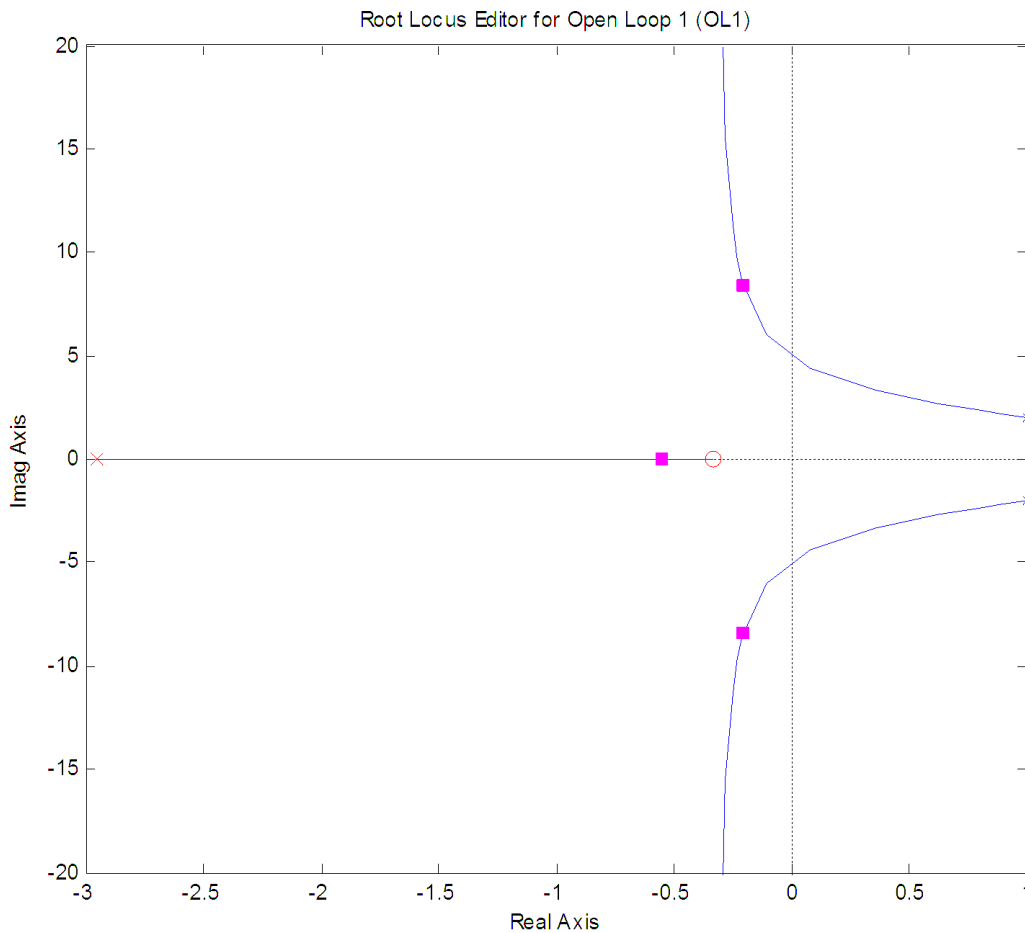


ECE-320, Quiz #3

Problems 1-3 refer to the following root locus plot for a unity feedback system with a plant and a controller.



1) Based on this root locus plot, the best estimate of the poles of the closed loop system are

