ECE-320: Linear Control Systems
Homework 1

Due: Monday March 14, 2016 *at the beginning of class*

1) For the following transfer functions, determine both the **impulse respons**e and the **unit step response**.

$$H(s) = \frac{s}{(s+1)(s+2)^2} \quad H(s) = \frac{1}{(2s+1)(3s+2)}$$

$$H(s) = \frac{2}{s^2+8s+25} \quad H(s) = \frac{s+2}{s^2+2s+4}$$

Scrambled Answers:

$$h(t) = \frac{2}{3}e^{-4t}\sin(3t)u(t),\ h(t) = -e^{-t}u(t) + e^{-2t}u(t) + 2te^{-2t}u(t),\ h(t) = e^{-t/2}u(t) - e^{-2t/3}u(t),$$

$$h(t) = e^{-t}\cos(\sqrt{3}t)u(t) + \frac{1}{\sqrt{3}}e^{-t}\sin(\sqrt{3}t)u(t),\ y(t) = \frac{1}{2}u(t) - 2e^{-t/2}u(t) + \frac{3}{2}e^{-2t/3}u(t),$$

$$y(t) = \frac{1}{2}u(t) + \frac{1}{2\sqrt{3}}e^{-t}\sin(\sqrt{3}t)u(t) - \frac{1}{2}e^{-t}\cos(\sqrt{3}t)u(t),\ y(t) = e^{-t}u(t) - e^{-2t}u(t) - te^{-2t}u(t),$$

$$y(t) = \frac{2}{25}u(t) - \frac{8}{75}e^{-4t}\sin(3t)u(t) - \frac{2}{25}e^{-4t}\cos(3t)u(t)$$

2) For the following transfer functions

$$H(s) = \frac{2}{s^2+2s+2} \quad H(s) = \frac{3}{s^2+4s+6} \quad H(s) = \frac{5}{s^2+6s+10}$$

$$H(s) = \frac{4}{s^2-4s+7} \quad H(s) = \frac{1}{s^2+4}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t) = \left[1 - e^{-t}\cos(t) - e^{-t}\sin(t)\right]u(t) \quad y(t) = \left[\frac{1}{2} - \frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t) - \frac{1}{2}e^{-2t}\cos(\sqrt{2}t)\right]u(t)$$

$$y(t) = \left[\frac{1}{2} - \frac{3}{2}e^{-3t}\sin(t) - \frac{1}{2}e^{-3t}\cos(t)\right]u(t) \quad y(t) = \left[\frac{4}{7} + \frac{8\sqrt{3}}{21}e^{2t}\sin(\sqrt{3}t) - \frac{4}{7}e^{2t}\cos(\sqrt{3}t)\right]u(t)$$
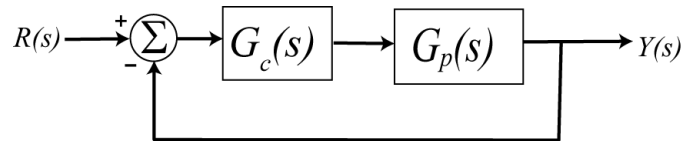
$$y(t) = \left[\frac{1}{4} - \frac{1}{4}\cos(2t)\right]u(t)$$

3) (Mason's Rule) For the block diagram shown below, determine a corresponding signal flow diagram and show that the closed loop transfer function is

