Name Solutions

ECE 320
Linear Control Systems

Exam 2
Winter 2015-2016

- This exam is closed-book in nature.
- No calculators or computers (except for music)
- Clearly indicate your answer and include units, labels, etc. as appropriate

Problem 1	______/18
Problem 2	______/20
Problem 3	______/13
Problem 4	______/16
Problems 5-15	______/33

Exam ~~3~~ 2 Score: _______ / 100

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1) For each of the following transfer functions determined from state variable models using state variable feedback, with two states, determine if the system is controllable or not

$$G_o(s) = \frac{1}{s^2 + k_2 s + 1}$$ N

$$G_o(s) = \frac{1}{(s - k_1)(s - k_2)}$$ Y

$$G_o(s) = \frac{1}{(s - k_1 k_2)^2}$$ N

$$G_o(s) = \frac{1}{s + k_2}$$ N

$$G_o(s) = \frac{1}{s^2 + k_1 k_2 s + (3 - k_1)}$$ Y

$$G_o(s) = \frac{(s + k_1)}{(s + k_1)(s + k_2 + 1)}$$ N

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2) For the following state variable system

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

with state variable feedback $u(t) = G_{pf} r(t) - Kx(t)$, determine the closed loop transfer function between the input $R(s)$ and the output $Y(s)$.

$$\tilde{A} = A - BK = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3-K_1 & -K_2 \end{bmatrix}$$

$$sI - \tilde{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3-K_1 & -K_2 \end{bmatrix} = \begin{bmatrix} s-1 & -2 \\ K_1-3 & s+K_2 \end{bmatrix}$$

$$(sI - \tilde{A})^{-1} = \frac{1}{(s-1)(s+K_2)+2(K_1-3)} \begin{bmatrix} s+K_2 & 2 \\ 3-K_1 & s-1 \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} s+K_2 & 2 \\ 3-K_1 & s-1 \end{bmatrix}$$

$$G_o(s) = C(sI-\tilde{A})^{-1}\tilde{B} = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix}}{\Delta(s)} \begin{bmatrix} s+K_2 & 2 \\ 3-K_1 & s-1 \end{bmatrix} \begin{bmatrix} 0 \\ G_{pf} \end{bmatrix} = \boxed{\frac{2\, G_{pf}}{(s-1)(s+K_2)+2(K_1-3)} = G_o(s)}$$

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3) Consider the following transfer function $G_o(s) = \dfrac{\alpha}{s^2 + 2\alpha s + \omega_o^2}$

a) Determine the sensitivity of this transfer function to variations in α. For full credit your answer must be the ratio of two polynomials.
b) Determine an expression for the sensitivity as a function of frequency.

$$S_\alpha^{G_o} = \frac{\alpha}{N}\frac{\partial N}{\partial \alpha} - \frac{\alpha}{D}\frac{\partial D}{\partial \alpha}$$

$$= \frac{\alpha}{\alpha}(1) - \frac{\alpha}{s^2+2\alpha s+\omega_o^2}(2s) = 1 - \frac{2\alpha s}{s^2+2\alpha s+\omega_o^2}$$

$$= \boxed{\frac{s^2+\omega_o^2}{s^2+2\alpha s+\omega_o^2} = S_\alpha^{G_o}}$$

$$S_\alpha^{G_o}(j\omega) = \frac{-\omega^2+\omega_o^2}{-\omega^2+2\alpha j\omega+\omega_o^2}$$

$$\boxed{\left| S_\alpha^{G_o}(j\omega) \right| = \frac{\sqrt{(\omega_o^2-\omega^2)^2}}{\sqrt{(\omega_o^2-\omega^2)^2+4\alpha\omega^2}}}$$

