ECE540 - Antenna Engineering - HW1W04

```
> restart:with(student):
```

Problem 1: The normalized field pattern is given by

```
> En:=piecewise(theta>0,sin(theta)/theta*sin(phi),sin(phi));
```

$$En := \begin{cases} \frac{\sin(\theta)\sin(\phi)}{\theta} & 0 < \theta \\ \sin(\phi) & otherwise \end{cases}$$

Part A: The beam is peaked at theta=0 and phi=0, pi. HPBW is calculated by evaluating the angles in the planes of theta=0 and phi=0 for which the electric field has a value of 0.707 (or the power density has a power of 1/2). The angle calculated is doubled

```
> HPBW_theta:=evalf(2*fsolve(subs(phi=Pi/2,En)=0.707,theta)/Pi*180);
```

$$HPBW\_theta := 159.4929260$$

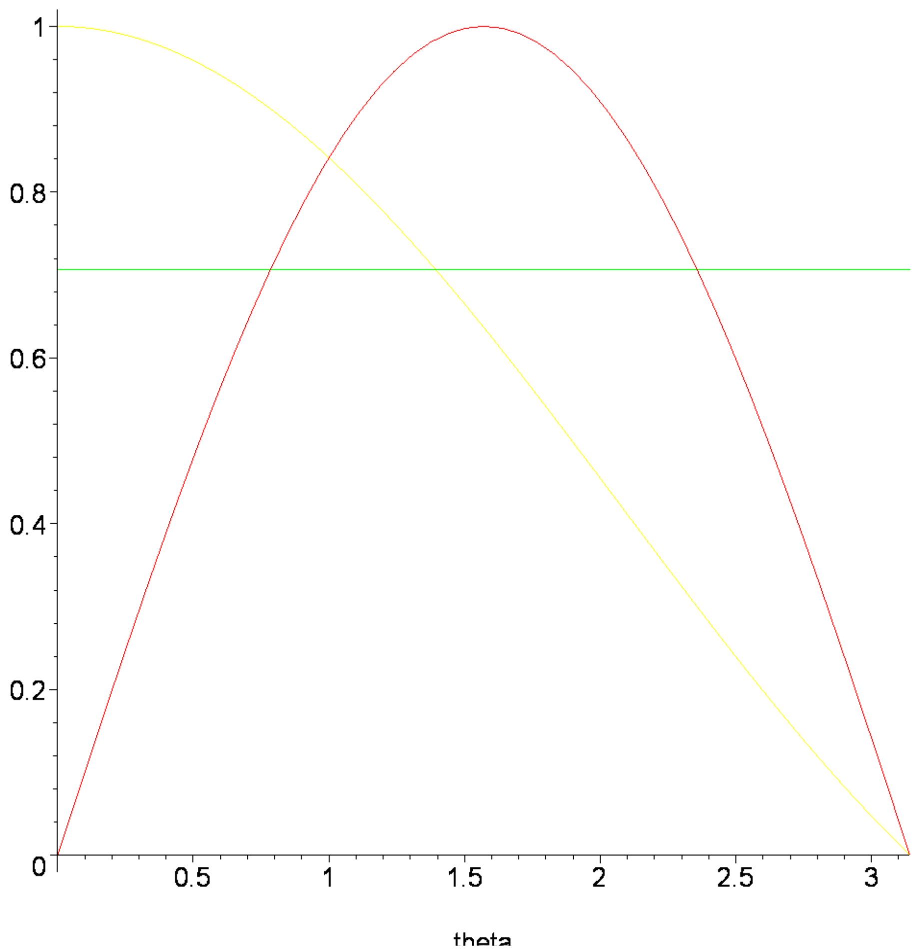
```
> Phi1:=evalf(fsolve(subs(theta=0.000001,En)=0.707,phi,2..3)/Pi*180)
;Phi2:=evalf(fsolve(subs(theta=0.000001,En)=0.707,phi,0.1..1)/Pi*1
80);HPBW_phi:=Phi1-Phi2;
```

$$\Phi1 := 135.0086516$$

$$\Phi2 := 44.99134832$$

$$HPBW\_phi := 90.01730328$$

```
> plot({sin(theta),subs(phi=Pi/2,En),0.707},theta=0..Pi);
```



Part B: Beam area is calculated as the integral of the normalized power pattern over all space (0<=theta<=Pi, 0<=phi<=2*Pi), see equation 2-4-5a

```
> BeamArea:=evalf(int(int(En^2*sin(theta),theta=0..Pi),phi=0..2*Pi))
  ;
```

$$BeamArea := 2.737096888$$

Part C: The beam efficiency is the ratio of the beam area in the desired direction for a specified BW to the total beam area over all space as calculated in part B. Note that this omits 1/2 of the region because the radiation is in the direction opposite to the main beam and is a back lobe. Unfortunately, it has the same magnitude and shape as the main beam so it radiates just as much power. One definition is to use the FNBW; FNBW_theta=Pi, FNBW_phi=Pi. So the BE is calculaated as

```
> BE_FNBW:=evalf(int(int(En^2*sin(theta),theta=0..Pi),phi=0..Pi))/Be
  amArea;
```

$$BE\_FNBW := 0.4999999996$$

An alternate definition is to use HPBW; HPBW_theta is between the limits of Pi/4 and 3Pi/4; HPBW_phi has identical limits. In this case the BE is given as

```
> BE_HPBW:=evalf(int(int(En^2*sin(theta),theta=Pi/4..3*Pi/4),phi=Pi/
  4..3*Pi/4))/BeamArea;
```

$$BE\_HPBW := 0.2790134259$$

Part D: Directivity is a more common measure (than BE) of how focused the radiation is in one direction. It is expressed as the beam area compared to the beam area of an isotropic radiator as

```
> Directivity:=evalf(4*Pi/BeamArea);
```

$$Directivity := 4.591131088$$

Problem 2: The Friis relationship, equation 2-11-5, can be used along with 2-11-4 to calculate the maximum power received over the link as

```
> Gain_trans:=10^(25/10);Gain_rec:=10^(20/10);frequency:=1e9;wavelen
  gth:=3e8/frequency;R:=500;Power_trans:=150;Area_trans:=Gain_trans*
  wavelength^2/4/Pi;Area_rec:=Gain_rec*wavelength^2/4/Pi;Power_rec:=
  evalf(Area_trans*Area_rec/R^2/wavelength^2);
```

$$Gain\_trans := 100\sqrt{10}$$

$$Gain\_rec := 100$$

$$frequency := 0.1\ 10^{10}$$

$$wavelength := 0.3$$

$$R := 500$$

$$Power\_trans := 150$$

$$Area\_trans := \frac{2.250000000\sqrt{10}}{\pi}$$

$$Area\_rec := \frac{2.250000000}{\pi}$$

$$Power\_rec := 0.00007209128596$$

Problem 3:

Part A: The wave is propagating out of the plane of the paper; when omea*t=0 the electric field points in the x-direction; when omega*t=Pi/2, in the -y-direction; when omega*t=Pi, in the -x-direction; when omega*t=3*Pi/2, in the y-direction. The electric field vector is rotating in the CW fashion. Since the two components are equal in magnitude the tip of the rotating E vector traces out a circle. So it is called **CW circularly polarized**.

Part B: with Ey=3 the tip of the rotating E vector traces out an ellipse with an **axial ratio of 2/3**; it still rotates in the CW fashion. The phase angle delta=time-phase angle by which Ey leads Ex=-Pi/2 in this case. From equation 2-17-3, we see that tan(2*tau)=tan(2*gamma)*cos(delta), but cos(-Pi/2)=0. So the **inclination angle tau=0**.