$$H_{system} = \frac{G_1 G_2 G_3 + G_4 (1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2)}{1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2}$$
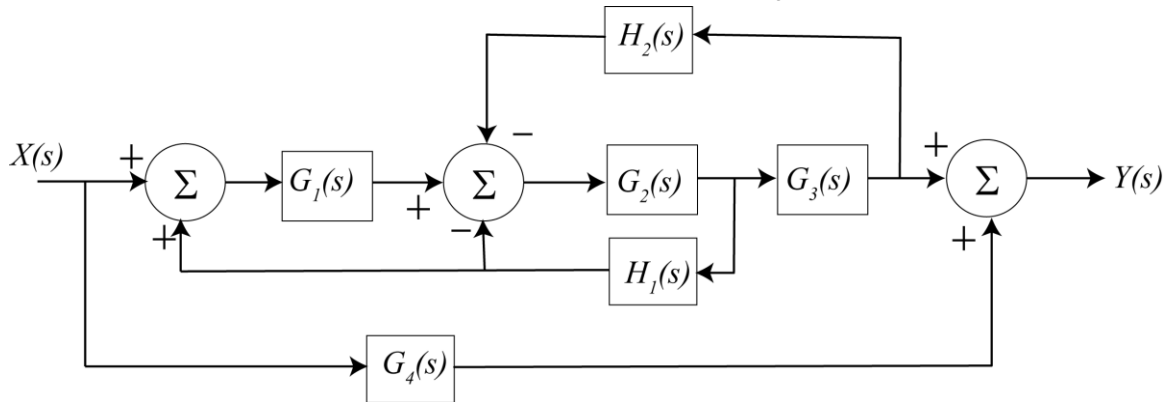
4) (Model Matching) Consider the following closed loop system, with plant $G_p(s)$ and controller $G_c(s)$.


One way to choose the controller is to try and make your closed loop system match a transfer function that you choose (hence the name model matching). Let's assume that our **desired** closed loop transfer function, $G_o(s)$, our plant can be written in terms of numerators and denominators as

$$G_o(s) = \frac{N_o(s)}{D_o(s)} \quad G_p(s) = \frac{N_p(s)}{D_p(s)}$$

Show that our controller is then $G_c(s) = \dfrac{N_o(s) D_p(s)}{N_p(s)[D_o(s) - N_o(s)]}$

Note that there are some restrictions here, in that for implementation purposes the controller must be stable, and it must be proper.

5) For the following system, with plant $G_p(s) = \dfrac{1}{s+1}$, and controller $G_c(s)$
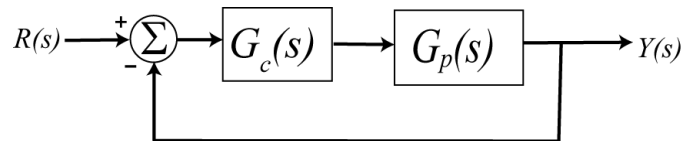
a) Using the results from problem 1, determine the controller so that the closed loop system matches a second order ITAE (**I**ntegral of **T**ime and **A**bsolute **E**rror) optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Anwes. $G_c(s) = \dfrac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

b) Show that the damping ratio for this system is 0.7, the closed loop poles of this system are at $-0.7\omega_0 \pm j0.714\omega_0$. For faster response should ω_0 be large or small?

c) Determine the controller so that the closed loop system matches a third order **deadbeat** system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

Ans. $G_c(s) = \dfrac{\omega_0^3(s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$, *note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.*

6) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.
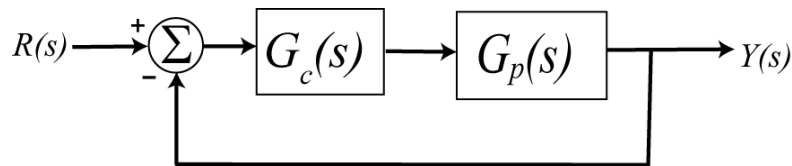
a) Determine the settling time and steady state error of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 0.1, and the corresponding settling time for the system.
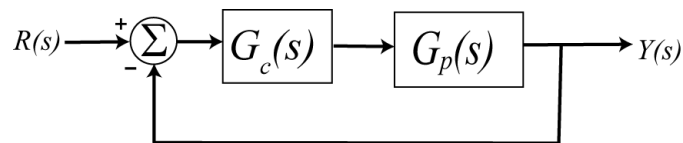
d) Assuming a proportional controller, $G_c(s) = k_p$,determine the value of k_p so the settling time is 0.5 seconds, and the corresponding steady state error.

e) Assuming an integral controller, $G_c(s) = k_i / s$, determine closed loop transfer function, $G_0(s)$

f) Assuming an integral controller, $G_c(s) = k_i / s$, determine the value of k_i so the steady state error for a unit step is less than 0.1 and the system is stable.

