TABLE 3.1 THE $Q(x)$ AND erfc (y) FUNCTIONS

Definition: $Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$

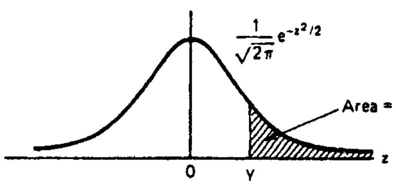
(1) $P(X > \mu_x + y\sigma_x) = Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$

(2) $Q(0) = \frac{1}{2}$; $Q(-y) = 1 - Q(y)$, when $y \geq 0$

(3) $Q(y) \approx \frac{1}{y\sqrt{2\pi}} e^{-y^2/2}$ when $y \gg 1$ (approximation may be used for $y > 4$)

(4) $\text{erfc}^*(y) \triangleq \frac{2}{\sqrt{\pi}} \int_y^\infty e^{-z^2}\,dz = 2Q(\sqrt{2}y), y > 0$

(5) $\text{erfc}(y) = 1 - \text{erf}(y)$



y	$Q(y)$	y	$Q(y)$	y	$Q(y)$	$Q(y)$	y
0.05	0.4801	1.05	0.1469	2.10	0.0179		
0.10	0.4602	1.10	0.1357	2.20	0.0139		
0.15	0.4405	1.15	0.1251	2.30	0.0107		
0.20	0.4207	1.20	0.1151	2.40	0.0082	10^{-3}	3.10
0.25	0.4013	1.25	0.0156	2.50	0.0062		
0.30	0.3821	1.30	0.0968	2.60	0.0047		
0.35	0.3632	1.35	0.0885	2.70	0.0035		
0.40	0.3446	1.40	0.0808	2.80	0.0026	$\frac{10^{-3}}{2}$	3.28
0.45	0.3264	1.45	0.0735	2.90	0.0019		
0.50	0.3085	1.50	0.0668	3.00	0.0013		
0.55	0.2912	1.55	0.0606	3.10	0.0010	10^{-4}	3.70
0.60	0.2743	1.60	0.0548	3.20	0.00069		
0.65	0.2578	1.65	0.0495	3.30	0.00048		
0.70	0.2420	1.70	0.0446	3.40	0.00034		
0.75	0.2266	1.75	0.0401	3.50	0.00023		
0.80	0.2119	1.80	0.0359	3.60	0.00016	$\frac{10^{-4}}{2}$	3.90
0.85	0.1977	1.85	0.0322	3.70	0.00010		
0.90	0.1841	1.90	0.0287	3.80	0.00007	10^{-5}	4.27
0.95	0.1711	1.95	0.0256	3.90	0.00005		
1.00	0.1587	2.00	0.0228	4.00	0.00003	10^{-6}	4.78

* In some references the error function is somewhat differently defined.

Source: [Shanmugam, 1979], with permission from John Wiley & Sons, Inc.

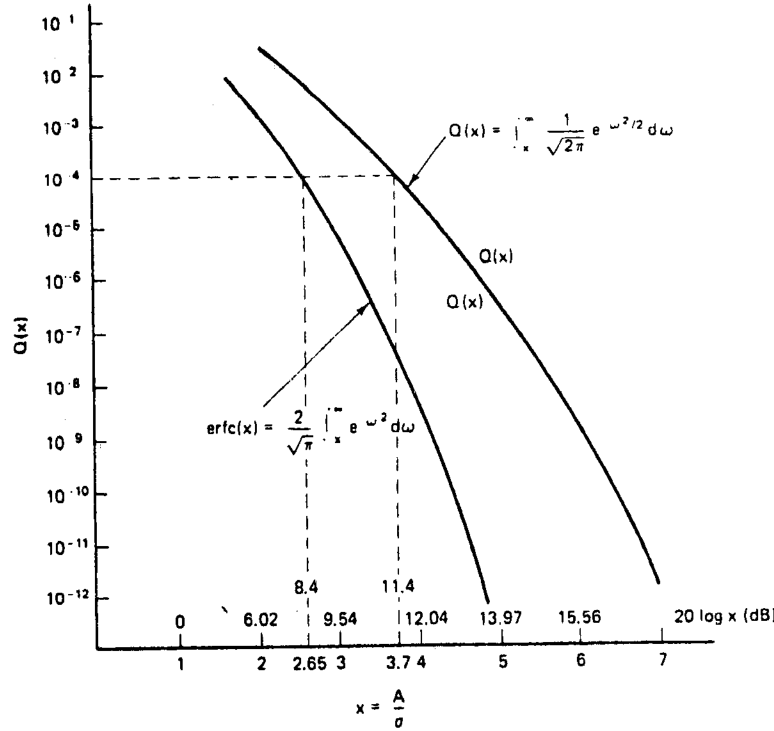


Figure 3.25 $P_e = f(S/N)$ of binary baseband signals. A, peak signal voltage; σ, rms noise voltage at threshold comparator input.

