station modulated with stereo multiplex. Note that the spectrum envelope resembles an FM signal with low modulation index. This is because the stereo modulation signal contains additional information in the frequency range of 23 to 53 kHz, far beyond the audio frequency limit of 15 kHz. Since the occupied bandwidth must not exceed the bandwidth of a transmitter modulated with a mono signal, the maximum frequency deviation of the carrier must be kept substantially lower.

Figure 32.

FM broadcast transmitter modulated with a stereo signal. 500 kHz span, 10 dB/Div, B=3 kHz, sweep time 50 ms/Div, approx. 200 sweeps.

Stereo Modulation Signal



3

Angular Modulation

FM Measurements with the Spectrum Analyzer

As with AM, it is possible to recover the modulating signal. The analyzer is used as a manually tuned receiver (zero span) with a wide IF bandwidth. However, in contrast to AM, the signal is not tuned into the passband center, but to one slope of the filter curve, as illustrated in Figure 33.

Here the frequency variations of the FM signal are converted into amplitude variations (FM to AM conversion). The resultant AM signal is then detected with the envelope detector. The detector output is displayed in the time domain and is also available as an audio output, for application to headphones or a speaker, at the video output terminal of the spectrum analyzer.

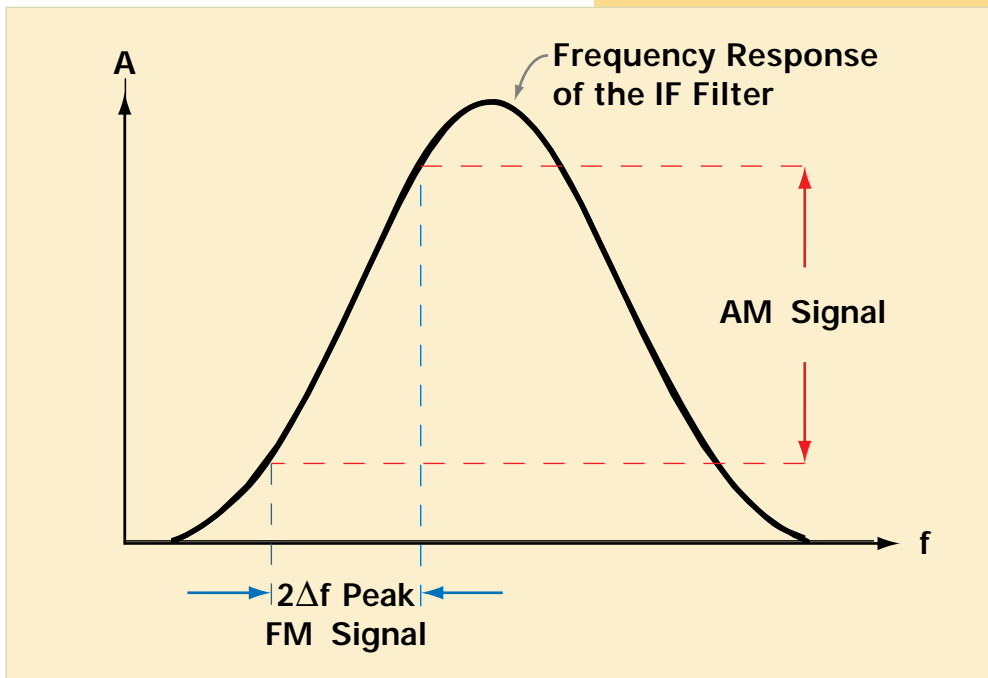


Figure 33.
Slope detection
of an FM signal.

3

Angular Modulation

FM Measurements with the Spectrum Analyzer



Spectrum Analysis 150-1
Amplitude and
Frequency Modulation

A disadvantage of this method is that the detector also responds to amplitude variations of the signal. Many of today's spectrum analyzers include an FM demodulator in addition to the AM demodulator. Therefore, we can again take advantage of the marker pause function to listen to an FM broadcast while in the swept-frequency mode.

We would set the frequency span to cover the desired range (that is, the FM broadcast band); set the active marker on the signal of interest, set the length of the pause (dwell time); and activate the FM demodulator. The analyzer sweeps to the marker and pauses for the set time, allowing us to listen to the signal during that pause interval. After the pause interval, the analyzer continues the sweep. If the marker is the active function, we can move it and listen to any other signal on the display.

3 Angular Modulation

AM Plus FM (Incidental FM)

Although AM and angular modulation are different methods of modulation, they have one property in common: they always produce a symmetrical sideband spectrum.

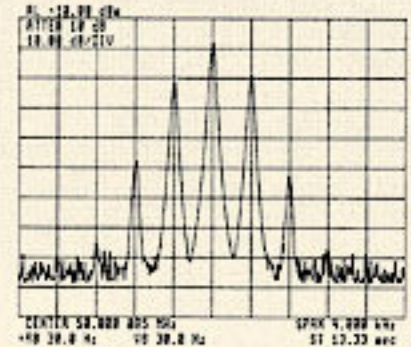
In Figure 34 we see a modulated carrier with asymmetrical sidebands. The only way this could occur is if both AM and FM, or if AM and phase modulation exist simultaneously at the same modulating frequency. The asymmetry indicates that the phase relations between carrier and sidebands are different for the AM and the angular modulation (see Appendix).

