ECE540 - Antenna Engineering - HW2W04

```
> restart:with(student):
```

Problem 1: (5-2-2 of textbook) - **Part A**: The element factor of a point source is 1, so all we need to find is the array factor. Assuming that we are in the far-field of the array we can write the array factor as

```
> AF1:=A1*exp(I*k*d/2*sin(phi))+A2*exp(-I*k*d/2*sin(phi));
```

$$AF1 := A1\, \mathbf{e}^{(1/2\, I\, k\, d\, \sin(\phi))} + A2\, \mathbf{e}^{(-1/2\, I\, k\, d\, \sin(\phi))}$$

We can substitute values for A1, A2,and d to calculate the array factor as

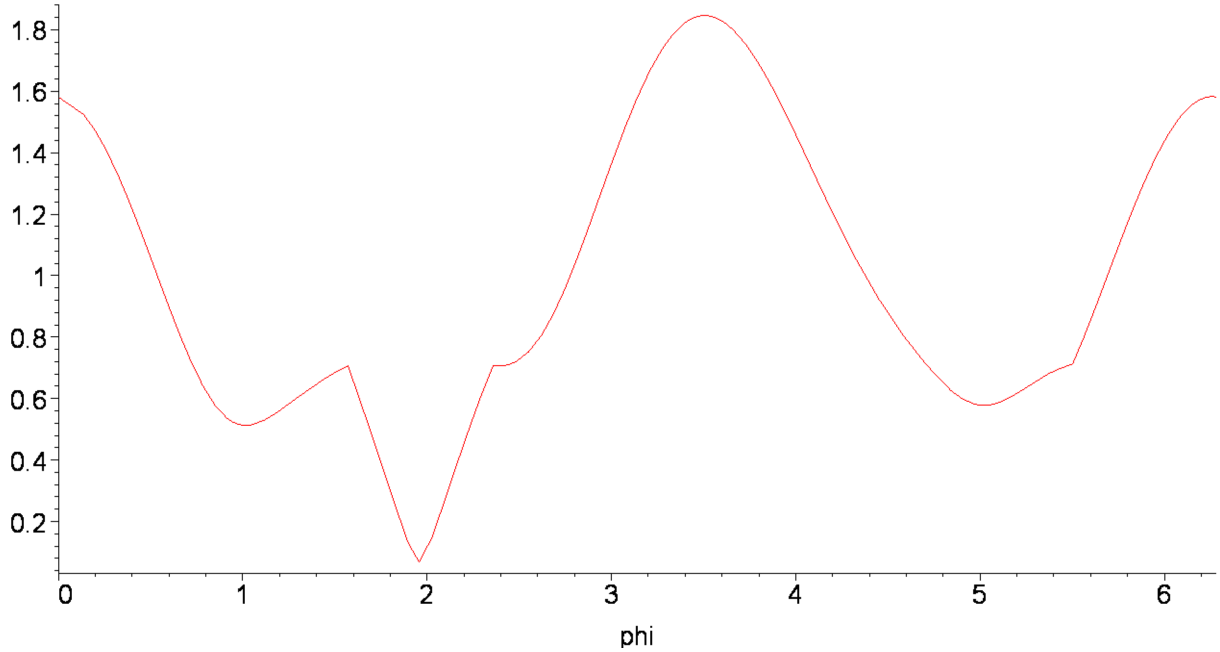
```
> AF1mag:=abs(subs({A1=abs(cos(phi))*exp(I*phi),A2=abs(cos(phi-Pi/4)
)*exp(I*(phi-Pi/4)),k=2*Pi/lambda,d=3*lambda/8},AF1));AF1arg:=argu
ment(subs({A1=abs(cos(phi))*exp(I*phi),A2=abs(cos(phi-Pi/4))*exp(I
*(phi-Pi/4)),k=2*Pi/lambda,d=3*lambda/8},AF1));
```