Partial Answers: $T_s = 1$, $e_{ss} = 0.5$, $k_p = 18$, $k_p = 2$, $T_s = 0.1$, $e_{ss} = 0.5$, $k_i > 0$

7) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{4}{s+7}$.

a) What is the bandwidth of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the bandwidth of the closed loop system is 27 rad/sec.

d) Assuming the proportional controller from problem c, determine the settling time and the steady state error for a unit step.

Partial Answers: 7, 5, 7/27, 4/27

8) (Matlab/Simulink) Download and uncompress the file **Model_Matching.rar** from the class website. The file **closedloop.s**lx is a Simulink file the for a simple closed loop system, as shown below in in Figure 3.
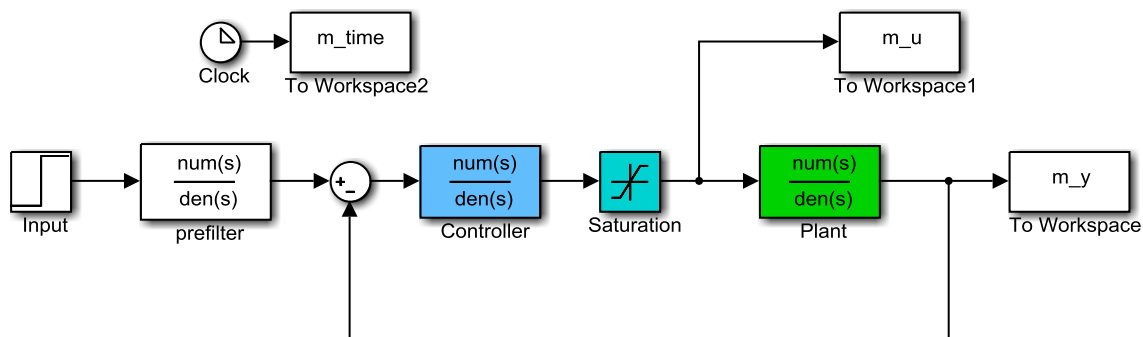
Figure 3. Simple feedback control system

In this system the plant is the thing we want to control, the controller modifies the behavior of the plant, and we have included a limit on the control effort (the Saturation block). Many real systems have limits on the control effort, as you will see.

The file **closedloop_driver.m** is the Matlab driver file that loads parameters and transfer functions into the Matlab workspace for the Simulink file to use. Using these two programs we can look at some model matching systems and some examples of what happens when the model does not match the real system. Also this gives us a review of Matlab and Simulink.

 If you run the programs as they are you should get a response like that shown in Figure 4 on the next page. This figure shows the response to the plant (the thing we want to control) in the top graph. The middle graph shows a control system using model matching for a 1 cm input. In this graph the red line shows the response of the closed loop transfer function we expect to get if the mathematical model of the plant is exact, and the red line shows the results using the model matching control system. You should note that the control system has a faster response, smaller overshoot, and a steady state error of zero (the output is equal to the input in steady state). The last graph shows the control effort of the system. Most practical systems have a limit on the allowed control effort since real systems (like op amps) tend to saturate.

a) Modify the parameter ω_0 to get the fastest response you can without saturating the control effort (the control effort should remain below 2). Note that the control effort is maximum at the beginning and then dies down, this is common for controllers. At this point our model of the plant and the true plant have the transfer function $G_p(s) = \dfrac{100}{s^2 + 2s + 10}$ .Turn in your graph.

In the next few parts we will look at examples when the model is not matched exactly. For each of these parts use the value of ω_0 you determined in part a.

b) Assume the true plant has the transfer function $G_p(s) = \dfrac{20}{s^2 + s + 20}$. This involves changing num_Gp and den_Gp. Then adjust Tf so the system reaches steady state. Rerun the simulation and turn in your results. You should notice that the model matching controller does not follow the ideal (the red model in the graph), but is does eventually converge to a systems with zero steady state error. Turn in your graph.

c) Assume the true plant has the transfer function $G_p(s) = \dfrac{25s + 50}{s^2 + s + 20}$. Rerun the simulation and turn in your results.

d) Assume the true plant has the transfer function $G_p(s) = \dfrac{100}{0.1s^3 + s^2 + s + 20}$. Rerun the simulation and turn in your results.