Since the sideband components of both modulation types add together vectorally, the resultant amplitude of one sideband might be reduced, while the amplitude of the other would be increased accordingly. The spectrum analyzer does not retain any phase information. So, in each case, it displays the absolute magnitude of the result.

Figure 34.

This CW signal is amplitude-modulated 80% at a 10 kHz rate. The harmonic distortion and incidental FM are clearly visible.

Asymmetrical Sidebands



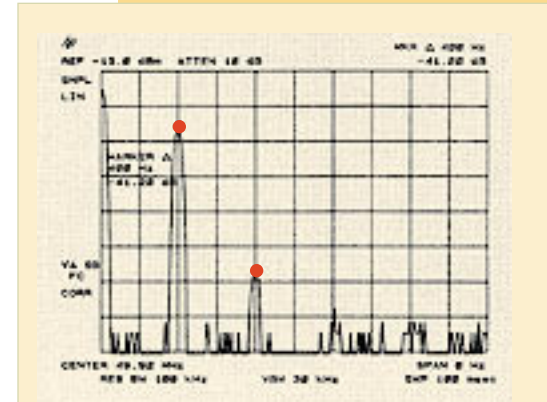
3 Angular Modulation

AM Plus FM (Incidental FM)

Provided that the peak deviation of the incidental FM is small relative to the maximum usable analyzer bandwidth, we can use the FFT capability of the analyzer (see Chapter 2) to remove the FM from the measurement. In contrast to Figure 33, which shows deliberate FM-to-AM conversion, here we tune the analyzer to center the signal in the IF passband. Then we choose a resolution bandwidth wide enough to negate the effect of the incidental FM and pass the AM components unattenuated. Using FFT then gives us just AM and AM-distortion data. Note that the apparent AM distortion in Figure 34 is higher than the true distortion shown in Figure 35.

For relatively low incidental FM, the amount of AM can be calculated with reasonable accuracy by taking the average amplitude of the first sideband pair. The amount of incidental FM can be calculated only if the phase relation between the AM and FM sideband vectors is known. It is not possible to measure $D_{f_{peak}}$ of the incidental FM using the slope detection method because of the simultaneously existing AM.

Figure 35.
True distortion, using
FFT to remove FM
from the measurement.



A Mathematics of Modulation

Amplitude Modulation

A sine wave carrier can be expressed by the general equation

$$e(t) = A \cdot \cos(\omega_c t + f_0) \quad (1-1)$$

In AM systems only A is varied. It is assumed that the modulating signal varies slowly compared to the carrier. This means that we can talk of an envelope variation or variation of the locus of the carrier peaks. The carrier, amplitude-modulated with the function f(t) (carrier angle f₀ arbitrarily set to zero), has the form

$$e(t) = A[1 + m \cdot f(t)] \cdot \cos \omega_c t \quad (m = \text{modulation degree}) \quad (1-2)$$

For f(t) = cos ω_mt (single sine wave) we get

$$e(t) = A \cdot (1 + m \cdot \cos \omega_m t) \cdot \cos \omega_c t \quad (1-3)$$

or

$$e(t) = A \cos \omega_c t + \frac{m \times A}{2} \cos(\omega_c + \omega_m)t + \frac{m \times A}{2} \cos(\omega_c - \omega_m)t \quad (1-4)$$

1530 AM

A e = COS(ωt + f)

A Mathematics of Modulation

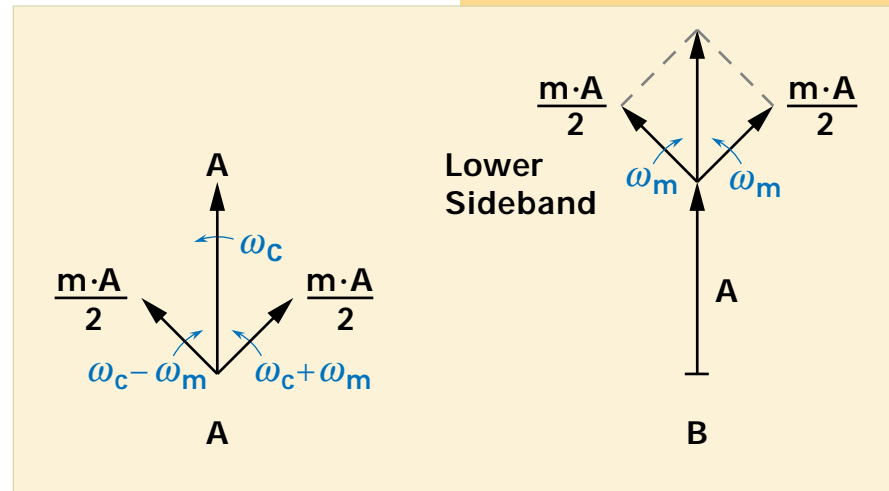
Amplitude Modulation

We get three steady-state components:

- (a) $A \cdot \cos \omega_c t$ Carrier
- (b) $\frac{m \cdot A}{2} \cos(\omega_c + \omega_m) t$ Upper Sideband
- (c) $\frac{m \cdot A}{2} \cos(\omega_c - \omega_m) t$ Lower Sideband

We can represent these components by three phasors rotating at different angular velocities (Figure A-1A). Assuming the carrier phasor A to be stationary, we obtain the angular velocities of the sideband phasors in relation to the carrier phasor (Figure A-1B).

Figure A-1.
Phasor diagram representations of the components of an AM signal.



A Mathematics of Modulation

Amplitude Modulation

We can see that the phase of the vector sum of the sideband phasors is always collinear with the carrier component; that is, their quadrature components always cancel. We can also see from Equation 1-3 and Figure A-1 that the modulation degree m cannot exceed the value of unity for linear modulation.

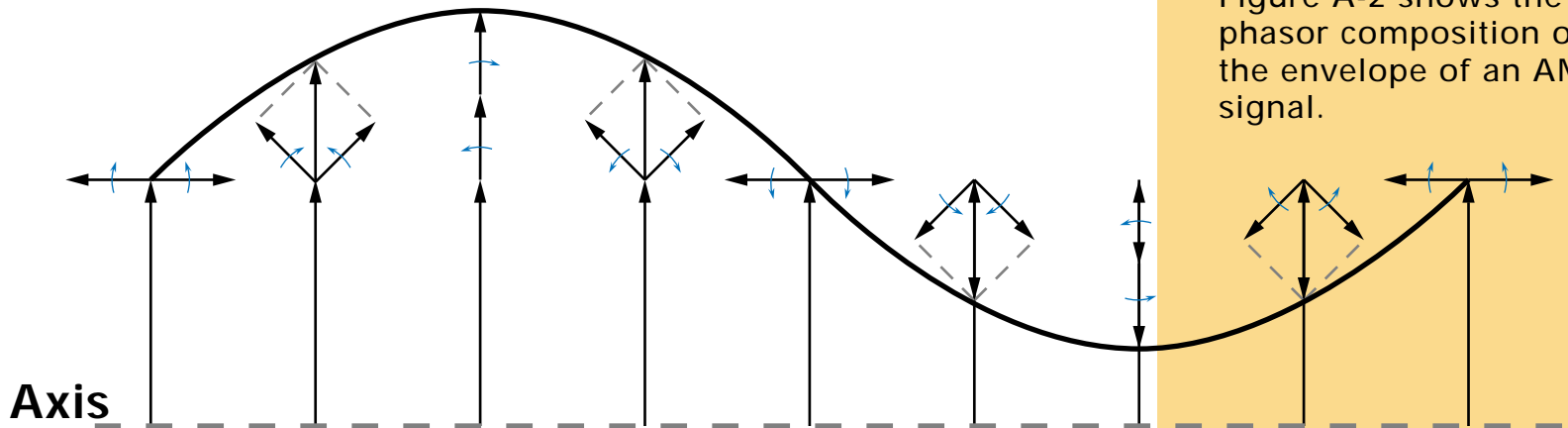


Figure A-2.

Figure A-2 shows the phasor composition of the envelope of an AM signal.

Mathematics of Modulation

Angular and Phase Modulation

The usual expression for a sine wave of angular frequency ω_c is

$$f(t) = \cos \phi(t) = \cos (\omega_c t + \phi_0) \quad (2-1)$$

We define the instantaneous radian frequency ω_i to be the derivative of the angle as a function of time:

$$\omega_i = \frac{d\phi}{dt} \quad (2-2)$$

This instantaneous frequency agrees with the ordinary use of the word frequency if

$$\phi(t) = \omega_c t + \phi_0.$$

If $f(t)$ in Equation 2-1 is made to vary in some manner with a modulating signal $f(t)$ the result is angular modulation.

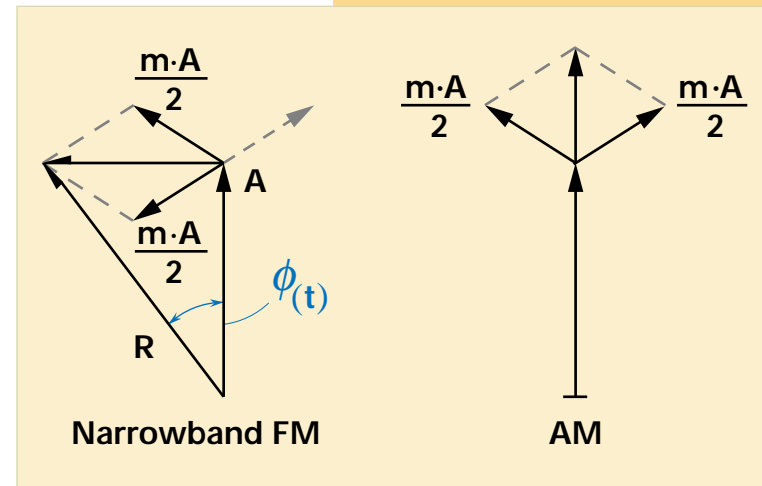
Phase Modulation In particular, when

$$\phi(t) = \omega_c t + \phi_0 + K_1 \cdot f(t) \quad (2-3)$$

we vary the phase of the carrier linearly with the modulation signal. K_1 is a constant of the system.