- a) $-0.3+j7$, $-0.3-j7$, -0.6 b) $1+j2$, $1-j2$, and -3

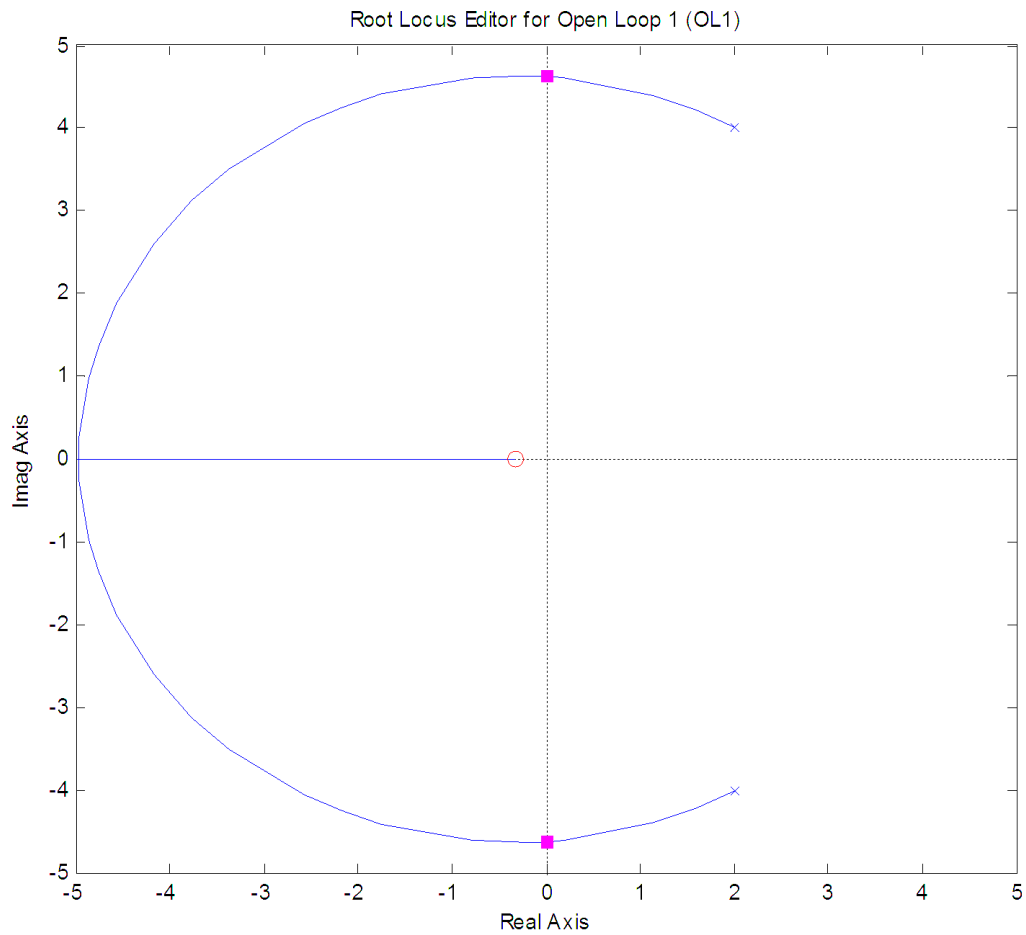
2) Is this a type one system? a) yes b) no

3) Is this a stable system for all values of the gain k ? a) yes b) no

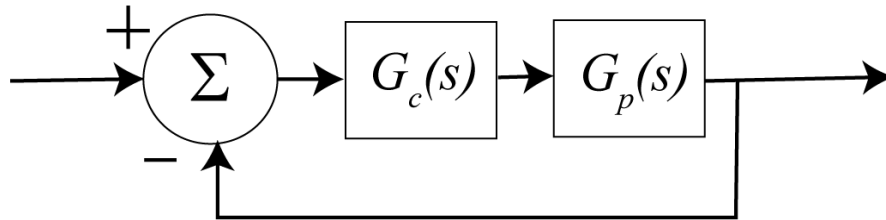
4) Consider the following root locus plot for a plant and controller in a unity feedback configuration.

If we want the system to be stable, should we

- a) increase the gain b) decrease the gain c) do nothing



Problems 5-10 refer to the following feedback system with plant $G_p(s) = \frac{1}{(s+3)(s+4)}$



5) If we use a proportional controller $G_c(s) = k_p$ will the system remain stable for all positive values of k_p ?

a) yes b) no

6) If we use a proportional controller $G_c(s) = k_p$ is there any value of k_p for which the settling time is less than 0.5 seconds?

a) yes b) no

7) If we use an integral controller $G_c(s) = \frac{k_i}{s}$ will the system remain stable for all positive values of k_i ?

a) yes b) no

8) If we use an integral controller $G_c(s) = \frac{k_i}{s}$ is there any value of k_i for which the settling time is less than 0.5 seconds?

a) yes b) no

9) For which of the following PID controllers will the settling time be smaller as $k \rightarrow \infty$

a) $G_c(s) = \frac{k(s+2+j)(s+2-j)}{s}$ b) $G_c(s) = \frac{k(s+4+2j)(s+4-2j)}{s}$

c) the results will be the same

10) For which of the following PD controllers will the settling time be smaller as $k \rightarrow \infty$

a) $G_c(s) = k(s+5)$ b) $G_c(s) = k(s+10)$ c) the results will be the same

11) The standard form for a PID controller is

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

For the following PID controller $G_c(s) = \frac{5(s^2 + 2s + 1)}{s}$ determine k_p , k_i , and k_d

For your ease, assume the form of convolution $y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$ in all of the following problems.

