ECE-320: Linear Control Systems Homework 3

Due: Monday March 28 at the beginning of class. **Exam #1, Friday April 1**

1) (*sisotool problem*) For the plant modeled by the transfer function

$$
G_1(s) = \frac{6000}{s^2 + 4s + 400}
$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$
PO \le 10\%
$$

\n
$$
T_s \le 2.5 \text{ sec}
$$

\n
$$
k_p \le 0.5
$$

\n
$$
k_i \le 5
$$

\n
$$
k_d \le 0.01
$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

2) (*sisotool problem*) For the plant modeled by the transfer function

$$
G_2(s) = \frac{6250}{s^2 + 0.5s + 625}
$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$
PO \le 10\%
$$

PI $T_s \le 15.0$ sec, PID $T_s \le 0.5$ sec
 $k_p \le 0.5$
 $k_i \le 5$
 $k_d \le 0.01$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

3) (Easy) Show that $\sum \delta(l) = u(n)$ *l n l* $\delta(l) = u(n)$ $=$ $\sum_{l=-\infty}^{l=n} \delta(l) = u(n)$ and $\sum_{l=-\infty}^{l=n} \delta(l-k) = u(n-k)$ *l* $\sum_{k=1}^{n} \delta(l-k) = u(n-k)$ $\sum_{k=-\infty}^{n} \delta(l-k) = u(n-k)$ **4) (Easy)** For impulse response $h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$ and input

$$
x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-2)
$$
, determine the output $y(n)$ (this should be easy).

5) For impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ 2 $h(n) = \left(\frac{1}{2}\right)^n u(n)$ $=\left(\frac{1}{2}\right) u(n)$ and input $x(n) = u(n)$, show that the system output is $(n) = 2 \left[1 - \left(\frac{1}{2} \right)^{n+1} \right] u(n)$ $y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right]u(n)$ $=2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]u(n)$

a) by evaluating the convolution sum $y(n) = \sum_{k=1}^{k=n} x(n-k)h(k)$ *k* $y(n) = \sum_{k = \infty}^{k = \infty} x(n-k)h(k)$ $=\sum_{k=-\infty}^{k=\infty}x(n-k).$ b) by evaluating the convolution sum $y(n) = \sum_{k=1}^{k} x(k)h(n-k)$ *k* $y(n) = \sum_{k = \infty}^{k = \infty} x(k)h(n-k)$ $=\sum_{k=-\infty}^{k=\infty}x(k)h(n-k)$ *Note that this is the unit step response of the system.*

6) For impulse response $(n) = \left(\frac{1}{2}\right)^{n-2} u(n-1)$ 3 $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$ and input $x(n) = \left(\frac{1}{2}\right)^{n-2}$ 2 $x(n) = \left(\frac{1}{2}\right)^n u(n-1)$ $=\left\lfloor \frac{1}{2} \right\rfloor$ $u(n \setminus$, show that the system output is $y(n) = 9 \left[\left(\frac{1}{2} \right)^{n-1} - \left(\frac{1}{3} \right)^{n-1} \right] u(n-2)$ $(n) = 9 \left[\frac{1}{2} \right]$ $y(n) = 9 \left[\left(\frac{1}{2} \right)^{n-1} - \left(\frac{1}{3} \right)^{n-1} \right] u(n-2)$ b $b = 9 \left[\left(\frac{1}{2} \right)^{n-1} - \left(\frac{1}{3} \right)^{n-1} \right] u(n-2)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$ *k* $y(n) = \sum_{n=1}^{\infty} x(n-k)h(k)$ $=$ $-\infty$ $=\sum_{n=1}^{\infty} x(n-k)$

7) For impulse response $(n) = \left(\frac{1}{2}\right)^{n-3} u(n-1)$ 2 $h(n) = \left(\frac{1}{2}\right)^{n-3} u(n)$ $=\left(\frac{1}{2}\right)$ $u(n-1)$ and input $(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$ 4 $x(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$, show that the system output is $(n) = \left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$ $\frac{1}{2}$ $\left(\frac{1}{4}\right)$ $y(n) = \left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^{n-1}\right]u(n)$ $\left[\left(\frac{1}{n}\right)^n - \left(\frac{1}{n}\right)^{n-1}\right]_{u(n)}$ $=\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$ t by evaluating the convolution sum $y(n) = \sum_{k=0}^{\infty} h(n-k)x(k)$ *k* $y(n) = \sum_{n=0}^{\infty} h(n-k)x(k)$ $=-\infty$ $= \sum_{n=1}^{\infty} h(n-k).$

8) For impulse response $(n) = \left(\frac{1}{2}\right)^{n+1} u(n-2)$ 3 $h(n) = \left(\frac{1}{2}\right)^{n+1} u(n)$ $=\left(\frac{1}{3}\right)$ $u(n-2)$ and input $(n) = \left(\frac{1}{2}\right)^{n-2} u(n+1)$ 2 $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n+1)$, show that the system output is $y(n) = \frac{16}{0} \left[\left(\frac{1}{2} \right)^n - \left(\frac{1}{2} \right)^n \right] u(n-1)$ $\frac{16}{9} \left(\frac{1}{2}\right) - \left(\frac{1}{3}\right)$ $y(n) = \frac{16}{0} \left[\left(\frac{1}{2} \right)^n - \left(\frac{1}{2} \right)^n \right] u(n-1)$ $=\frac{16}{9}\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n\right]u(n-1)$ b by evaluating the convolution sum $y(n) = \sum_{k=0}^{\infty} h(n-k)x(k)$ *k* $y(n) = \sum_{n=0}^{\infty} h(n-k)x(k)$ $=$ $-\infty$ $= \sum_{n=1}^{\infty} h(n-k).$