Figure A-3.

Phase and frequency modulation are both special cases of angular modulation.



Mathematics of Modulation

Frequency Modulation

Now we let the instantaneous frequency, as defined in Equation 2-2, vary linearly with the modulating signal

$$\omega(t) = \omega_c + K_2 \cdot f(t)$$

Then

$$\phi(t) = \int \omega(t) dt = \omega_c t + \phi_0 + K_2 \cdot \int f(t) dt \quad (2-4)$$

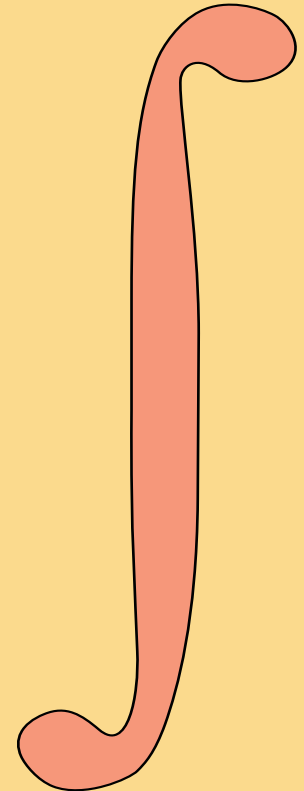
In the case of phase modulation, the phase of the carrier varies with the modulation signal, and in the case of FM the phase of the carrier varies with the integral of the modulating signal. Thus, there is no essential difference between phase and frequency modulation. We shall use the term FM generally to include both modulation types. For further analysis we assume a sinusoidal modulation signal at the frequency f_m :

$$f(t) = a \cdot \cos \omega_m t$$

The instantaneous radian frequency ω_i is

$$\omega_i = \omega_c + \Delta \omega_{\text{peak}} \cdot \cos \omega_m t, \quad \Delta \omega_{\text{peak}} \ll \omega_c \quad (2-5)$$

$\Delta \omega_{\text{peak}}$ is a constant depending on the amplitude, a , of the modulating signal and on the properties of the modulating system.



A Mathematics of Modulation

Frequency Modulation

The phase $\phi(t)$ is then

$$\phi(t) = \int \omega_i dt = \omega_c t + \frac{\Delta\omega_{\text{peak}}}{\omega_m} \sin \omega_m t + \phi_0$$

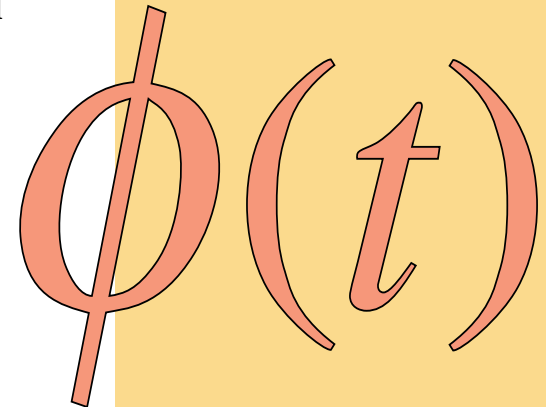
We can take ϕ_0 as zero by referring to an appropriate phase reference. The frequency modulated carrier is then expressed by

$$e(t) = A \cdot \cos(\omega_c t + m \cdot \sin \omega_m t) \quad (2-6)$$

For

$$m = \frac{\Delta\omega_{\text{peak}}}{\omega_m} = \frac{\Delta f_{\text{peak}}}{f_m} \quad (2-7)$$

In frequency modulation, m is the modulation index and represents the maximum phase shift of the carrier; Δf_{peak} is the maximum frequency deviation of the carrier.



