```
> with(student):with(linalg):with(plottools):restart:
Warning, the protected names norm and trace have been redefined and unprotected
```

ECE540 - W04 - TakeHome Exam I

Problem 1, Part A: The solid angle of an antenna is calculated via equation 2-4-5. The normalized field is proportional to sin(phi) so the normalized power is equal to sin(phi)^2 and the solid angle is calculated as

```
> E:=sin(phi);Solid_angle_antenna:=int(int(E^2*sin(phi),phi=0..Pi),theta=0..2*Pi);evalf(%);
```

$$E := \sin(\phi)$$

$$Solid\_angle\_antenna := \frac{8\,\pi}{3}$$

$$8.377580412$$

The directivity is calculated from equation 2-7-4 as

```
> directivity_antenna:=4*Pi/Solid_angle_antenna;
```

$$directivity\_antenna := \frac{3}{2}$$

Part B: The 4 antennas form an vertical array; assume high enough above the earth to neglect images. So array theory can be used to calculate the normalized AF according to equation 5-6-9 as

```
> f:=1e8;lambda:=3e8/f;d:=1.5;k:=2*Pi/lambda;psi:=k*d*cos(phi);AF:=1/4*abs(sin(4*psi/2)/sin(psi/2));
```

$$f := 0.1\;10^{9}$$

$$\lambda := 3.$$

$$d := 1.5$$

$$k := 0.6666666666\,\pi$$

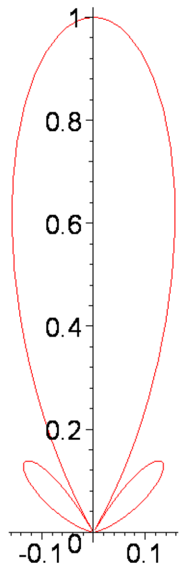
$$\psi := 0.9999999999\,\pi\cos(\phi)$$

$$AF := \frac{1}{4}\left|\frac{\sin(2.000000000\,\pi\cos(\phi))}{\sin(0.5000000000\,\pi\cos(\phi))}\right|$$

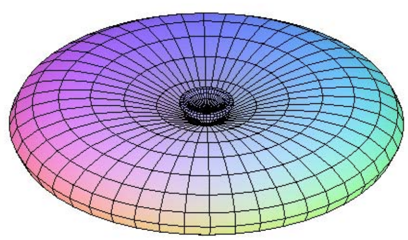
```
> power_pattern_array:=E*AF;
```

$$power\_pattern\_array := \frac{1}{4}\sin(\phi)\left|\frac{\sin(2.000000000\,\pi\cos(\phi))}{\sin(0.5000000000\,\pi\cos(\phi))}\right|$$

```
> plot(power_pattern_array,phi=0..Pi,coords=polar,scaling=CONSTRAINED);
```




```
> plot3d(power_pattern_array,theta=0..2*Pi,phi=0..Pi,coords=spherical,grid=[49,49],scaling=constrained);
```



Part c: The solid angle and directivity of the array are calculated by the method used for the single antenna of part a, but with the array power pattern described in part b.

```
> Solid_angle_array:=int(int(power_pattern_array*sin(phi),phi=0.01..Pi-0.01),theta=0..2*Pi);directivity_array:=4*Pi/Solid_angle_array;evalf(%);
```

$$\mathit{Solid\_angle\_array} := 4.400670309$$

$$\mathit{directivity\_array} := 0.9089524364\,\pi$$

$$2.855558297$$

Problem 2: Problem 13-6-3 of textbook The three antenna system can be characterized by a 3-port network that is represented as a 3x3 matrix where Ra=Rb=Rc=100 and Rab=Rbc=40 while Rac=-10. Due to reciprocity Rab=Rba=Rbc=Rcb and Rca=Rac. In addition, Ia=Ic=-Ib. This leads to

```
> Ra:=100:Rb:=100:Rc:=100:Rab:=40:Rba:=40:Rbc:=40:Rcb:=40:Rac:=-10:Rca:=-10:Ic:=Ia:Ib:=-Ia:
Rain:=(Ra*Ia+Rab*Ib+Rac*Ic)/Ia;Rbin:=(Rba*Ia+Rb*Ib+Rbc*Ic)/Ib;Rcin:=(Rca*Ia+Rcb*Ib+Rc*Ic)/Ic;
```

$$Rain := 50$$

$$Rbin := 20$$

$$Rcin := 50$$

Problem 3: Problem 13-10-1 of textbook. The Z-parameter form of a network is represented by a T of Z11-Zm, Zm, and Z22-Zm.