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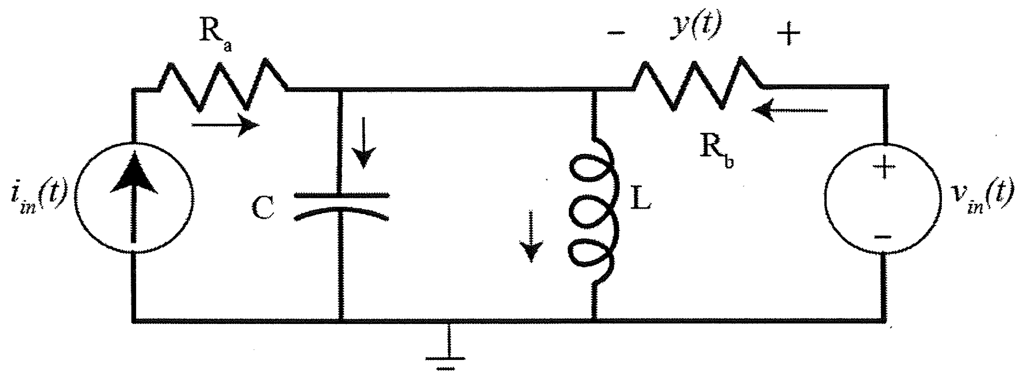
4) For the following circuit, determine a state variable representation. Assume the currents flow in the directions shown, the output is $y(t)$, the first state is $v_C(t)$ and the second state is $i_L(t)$.

Recall that $i_C(t) = C\dfrac{dv_C(t)}{dt}$ $\quad v_L(t) = L\dfrac{di_L(t)}{dt}$



$$i_{in}(t) + \frac{V_{in}(t) - V_c(t)}{R_b} = C\frac{dV_c(t)}{dt} + i_L(t) \qquad L\frac{di_L(t)}{dt} = V_c(t) \qquad y(t) = V_{in}(t) - V_c(t)$$

$$\frac{dV_c(t)}{dt} = -\frac{1}{R_bC}V_c(t) - \frac{1}{C}i_L(t) + \frac{1}{C}i_{in}(t) + \frac{1}{R_bC}V_{in}(t) \qquad \frac{di_L(t)}{dt} = \frac{1}{L}V_c(t)$$

$$\frac{d}{dt}\begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{R_bC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}}_{A} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{R_bC} & \frac{1}{C} \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} V_{in}(t) \\ i_{in}(t) \end{bmatrix}$$

$$y(t) = \underbrace{\begin{bmatrix} -1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{D} \begin{bmatrix} V_{in}(t) \\ i_{in}(t) \end{bmatrix}$$

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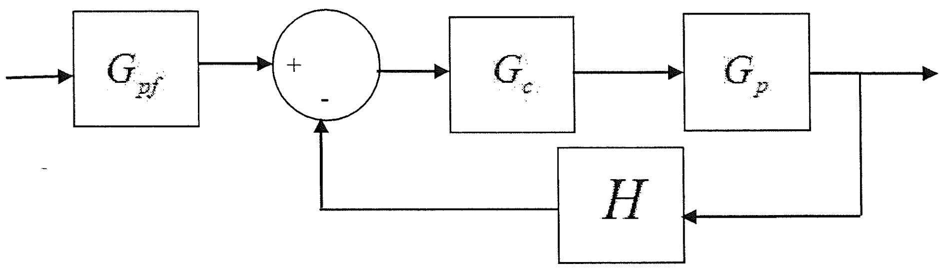
Problems 5 and 6 refer to the following system



5) To reduce the sensitivity of the closed loop transfer function variations in the plant G_p, we should