A Mathematics of Modulation

Narrowband FM

To simplify the analysis of FM, we first assume that $m \ll \frac{\pi}{2}$ (usually $m < 0.2$). We have

$$e(t) = A \cos(\omega_c t + m \sin \omega_m t)$$

$$= A \left[\cos \omega_c t \cos(m \sin \omega_m t) - \sin \omega_c t \sin(m \sin \omega_m t) \right]$$

for $m \ll \frac{\pi}{2}$

$$\cos(m \cdot \sin \omega_m t) = 1 \text{ and } \sin(m \cdot \sin \omega_m t) = m \cdot \sin \omega_m t,$$

thus

$$e(t) = A \cdot (\cos \omega_c t - m \cdot \sin \omega_m t \cdot \sin \omega_c t) \quad (2-8)$$

Written in sideband form

$$e(t) = A \cos \omega_c t + \frac{m \times A}{2} \cos(\omega_c + \omega_m)t - \frac{m \times A}{2} \cos(\omega_c - \omega_m)t. \quad (2-9)$$

$$m \ll \frac{\pi}{2}$$

A Mathematics of Modulation

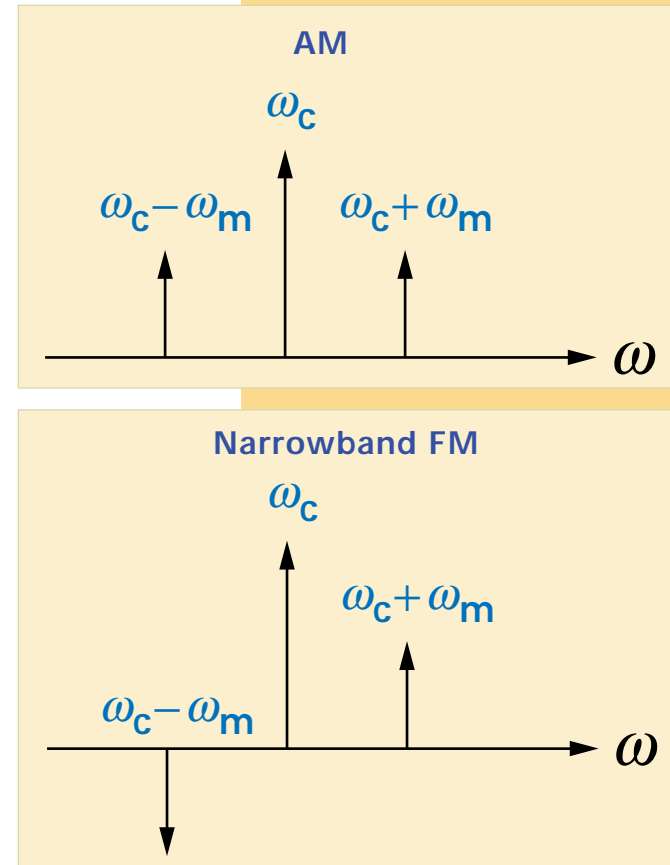
Narrowband FM

This resembles the AM case in Equation 1-4, except that in narrowband FM the phase of the lower sideband is reversed and the resultant sideband vector sum is always in phase quadrature with the carrier.

FM thus gives rise to phase variations with very small amplitude change ($m \ll \frac{\pi}{2}$), while AM gives amplitude variations with no phase deviation.

Figure A-4 shows the spectra of an AM and a narrowband FM signal. However, on a spectrum analyzer the FM sidebands appear as they do in AM because the analyzer does not retain phase information.

Figure A-4.
Signal spectra.



A Mathematics of Modulation

Wideband FM

$$\begin{aligned}
 e(t) &= A \cdot \cos(\omega_c t + m \cdot \sin \omega_m t) \quad \text{For } m \text{ not small} \\
 &= A \cdot [\cos \omega_c t \cdot \cos(m \cdot \sin \omega_m t) - \sin \omega_c t \cdot \sin(m \cdot \sin \omega_m t)] \\
 \cos(m \cdot \sin \omega_m t) &= \\
 J_0(m) + 2J_2(m) \cos 2\omega_m t + 2J_4(m) \cos 4\omega_m t + \dots
 \end{aligned} \tag{2-10}$$

$$\begin{aligned}
 \sin(m \cdot \sin \omega_m t) &= 2J_1(m) \cdot \sin \omega_m t \\
 &\quad + 2J_3(m) \cdot \sin 3\omega_m t + \dots
 \end{aligned} \tag{2-11}$$

When $J_n(m)$ is the n^{th} -order Bessel function of the first kind, we get

$$\begin{aligned}
 e(t) &= \\
 J_0(m) \cos \omega_c t &- J_1(m) [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] \\
 + J_2(m) [\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] \\
 - J_3(m) [\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t] \\
 + \dots
 \end{aligned} \tag{2-12}$$

e(t)