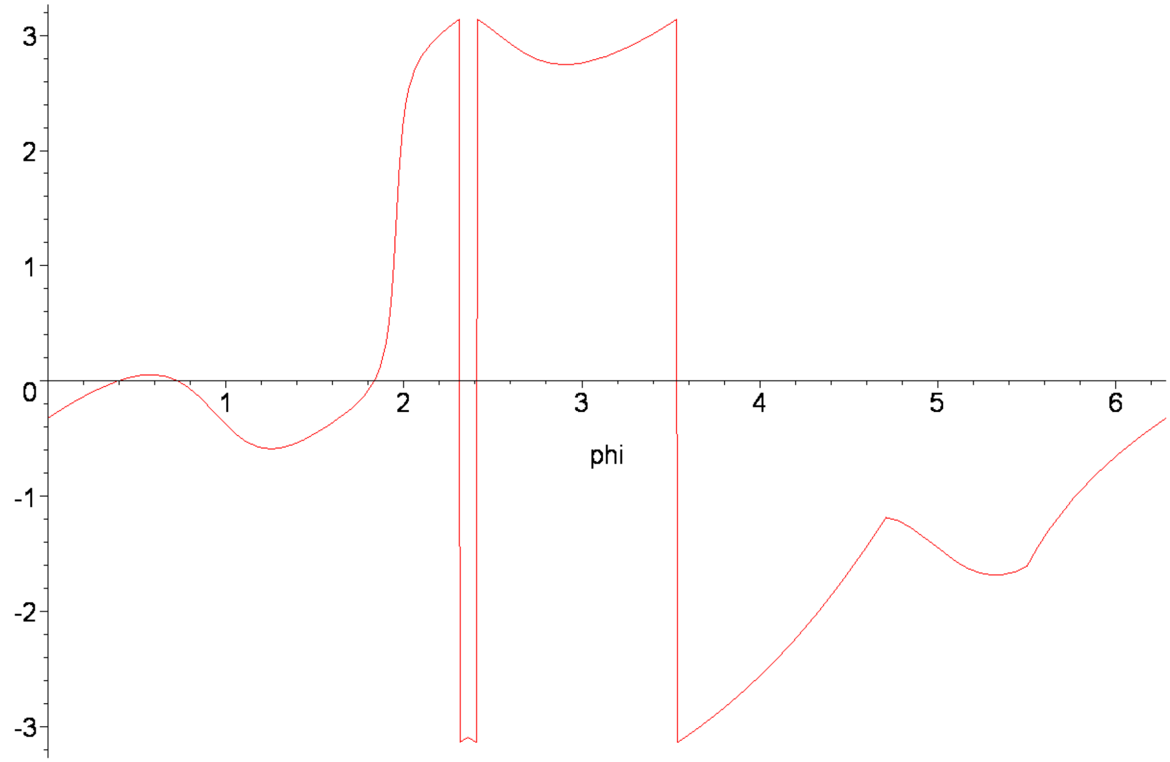
$$AF1mag := \left| \left|\cos(\phi)\right| \mathbf{e}^{(\phi I)}\, \mathbf{e}^{(3/8\, I\, \pi \sin(\phi))} + \left|\sin\left(\phi + \frac{\pi}{4}\right)\right| \mathbf{e}^{\left(\left(\phi - \frac{\pi}{4}\right) I\right)}\, \mathbf{e}^{(-3/8\, I\, \pi \sin(\phi))} \right|$$

$$AF1arg := \mathrm{argument}\left(\left|\cos(\phi)\right| \mathbf{e}^{(\phi I)}\, \mathbf{e}^{(3/8\, I\, \pi \sin(\phi))} + \left|\sin\left(\phi + \frac{\pi}{4}\right)\right| \mathbf{e}^{\left(\left(\phi - \frac{\pi}{4}\right) I\right)}\, \mathbf{e}^{(-3/8\, I\, \pi \sin(\phi))} \right)$$

Part B: The magnitude expression seems more useful to me; let's plot it.

```
> plot(AF1mag,phi=0..2*Pi);plot(AF1arg,phi=0..2*Pi);
```





Problem 2: (5-2-3 of textbook) - **Part A**: Since the relative E-field is given (with a peak value of 1), the spacing really doesn't enter into this computation. Using the standard equation for directivity (equation 2-7-1 or 2-7-4)

```
> E2:=cos(Pi/2*cos(theta));D2A:=1/(int(int(E2*conjugate(E2)*sin(theta),theta=0..Pi),phi=0..2*Pi)/4/Pi);
```

$$E2 := \cos\left(\frac{1}{2}\pi\cos(\theta)\right)$$

$$D2A := 2$$

Part B: The relative electric field for two identical, isotropic in-phase point radiators is given by

```
> E2B:=exp(I*2*Pi/lambda*d/2*cos(theta))+exp(-I*2*Pi/lambda*d/2*cos(theta));
```

$$E2B := \mathbf{e}^{\left(\frac{\pi\, d \cos(\theta)\, I}{\lambda}\right)} + \mathbf{e}^{\left(\frac{-I\,\pi\, d\cos(\theta)}{\lambda}\right)}$$

```
> assume(d>0):assume(lambda>0):D2B:=1/(int(int(E2B*conjugate(E2B)*sin(theta),theta=0..Pi),phi=0..2*Pi)/4/Pi);
```

$$D2B := -\frac{2\, d\tilde{}\, \pi}{(-1)^{\left(\frac{2\, d\tilde{}}{\lambda\tilde{}}\right)}\lambda\tilde{}\, I - 4\,\pi\, d\tilde{} - (-1)^{\left(-\frac{2\, d\tilde{}}{\lambda\tilde{}}\right)}\lambda\tilde{}\, I}$$