e) Assume the true plant has the transfer function $G_p(s) = \dfrac{100}{s^2 - 2s - 20}$. Set the final time to 3.5 seconds (Tf = 3.5) and rerun the simulation and turn in your results. What happens in this case?
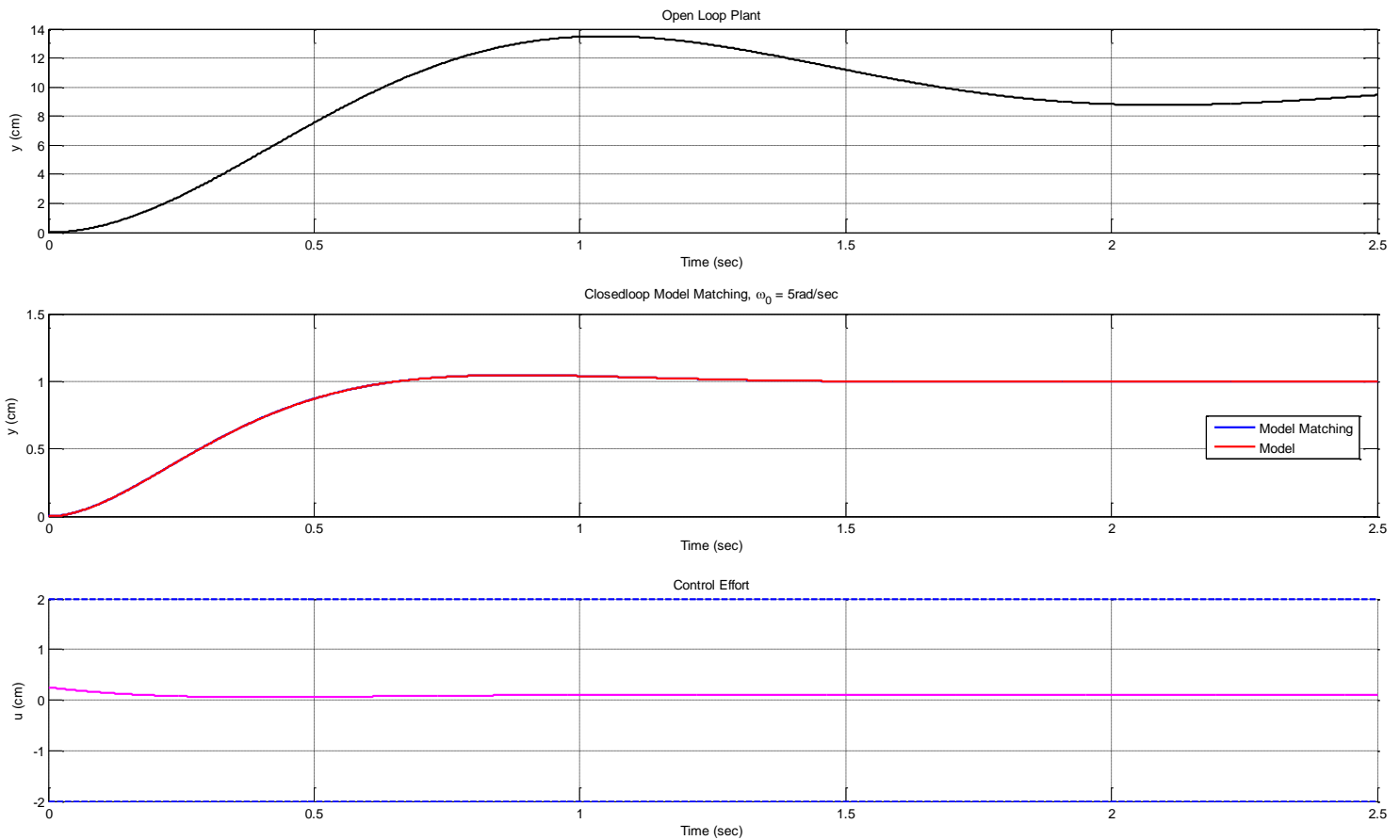
Figure 4. Response of a plant and an ITAE closed loop model matching system.