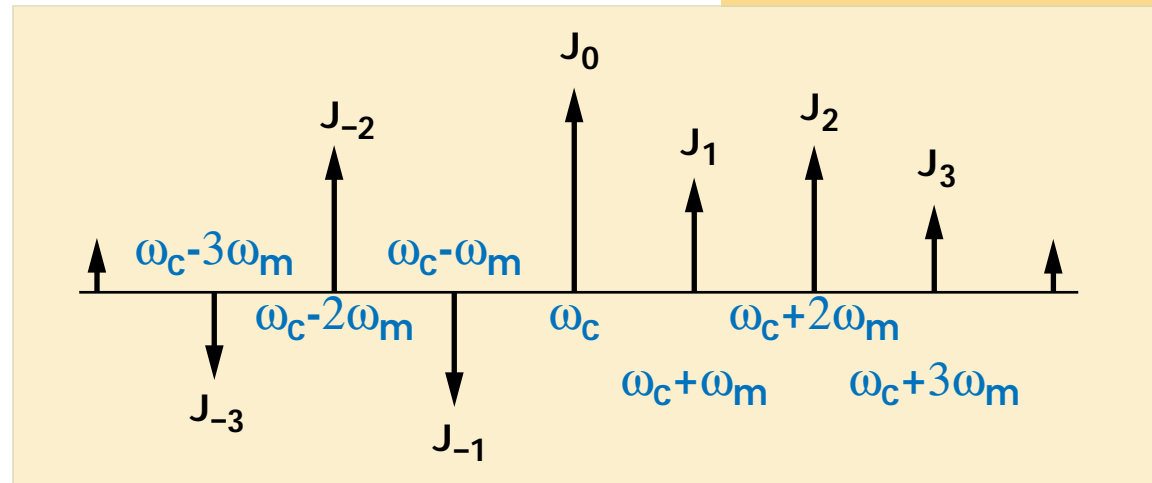
A Mathematics of Modulation

Wideband FM

We thus have a time function consisting of a carrier and an infinite number of sidebands whose amplitudes are proportional to $J_n(m)$. We can see that

- (a) The vector sums of the odd-order sideband pairs are always in quadrature with the carrier component;
- (b) The vector sums of the even-order sideband pairs are always collinear with the carrier component.

Figure A-5.
Composition of an FM
wave into sidebands.



Mathematics of Modulation

Phasor Diagrams

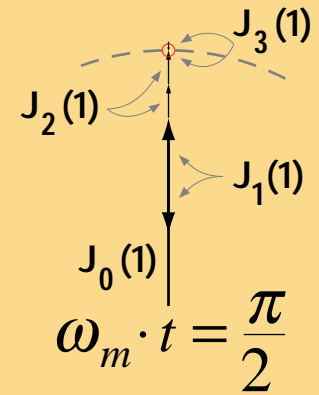
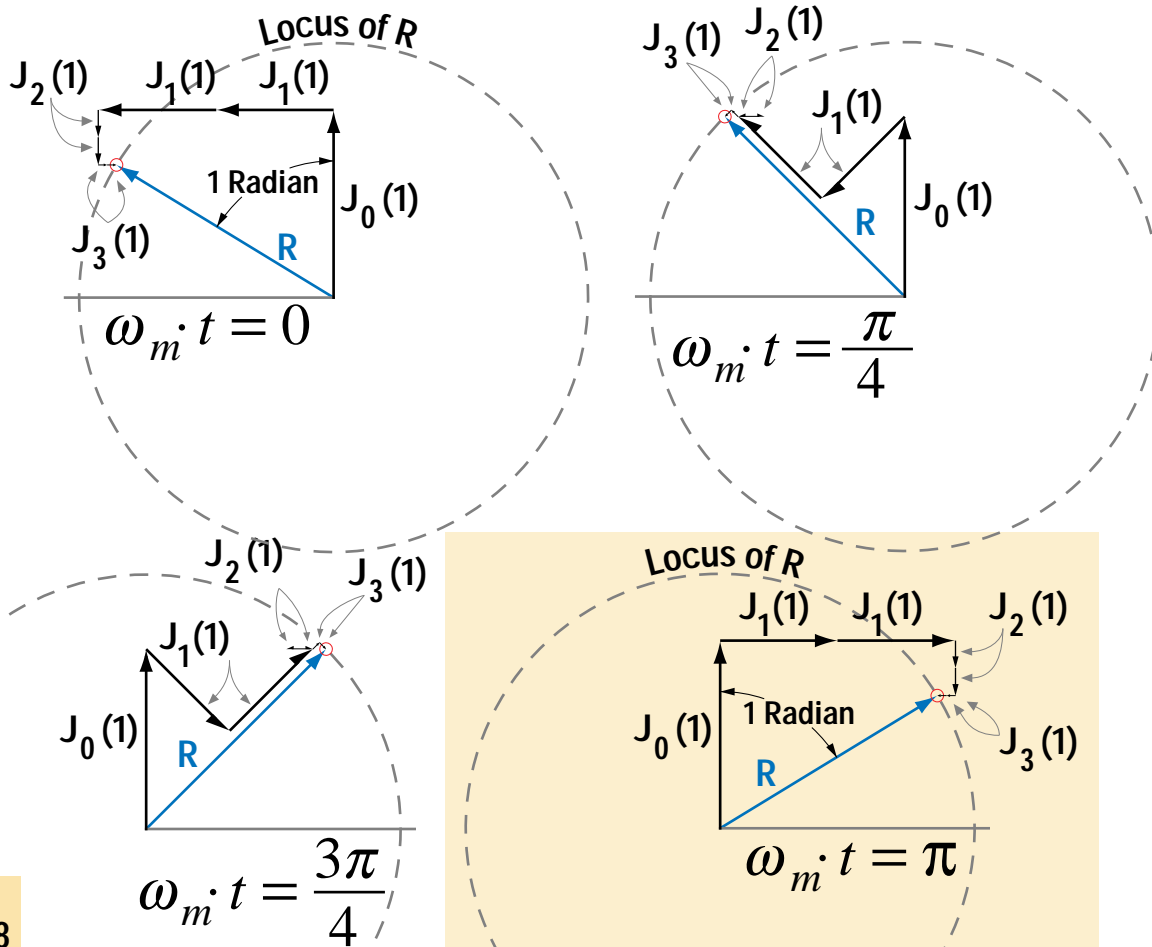