This is a rather messy form, but if we recognize the E2B is in the form of a 2*cosine term (and is normalized to a peak value of 1 by dividing by 2). The integration is as before to give

```
> E2B_alt:=cos(Pi/lambda*d*cos(theta));D2B_alt:=1/(int(int(E2B_alt*conjugate(E2B_alt)*sin(theta),theta=0..Pi),phi=0..2*Pi)/4/Pi);
```

$$E2B\_alt := \cos\left(\frac{\pi\, d\sim \cos(\theta)}{\lambda\sim}\right)$$

$$D2B\_alt := \frac{2\, d\sim \pi}{\cos\left(\frac{\pi\, d\sim}{\lambda\sim}\right) \sin\left(\frac{\pi\, d\sim}{\lambda\sim}\right) \lambda\sim + \pi\, d\sim}$$

Still doesn't look quite the same as the textbook result, but using the trig identity sina*cosa=sin(2a)/2 and dividing both numerator and denominator by Pi*d puts into the same form as the textbook.

Problem 3: (5-2-8 of textbook) - I think that it is most convenient to use the center of the array as the phase reference. Then I would consider that there are two separate arrays: elements 1 & 2 that are in phase comprise array A and elements 3 & 4 that are out of phase comprise array B. The out of phase array has a minus sign to indicate the opposite phase. Note also that array A has an exponential phase dependence with sin(phi); array B has an exponential dependence with cos(phi). The sum of the two arrays is the total field.

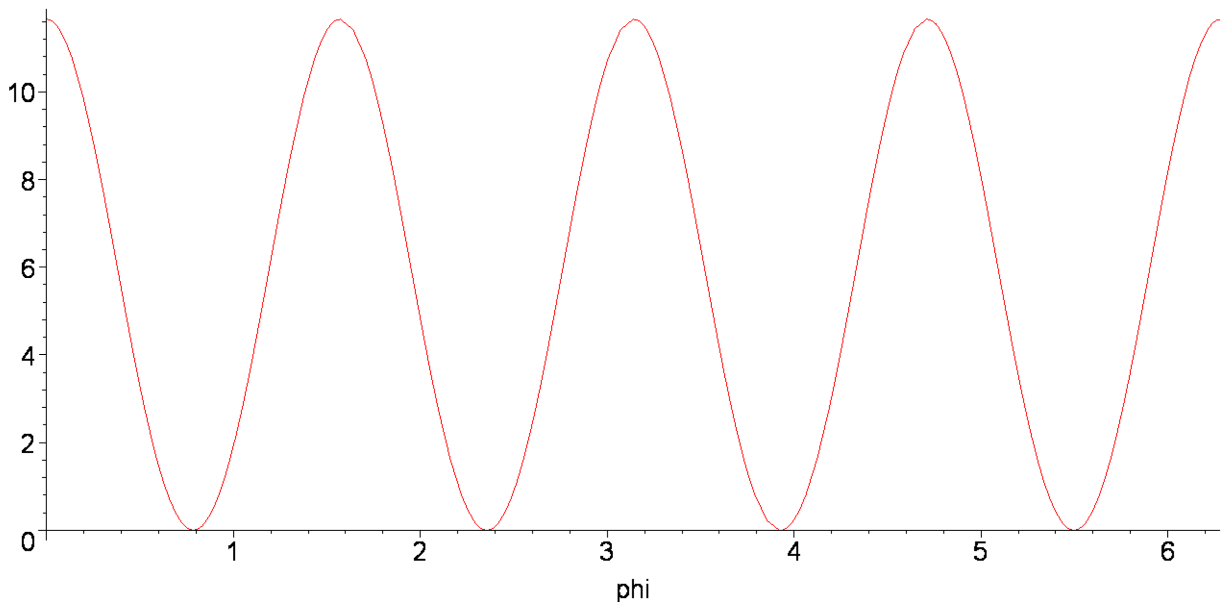
```
> k:=2*Pi/lambda:d:=3*lambda/8:E3A:=exp(I*k*d*cos(phi))+exp(-I*k*d*cos(phi));E3B:=-(exp(I*k*d*sin(phi))+exp(-I*k*d*sin(phi)));AF3:=abs(E3A+E3B)^2;
```