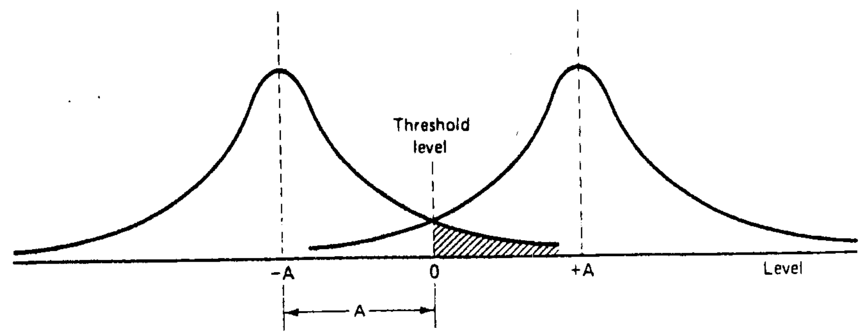
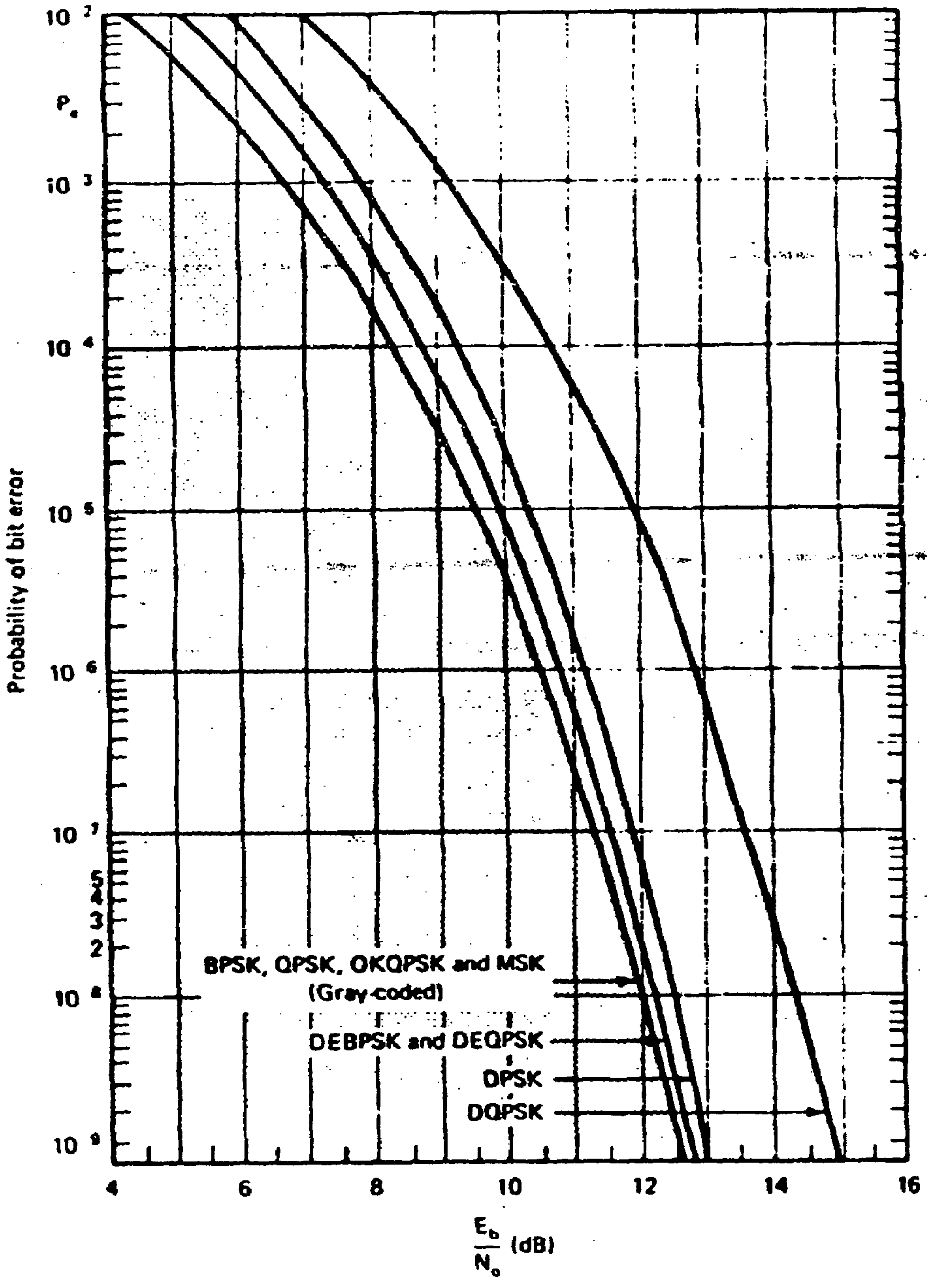


Figure 3.26 Probability of error distribution—baseband systems. A more general derivation of optimum receivers is given in Section 4.4 (see Figs. 4.9 and 4.10). Shaded area represents P_e surface for a transmitted −1 state. If there were no noise on the line, the received signal sample would equal $-A$ volts. To make an error, the noise has to be sufficiently large to change the polarity of the signal; that is, it has to be positive and larger than A.


Probability of bit error
P_e
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
5
4
3
2
10^{-8}
10^{-9}
BPSK, QPSK, OKQPSK and MSK (Gray-coded)
DEBPSK and DEQPSK
DPSK
DQPSK
4
6
8
10
12
14
16
$\frac{E_b}{N_o}$ (dB)