Figure A-6. Phasor diagrams of an FM signal with a modulation index $m=1$. Different diagrams correspond to different points in the cycle of the sinusoidal modulating wave.

You can zoom in for greater detail using the magnifying glass found in the toolbar.

Broadcast Radio

Frequency Modulation in AM/FM Radio

In AM and FM commercial radio the modulating signal is not just a simple sinusoid as discussed in the previous chapters, but a more complex audio signal such as speech or music. Each radio station modulates its specific carrier by an audio signal producing a modulated signal that occupies a small band of frequencies centered about the station's carrier frequency. As shown in Figure B1, the signal received by a radio consists of the signals sent from all stations, whose signals are spaced far enough apart to prevent overlap.

In the United States, AM radio stations are spaced 20 kHz apart and FM radio stations are spaced 200 kHz apart. Tuning of the radio dial selects one of the small frequency bands of a given station. A demodulator in the radio extracts the modulating audio signal from the received signal. FM radio has a number of performance advantages over AM radio, including better power efficiency and noise rejection, but FM radio provides these advantages at the expense of using a larger channel bandwidth.

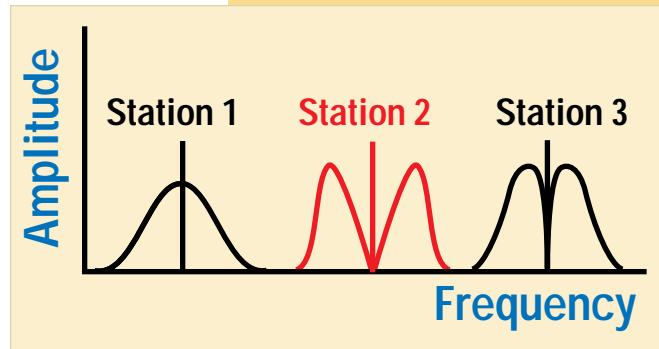


Figure B1.
A radio receives signals from all stations within its range. Stations are separated in frequency to prevent their signals from overlapping. Tuning the radio to a specific station selects the narrow band of frequencies transmitted by that station.



Relevant Products

HP 8648A Synthesized Signal Generator

The HP 8648A synthesized RF signal generator is ideal for manufacturing high-volume products such as cordless phones, pagers and two-way radios.

Designed for semi-automated receiver test and a variety of general purpose applications, the HP 8648A adds enhanced residual FM, improved level accuracy, and superior phase noise performance to an extensive suite of basic capabilities. The affordable HP 8648A is built to stringent quality standards, and offers an all-electronic attenuator to promote measurement consistency and repeatability. The simplified design of its front panel shortens the user's learning curve and increases productivity. With 300 storage registers and ten user-definable sequences (accessible from a remote keypad), the HP 8648A easily adapts to any test procedure.

Thousands of up-to-date Test & Measurement product datasheets may be found at the HP World Wide Web.



Product Features

- Economical synthesized signal generator
- Electronic attenuator
- Superior level accuracy
- Wide frequency and power coverage
- Remote and memory interfaces for semi-automated testing
- Simple, dependable operation

<http://www.hp.com/go/tmdatasheets>

Relevant Products

HP 71100C RF Spectrum Analyzer

The HP 71100C RF spectrum analyzer operates from 100 Hz to 2.9 GHz.

This analyzer uses frequency synthesis to offer very precise, high-speed tuning, with excellent frequency accuracy, amplitude accuracy, and repeatability.

Because the HP 71100C is part of the Modular Measurement System (MMS) family, you can choose a configuration that best meets your performance, size, and price requirements. Accessories such as preamplifiers or tracking generators are easily added, for enhanced versatility.

The HP 70000 series of spectrum analyzers is ideal for automated testing. Their proven reliability and repeatability allow a three-year calibration cycle, giving you a very low lifetime cost.

Thousands of up-to-date Test & Measurement product datasheets may be found at the HP World Wide Web.



Product Features

- 100 Hz to 2.9 GHz
- 134 dBm to -131 dBm sensitivity; -156 dBm with optional preamp
- HP 8566B programming language compatible
- Three-year calibration cycle

<http://www.hp.com/go/tmdatashets>

Relevant Products

HP 33120A 15MHz Function/Arbitrary Waveform Generator

Get custom waveform generation in a function generator at an affordable price that fits many limited budgets. Now you can produce the exact test waveforms you need.