$$E3A := \mathbf{e}^{(3/4\, I\, \pi \cos(\phi))} + \mathbf{e}^{(-3/4\, I\, \pi \cos(\phi))}$$

$$E3B := -\mathbf{e}^{(3/4\, I\, \pi \sin(\phi))} - \mathbf{e}^{(-3/4\, I\, \pi \sin(\phi))}$$

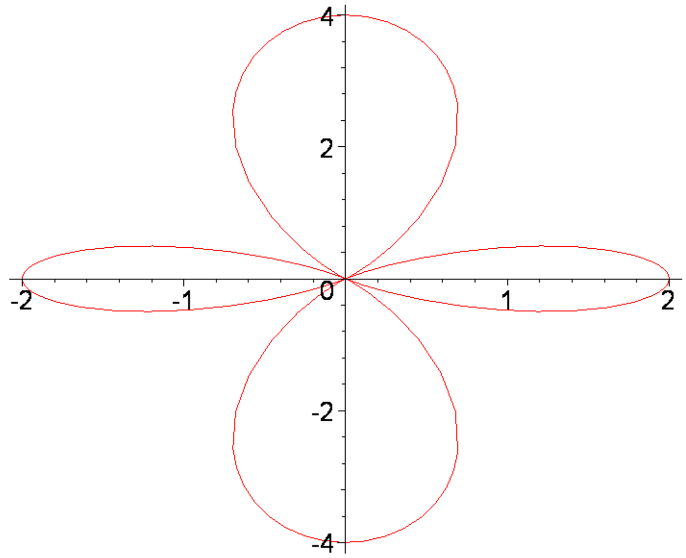
$$AF3 := \left| \mathbf{e}^{(3/4\, I\, \pi \cos(\phi))} + \mathbf{e}^{(-3/4\, I\, \pi \cos(\phi))} - \mathbf{e}^{(3/4\, I\, \pi \sin(\phi))} - \mathbf{e}^{(-3/4\, I\, \pi \sin(\phi))} \right|^2$$

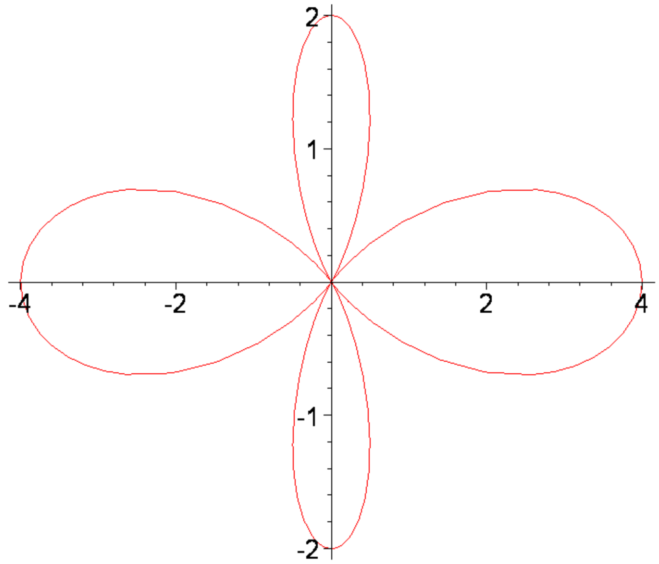
```
> plot(abs(AF3),phi=0..2*Pi);
```

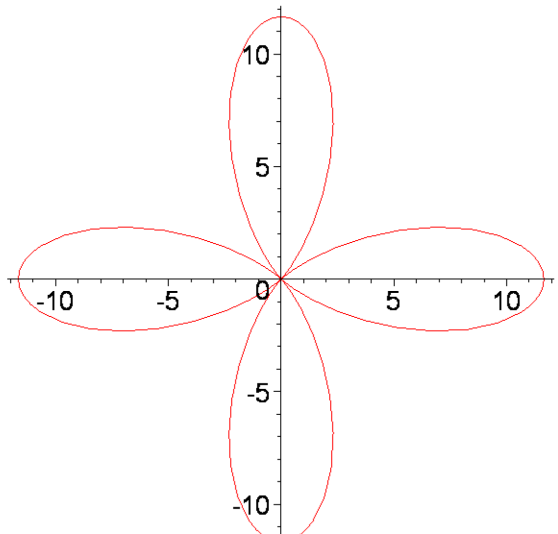


This would look much better in polar form, but let's also plot the separate AF3A and AF3B

```
> plot([E3A(phi)^2,phi,phi=0..2*Pi],coords=polar);plot([E3B(phi)^2,phi,phi=0..2*Pi],coords=polar);plot([AF3(phi),phi,phi=0..2*Pi],coords=polar);
```