a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small

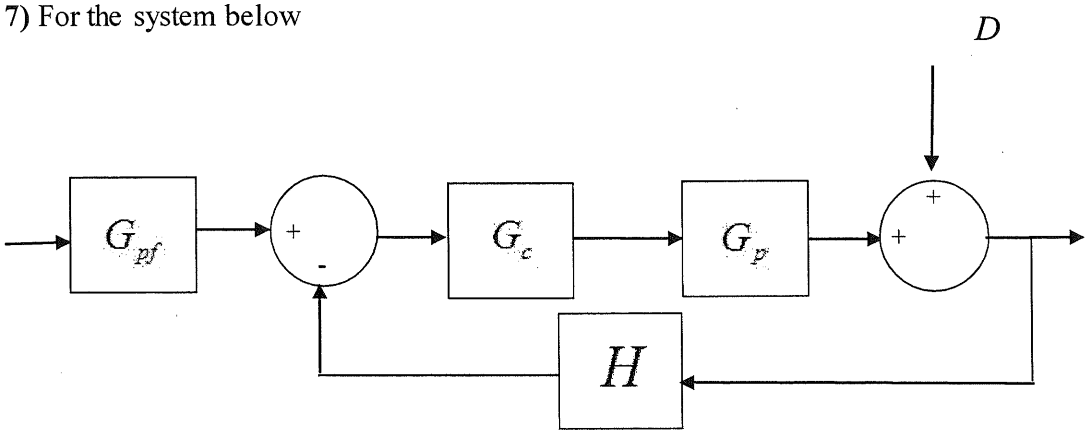
c) make G_{pf} large d) do nothing, we cannot change the sensitivity

6) To reduce the sensitivity of the closed loop transfer function to variations in the sensor H, we should

a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small

c) make G_{pf} large d) do nothing, we cannot change the sensitivity

7) For the system below



to reduce the effects of the external disturbance D on the system output, we should

a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small

c) make G_{pf} large d) do nothing, we cannot change the sensitivity

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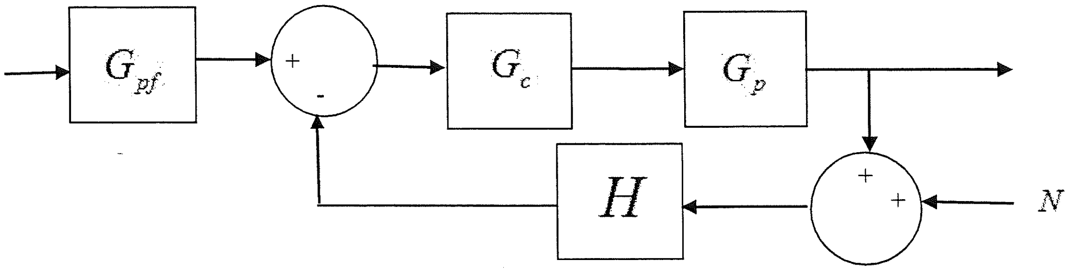
8) For the system below



to reduce the effects of sensor noise N on the closed loop system , we should

a) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ large b) make $|G_c(j\omega)G_p(j\omega)H(j\omega)|$ small

c) make $|H(j\omega)|$ large d) do nothing, we cannot change the sensitivity

9) Consider a plant that is unstable but is a controllable system. Is it possible to use state variable feedback to make this system stable?

a) Yes b) No

10) Is it possible for a system with state variable feedback to change the zeros of the plant (other than by pole-zero cancellation) ?

a) Yes b) No

11) Is it possible for a system with state variable feedback to introduce zeros into the closed loop system?

a) Yes b) No

12) If a plant has n poles, then a system with state variable feedback with no pole-zero cancellations will have

a) more than n poles b) less than n poles c) n poles d) it is not possible to tell