The HP 33120A function/arb generator delivers more than a half-dozen standard waveforms, plus any arbitrary waveform you can dream up. Create signals that mimic noise, vibration, control pulses – whatever it takes to test your circuits with realistic signals. (Using the HP BenchLink software, you can even capture signals with an oscilloscope, then modify and play them back through the HP 33120A.) And the flexibility this signal source provides is backed up with great performance capabilities.



Product Features

- 15 MHz Sine and Square Waves
- Wide selection of built-in waveforms
- Linear and log sweeps
- Four built-in modulation methods
- Phase lock option

Thousands of up-to-date Test & Measurement product datasheets may be found at the HP World Wide Web.

<http://www.hp.com/go/tmdatasheets>

Relevant Products

HP 8591E 1.8 GHz Portable Spectrum Analyzer

The HP 8591E RF spectrum analyzer is easy to use.

The HP 8591E is an easy-to-use RF spectrum analyzer that offers a wide range of performance levels, features, and optional capabilities to meet your measurement needs. Down-loadable measurement personalities combine with plug-in circuit-card options to provide superior test solutions that you can tailor to the specific needs of your application.

If your needs call for basic spectrum analysis, you might consider the HP 8590L, or, for higher performance, the HP 8560E.

Thousands of up-to-date Test & Measurement product datasheets may be found at the HP World Wide Web.



Product Features

- Measurement personalities
- Four-slot cardcage
- Split-screen display
- One-button measurement routines
- Advanced measurement functions
- Dual interfaces
- Tracking generator

<http://www.hp.com/go/tmdatasheets>

Relevant Services

RF and microwave education and training

HP offers an extensive curriculum of education services at locations worldwide. To help design engineers learn how to best use their tools, we provide training courses taught by experts. The HP training program focuses on how to make accurate measurements using HP instruments and systems, including spectrum and network analyzers, power meters, vector signal analyzers, and phase noise systems.



HP education classes are scheduled regularly. The curriculum can be tailored to a company's special needs, and training sessions can be conducted at its site.

Visit HP's World Wide Website for the complete and continuously updated list of Test & Measurement class schedules and locations around the world. You can register directly online! Click on the URL below or type the address in your browser.



<http://www.hp.com/go/tmeducation>

Worldwide Contacts

Hewlett-Packard Test & Measurement Offices

Asia-Pacific

Hewlett-Packard Asia Pacific Ltd.
Hewlett-Packard Hong Kong Ltd.
17-21/F Shell Tower, Times Square
1 Matheson Street, Causeway Bay
Hong Kong

85 2 599 7777

For Japan: **0120-421-345**

Australia/New Zealand

Hewlett-Packard Australia Ltd.
31-41 Joseph Street
Blackburn, Victoria 3130
Australia

1 800 627 485

Canada

Hewlett-Packard Ltd.
5150 Spectrum Way
Mississauga, Ontario L4W 5G1
905-206-4725

Europe/Middle East/Africa

Hewlett-Packard S.A.
150, Route du Nant-d'Avril
1217 Meyrin 1
Geneva, Switzerland

For Europe:
41 22 780 81 11

For Middle East/Africa:
41 22 780 41 11

Latin America

Latin American Region
Headquarters
Hewlett-Packard Company
Waterford Building, Suite 950
5200 Blue Lagoon
Miami, FL 33126
305-267-4245

United States of America

Hewlett-Packard Company
800-452-4844

Visit HP's World Wide Website for the complete and up to date listing of Test & Measurement sales offices addresses and call centers phone numbers worldwide.

<http://www.hp.com/go/tmdir>



Copyrights and Credits

Authors, contributors and producers

Authors & Contributors

Blake Peterson – 1989 revision and update; original application note by HP technical staff, 1971.

Lee Smith – 1996 revised & updated content, technical illustrations, mathematical programming, and animations.

Jeff Gruszynski – expert domain consultant, content perspectives.

Walt Patstone – technical content review & proof-reading.

Graphic & Interaction Design

Kathy Cunningham – Quark guru and page layout design.

Christina Bangle – free-style illustration and graphic reviews.

Ev Shafrir – original concept, art direction, interaction design, digital publishing & production, content supervision, overall direction, & project management.

Business Manager

Nyna Casey – funding, support, excitement, and enthusiastic encouragement of innovation in interactive publishing.

© Copyright Hewlett-Packard Company 1989, 1996 All Rights Reserved. Adaptation, reproduction, translation, extraction, dissemination or disassembly without prior written permission is prohibited except as allowed under the copyright laws. *Spectrum Analysis Amplitude and Frequency Modulation* is electronically published as part of the HP Test & Measurement Digital Application Note Library for the World Wide Web, April 1996. Original printed publication Number 5954-9130.