The pattern has the 4-petalled look of a lemniscate (I think that's the name of this curve.)

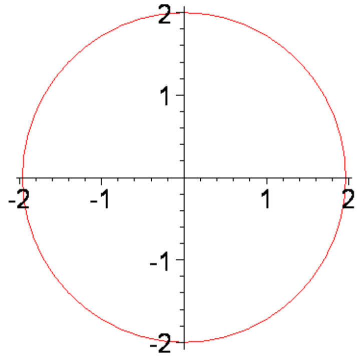
Problem 4: (5-3-1 of textbook) - The AF for two identical, in-phase, isotropic radiators (let's align them along the z-axis so that the polar angle theta describes the pattern; note there will be no variation with the azimuthal angle phi) is given by.

```
> AF4:=abs(exp(I*k*d4/2*cos(theta))+exp(-I*k*d4/2*cos(theta)));
```

$$AF4 := \left| e^{\left(\frac{\pi\, d4 \cos(\theta)\, I}{\lambda\sim}\right)} + e^{\left(\frac{-I\, \pi\, d4 \cos(\theta)}{\lambda\sim}\right)} \right|$$

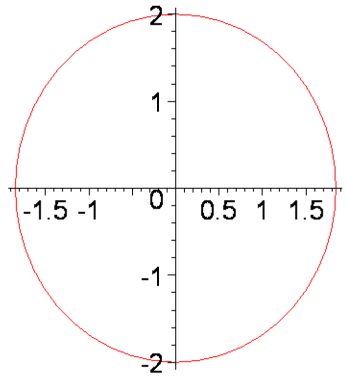
I can't seem to make the polar plot work for a sequence of values of d so I will plot each one separately.

```
> AF4A:=subs(d4=lambda/16,AF4):plot([AF4A(theta),theta,theta=0..2*Pi
  ],coords=polar,scaling=constrained);
```



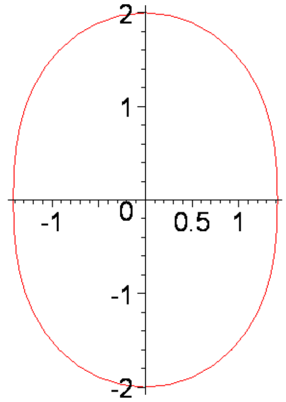
Note, the pattern deviates very slightly from a circle with d=lambda/16..

```
> AF4B:=subs(d4=lambda/8,AF4):plot([AF4B(theta),theta,theta=0..2*Pi]
  ,coords=polar,scaling=constrained);
```



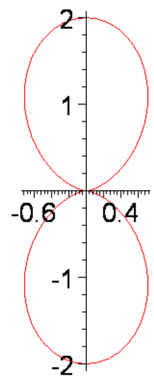
Somewhat more deviation from circular with d=lambda/8.

```
> AF4C:=subs(d4=lambda/4,AF4):plot([AF4C(theta),theta,theta=0..2*Pi]
  ,coords=polar,scaling=constrained);
```



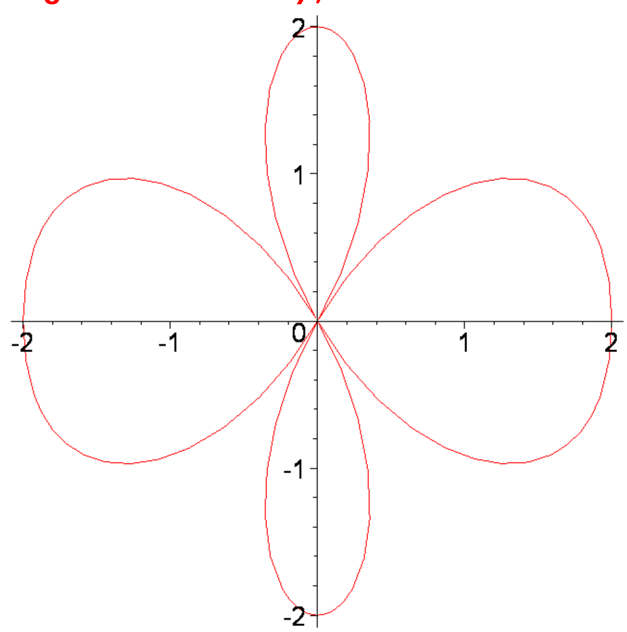
Much more deviation, approaching a "rectangle" with d=lambda/4.

```
> AF4D:=subs(d4=lambda/2,AF4):plot([AF4D(theta),theta,theta=0..2*Pi],coords=polar,scaling=constrained);
```