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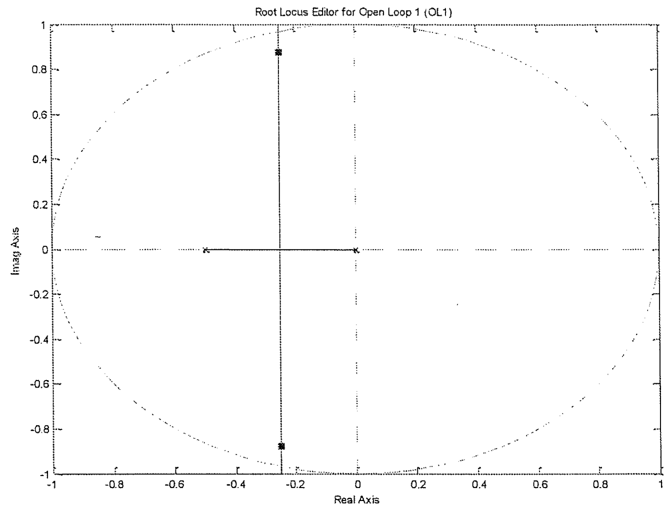
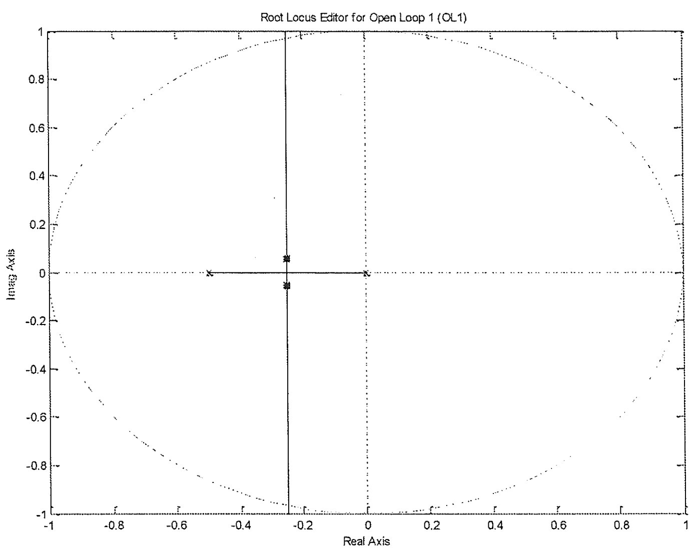
Problems 13 and 14 refer to the following two root locus plot for a discrete-time system





13) For which system is the settling time likely to be smallest?

a) The system on the top b) the system on the bottom c) the settling time will be the same

14) Is this a type 1 system?

a) yes b) no c) not enough information

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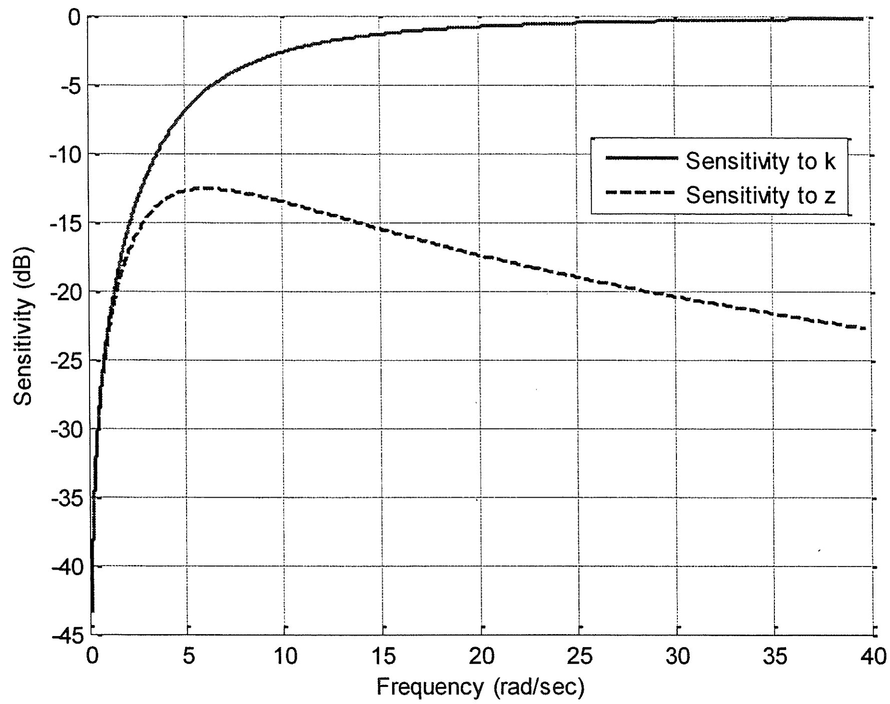
15) The graph below shows a plot of the sensitivities to two parameters. Over this frequency range, the system is more sensitive to which parameter?



more sensitive to k