Part a: The Thevenin impedance seen at port 2 is the series combination of Zthev=Z22-Zm+(Z11-Zm)||Zm while the open circuit voltage at port 2 is given by V1*Zm/(Z11-Zm+Zm) or

```
> Zthev:=expand(((Z22-Zm)*(Z11-Zm+Zm)+(Z11-Zm)*Zm)/(Z11-Zm+Zm));Vthev:=V1*Zm/(Z11-Zm+Zm);
```

$$Zthev := Z22 - \frac{Zm^2}{Z11}$$

$$Vthev := \frac{V1\ Zm}{Z11}$$

Part b: Maximum power transfer occurs when the load is the complex conjugate of the Thevenin impedance seen at those terminals.

```
> ZLopt:=conjugate(Zthev);
```

$$ZLopt := \overline{\left(Z22 - \frac{Zm^2}{Z11}\right)}$$

Problem 4: Part a: An antenna parallel to and a distance d from a ground plane produces a image with a parallel and oppositely directed current located a distance -d from the ground plane. This image is used by replacing the ground plane with the image; the total field is the sum of the actual antenna and the image. The self impedance of a resonant half-wave dipole is given in Figure 13-3 with d=0 as Z11=73+j42.5 ohms; the mutual impedance of the image antenna located a distance of 2*lambda/4=lambda/2 is given as Z12=-12-j28 ohms. Zin=V1/I1=Z11*I1+Z12*I2 or

```
> Z11:=73+I*42.5:Z12:=-12-I*28:I2:=-I1:Zin:=(Z11*I1+Z12*I2)/I1;
```

$$Zin := 85. + 70.5\ I$$

Part b: The antenna and it's image form a 2-element array with the progressive phase shift of Pi. Using equations 5-6-2 and 5-6-9 to calculate the normalized AF and the normalized pattern of the half-wave dipole equation 6-4-4. So the power pattern is calculated as

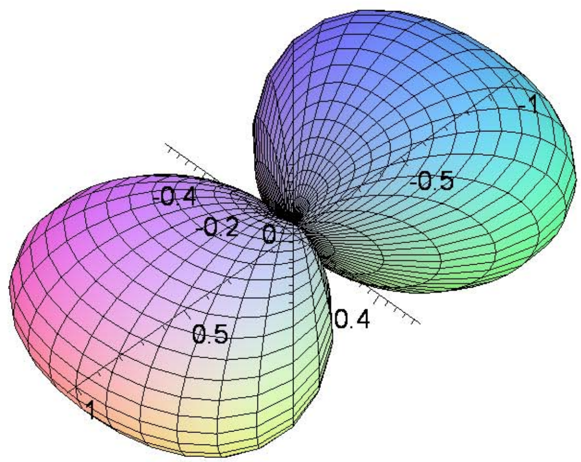
```
> Eant4:=cos(Pi/2*cos(phi))/sin(phi):delta:=Pi:d:=lambda4/2:psi:=2*Pi/lambda4*d*cos(theta)+delta:AF4:=1/2*sin(2*psi/2)/sin(psi/2);Etot:=Eant4*AF4;power_pattern_array4:=abs(Etot)^2;
```

$$AF4 := -\frac{1}{2}\frac{\sin(\pi\cos(\theta))}{\cos\left(\frac{1}{2}\pi\cos(\theta)\right)}$$

$$Etot := -\frac{1}{2}\frac{\cos\left(\frac{1}{2}\pi\cos(\phi)\right)\sin(\pi\cos(\theta))}{\sin(\phi)\cos\left(\frac{1}{2}\pi\cos(\theta)\right)}$$

$$power\_pattern\_array4 := \frac{1}{4}\left|\frac{\cos\left(\frac{1}{2}\pi\cos(\phi)\right)\sin(\pi\cos(\theta))}{\sin(\phi)\cos\left(\frac{1}{2}\pi\cos(\theta)\right)}\right|^2$$

```
> plot3d(power_pattern_array4,theta=0..2*Pi,phi=0..Pi,coords=spheric
  al,grid=[49,49],scaling=constrained,axes=normal);
>
```


-1
-0.5
-0.4
-0.2
0
0.4
0.5
1

>