Very directional with d=lambda/2.

```
> AF4E:=subs(d4=lambda,AF4):plot([AF4E(theta),theta,theta=0..2*Pi],coords=polar,scaling=constrained);
```



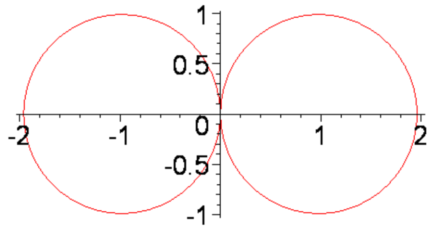
Problem 5: (5-3-2 of textbook) - The problem is ambiquous since the angle used in the previous problem was unspecified. Based upon this problem and the next one, I think that the authors assumed that the elements were along the x-axis. So for the elements along the z-axis the angular variation is with theta instead of phi. For this problem we have an element factor and an array factor and the total field is the product of the two, E=EF*AF and the power pattern is proportional to |E|^2.

```
> E5:=cos(theta)*(exp(I*k*d5/2*cos(theta))+exp(-I*k*d5/2*cos(theta)));
```

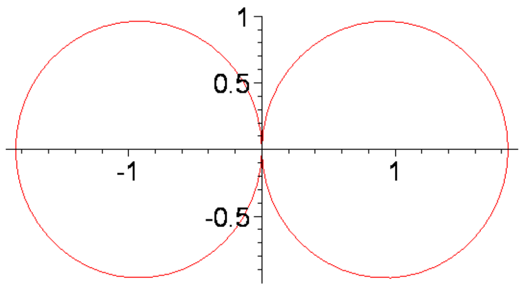
$$E5 := \cos(\theta)\left(\mathbf{e}^{\left(\frac{\pi\, d5 \cos(\theta)\, I}{\lambda\sim}\right)} + \mathbf{e}^{\left(\frac{-I\, \pi\, d5 \cos(\theta)}{\lambda\sim}\right)}\right)$$

```
> E5A:=abs(subs(d5=lambda/16,E5)):plot([E5A(theta),theta,theta=0..2*
```

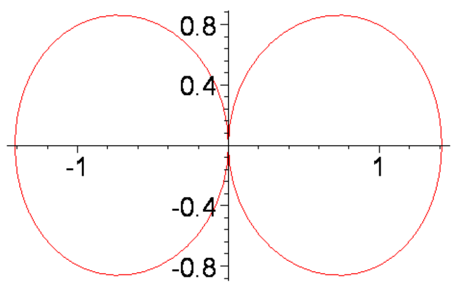
```
Pi],coords=polar,scaling=constrained);
```



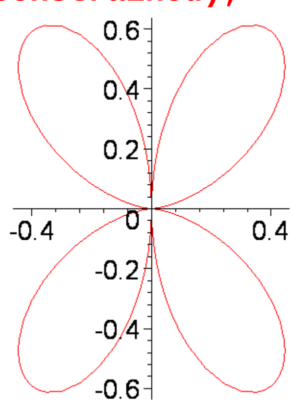

```
> E5B:=abs(subs(d5=lambda/8,E5)):plot([E5B(theta),theta,theta=0..2*P
i],coords=polar,scaling=constrained);
```




```
> E5C:=abs(subs(d5=lambda/4,E5)):plot([E5C(theta),theta,theta=0..2*P
i],coords=polar,scaling=constrained);
```

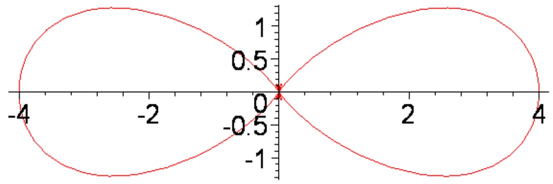



```
> E5D:=abs(subs(d5=lambda/2,E5)):plot([E5D(theta),theta,theta=0..2*P
i],coords=polar,scaling=constrained);
```




```
> E5E:=(subs(d5=lambda,E5)):plot([E5E(theta)^2,theta,theta=0..2*Pi],
```

```
coords=polar,scaling=constrained);
```