12) The finite summation $S_N = \sum_{k=0}^N a^k$ is equal to

- a) $\frac{1-a^N}{1-a}$ b) $\frac{1-a^{N-1}}{1-a}$ c) $\frac{1-a^{N+1}}{1-a}$ d) $\frac{1+a^{N+1}}{1-a}$ e) none of these

13) The finite summation $S = \sum_{k=-1}^{N+2} a^k$ is equal to

- a) $a^{-1} \frac{1-a^{N+3}}{1-a}$ b) $a^1 \left(\frac{1-a^{N+4}}{1-a} \right)$ c) $a^{-1} \left(\frac{1-a^{N+4}}{1-a} \right)$ d) $a^{-1} \left(\frac{1-a^{N-4}}{1-a} \right)$ e) none of these

14) For a discrete time system, $\delta(0)$ is equal to

- a) 0 b) 1 c) ∞ d) it is not defined

15) If an LTI system with impulse response $h(n) = 4^{n-1}u(n-1)$ has input $x(n) = \delta(n)$, the output of the system is

- a) $y(n) = 4^{n-1}u(n-1)\delta(n)$ b) $y(n) = 4^{n-1}u(n)$ c) $y(n) = 4^{n-1}u(n-1)$ d) none of these

16) If an LTI system with impulse response $h(n) = 3^{n+1}u(n)$ has input $x(n) = 3\delta(n-1)$, the output of the system is

- a) $y(n) = 3^{n+1}u(n-1)$ b) $y(n) = 3^n u(n-1)$ c) $y(n) = 3^n u(n)$ d) none of these

17) If an LTI system with impulse response $h(n) = 2^{n-1}u(n-1)$ has input $x(n) = 2\delta(n-1)$, the output of the system is

- a) $y(n) = 2^{n-2}u(n-2)$ b) $y(n) = 2^n u(n-2)$ c) $y(n) = 2^{n-1}u(n-2)$ d) none of these

18) If an LTI system with impulse response $h(n) = 3\delta(n-1)$ has input $x(n) = 2\delta(n-1)$, the output of the system is

- a) $y(n) = 3 \times 2u(n-2)$ b) $y(n) = 3 \times 2\delta(n-1)$ c) $y(n) = 3 \times 2\delta(n-2)$ d) none of these

19) If an LTI system with impulse response $h(n) = 3^n u(n)$ has input $x(n) = u(n)$, the output of the system is

- a) $y(n) = 3^n u(n)$ b) $y(n) = 3^{n+1} u(n)$ c) $y(n) = \frac{1-3^{n+1}}{1-3} u(n)$ d) $y(n) = \frac{1-3^{n-1}}{1-3} u(n)$
 e) none of these

20) If an LTI system with impulse response $h(n) = 3^n u(n)$ has input $x(n) = 2^n u(n)$, the output of the system is

- a) $y(n) = 3^n 2^n u(n)$ b) $y(n) = 3^n \frac{1 - \left(\frac{2}{3}\right)^{n+1}}{1 - \frac{2}{3}} u(n)$ c) $y(n) = 2^n \frac{1 - \left(\frac{3}{2}\right)^{n+1}}{1 - \frac{3}{2}} u(n)$

- d) $y(n) = \left[\frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} \right] \left[\frac{1 - \left(\frac{1}{3}\right)^{n+1}}{1 - \frac{1}{3}} \right] u(n)$ e) none of these

21) The sum $S = \sum_{k=0}^{\infty} a^k$ will converge provided

- a) $|a| > 1$ b) $|a| < 1$

22) If the sum $S = \sum_{k=0}^{\infty} a^k$ converges, it is equal to

- a) $\frac{1}{1+a}$ b) $\frac{1}{1-a}$ c) $\frac{a}{1-a}$ d) $\frac{a}{1+a}$ e) none of these

Root Locus Construction

Once each pole has been paired with a zero, we are done

1. Loci Branches

$$\underset{k=0}{\text{poles}} \rightarrow \underset{k=\infty}{\text{zeros}}$$

Continuous curves, which comprise the locus, start at each of the n poles of $G(s)$ for which $k = 0$. As k approaches ∞ , the branches of the locus approach the m zeros of $G(s)$. Locus branches for excess poles extend to infinity.

The root locus is **symmetric** about the real axis.

2. Real Axis Segments

The root locus includes all points along the real axis to the left of an odd number of poles plus zeros of $G(s)$.

3. Asymptotic Angles

As $k \rightarrow \infty$, the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180^\circ + i360^\circ}{n - m}, i = 0, \pm 1, \pm 2, \dots$$

until all $(n - m)$ angles not differing by multiples of 360° are obtained. n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$.

4. Centroid of the Asymptotes

The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma_c = \frac{\sum_i p_i - \sum_j z_j}{n - m}$$

where p_i is the i^{th} pole of $G(s)$, z_j is the j^{th} zero of $G(s)$, n is the number of poles of $G(s)$ and m is the number of zeros of $G(s)$. This point is termed the *centroid of the asymptotes*.

5. Leaving/Entering the Real Axis

When two branches of the root locus leave or enter the real axis, they usually do so at angles of $\pm 90^\circ$.