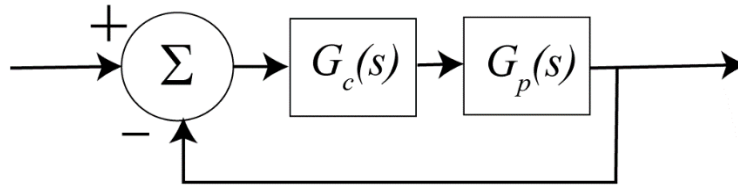


ECE-320,**Quiz #4**

For this problem assume the closed loop system below.

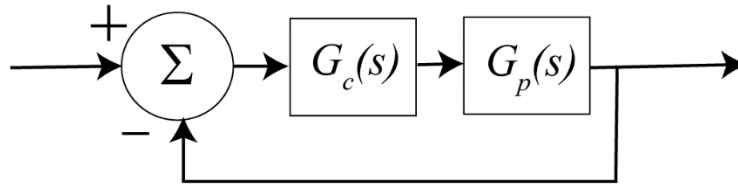


$$\text{Assume } G_p(s) = \frac{1}{(s+1+j)(s+1-j)}$$

For each of the following problems you should sketch the root locus, and where necessary, compute the centroid of the asymptotes and the angles of the asymptotes. Be sure to indicate the direction along the root locus as the free parameter increases.

- 1) Assume controller is a proportional controller, $G_c(s) = k_p$
- 2) Assume the controller is an integral controller, $G_c(s) = \frac{k_i}{s}$
- 3) Assume the controller is the PI controller $G_c(s) = \frac{k(s+10)}{s}$
- 4) Assume the controller is the PD controller $G_c(s) = k(s+10)$

For this problem assume the closed loop system below.

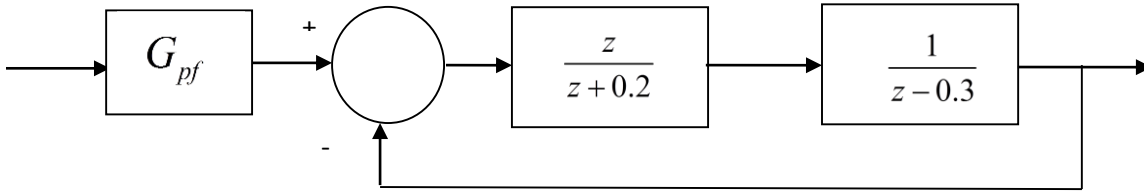


Assume $G_p(s) = \frac{1}{(s+1)(s+2)}$

For each of the following problems you should sketch the root locus, and where necessary, compute the centroid of the asymptotes and the angles of the asymptotes. Be sure to indicate the direction along the root locus as the free parameter increases.

- 5) Assume the controller is an integral controller, $G_c(s) = \frac{k_i}{s}$
- 6) Assume the controller is the PI controller $G_c(s) = \frac{k(s+10)}{s}$
- 7) Assume the controller is the PD controller $G_c(s) = k(s+10)$
- 8) Assume the controller is the PID controller $G_c(s) = \frac{k(s+3+j)(s+3-j)}{s}$

Problems 9 - 11 refer to the following system:

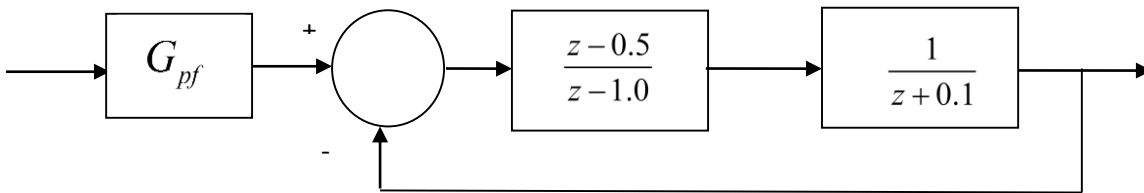


- 9) Assuming the prefilter G_{pf} is 1, estimate the **position error constant** K_p

- 10) Assuming the prefilter G_{pf} is 1, estimate the **steady state error** for a unit step as

- 11) Estimate the value of the prefilter G_{pf} that produces a **steady state error** of zero for a step input

Problems 12- 13 refer to the following system with a sampling interval $T = 0.1$ seconds :



- 12) Assuming the prefilter G_{pf} is 1, estimate the **velocity error constant** K_v

- 13) Assuming the prefilter G_{pf} is 1, estimate the **steady state error** for a unit ramp

Root Locus Construction

Once each pole has been paired with a zero, we are done

1. *Loci Branches*

$$\underset{k=0}{\text{poles}} \rightarrow \underset{k=\infty}{\text{zeros}}$$

Continuous curves, which comprise the locus, start at each of the n poles of $G(s)$ for which $k = 0$. As k approaches ∞ , the branches of the locus approach the m zeros of $G(s)$. Locus branches for excess poles extend to infinity.

The root locus is **symmetric** about the real axis.

2. *Real Axis Segments*

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of $G(s)$.

3. *Asymptotic Angles*

As $k \rightarrow \infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^\circ + i360^\circ}{n - m}, i = 0, \pm 1, \pm 2, \dots$$

until all $(n - m)$ angles not differing by multiples of 360° are obtained. n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$.

4. *Centroid of the Asymptotes*

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n - m}$$

where p_i is the i^{th} pole of $G(s)$, z_j is the j^{th} zero of $G(s)$, n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$. This point is termed the *centroid of the asymptotes*.

5. *Leaving/Entering the Real Axis*

When two branches of the root locus leave or enter the real axis, they usually do so at angles of $\pm 90^\circ$.