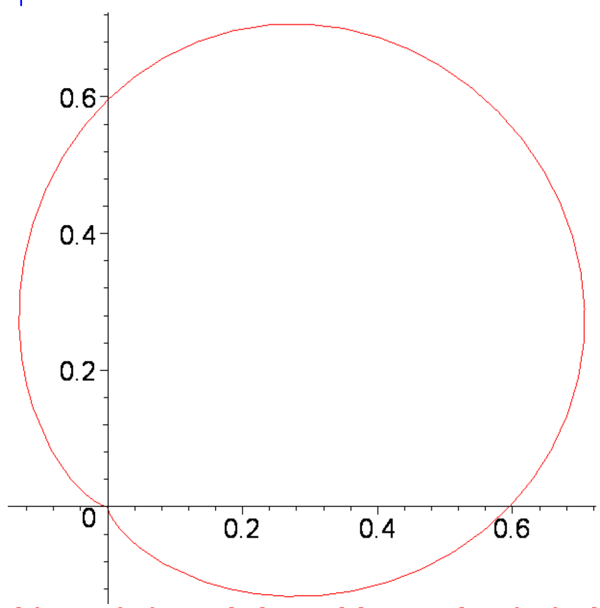


Problem 6: (5-4-1 of textbook) - This takes a bit of experimentation (and good luck). Look at Figure 5-13, p104 of textbook. This would be an acceptable pattern if the radiators were oriented along the NE-SW axis. This array is composed of a primary array of 2 elements separated by d=0.3*lambda and a phase difference of delta=1.425*Pi serves as a starting point. With some experimentation a separation of d=0.2*lambda and a phase difference of delta=1.4*Pi gives reasonable result. A secondary array that has two broad beams oppositely directed along the axis of the desired radiation. This is achieved by two radiators (each radiator is actually a primary array) separated by d=0.6*lambda and a phase difference of delta=Pi.

```
> E_pri:=(exp(I*2*Pi*0.2/2*cos(theta)+I*1.4*Pi)+exp(-I*2*Pi*0.2/2*cos(theta)))/2;AF_pri:=abs(E_pri)^2;plot([AF_pri(theta+Pi/4)^2,theta+Pi/4,theta=0..2*Pi],coords=polar,scaling=constrained);
```

$$E\_pri := \frac{1}{2}\mathbf{e}^{(0.2000000000\, I\, \pi \cos(\theta) + 1.4\, I\, \pi)} + \frac{1}{2}\mathbf{e}^{(-0.2000000000\, I\, \pi \cos(\theta))}$$

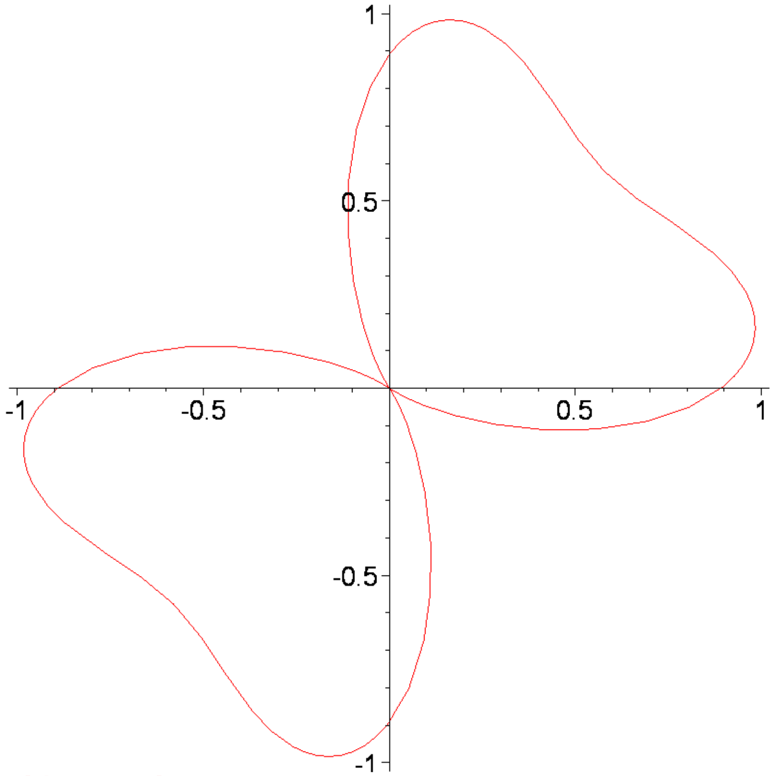
$$AF\_pri := \left|\frac{1}{2}\mathbf{e}^{(0.2000000000\, I\, \pi \cos(\theta) + 1.4\, I\, \pi)} + \frac{1}{2}\mathbf{e}^{(-0.2000000000\, I\, \pi \cos(\theta))}\right|^2$$




```
> E_sec:=(exp(I*2*Pi*0.6/2*cos(theta))-exp(-I*2*Pi*0.6/2*cos(theta)))/2;AF_sec:=abs(E_sec)^2;plot([AF_sec(theta+Pi/4)^2,theta+Pi/4,theta=0..2*Pi],coords=polar,scaling=constrained);
```

$$E\_sec := \frac{1}{2} e^{(0.6000000000\, I\, \pi \cos(\theta))} - \frac{1}{2} e^{(-0.6000000000\, I\, \pi \cos(\theta))}$$

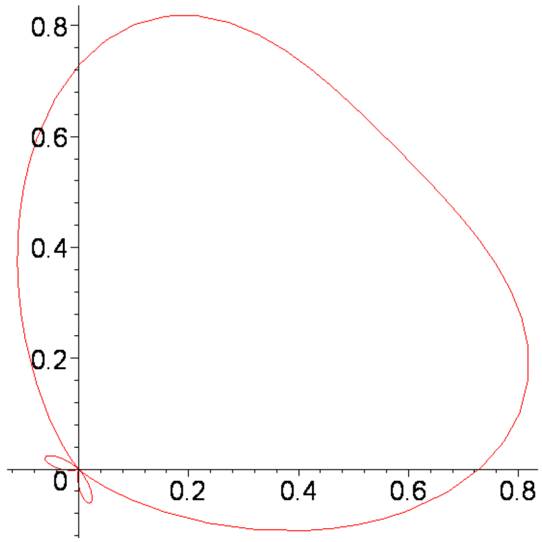
$$AF\_sec := \left| -\frac{1}{2} e^{(0.6000000000\, I\, \pi \cos(\theta))} + \frac{1}{2} e^{(-0.6000000000\, I\, \pi \cos(\theta))} \right|^2$$




```
> AF6:=abs(E_pri*E_sec)^2;
```

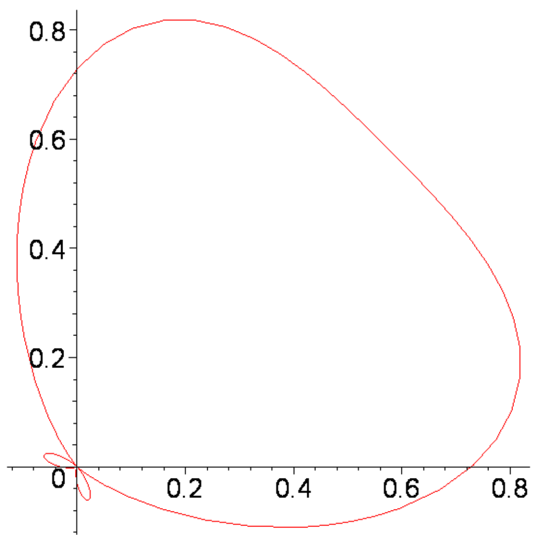
$$AF6 := \left| \left(\frac{1}{2} e^{(0.2000000000\, I\, \pi \cos(\theta) + 1.4\, I\, \pi)} + \frac{1}{2} e^{(-0.2000000000\, I\, \pi \cos(\theta))} \right) \left(\frac{1}{2} e^{(0.6000000000\, I\, \pi \cos(\theta))} - \frac{1}{2} e^{(-0.6000000000\, I\, \pi \cos(\theta))} \right) \right|^2$$

```
> plot([AF6(theta+Pi/4),theta+Pi/4,theta=0..2*Pi],coords=polar,scali
  ng=constrained);
```



The array factor could also could be represented in trigonometric form as shown below; the plots confirm this equality.

```
> plot([cos(Pi*(0.2*cos(theta)+0.7))^2*sin(0.6*Pi*cos(theta))^2,theta+Pi/4,theta=0..2*Pi],coords=polar,scaling=constrained);
```




```
>
```