

**ECE-320: Linear Control Systems**  
Homework 3

Due: Tuesday September 20 at 1 PM

1) For the following transfer functions, determine

- the characteristic polynomial
- the characteristic modes
- if the system is stable, unstable, or marginally stable

a)  $H(s) = \frac{s-1}{s(s+2)(s+10)}$

b)  $H(s) = \frac{s(s-1)}{(s+1)^2(s^2+s+1)}$

c)  $H(s) = \frac{1}{s^2(s+1)}$

d)  $H(s) = \frac{s^2-1}{(s-1)(s+2)(s^2+1)}$

e)  $H(s) = \frac{1}{(s^2+2)(s+1)}$

2) For a system with the following pole locations, estimate the settling time and determine the dominant poles

a) -1, -1, -4, -5

b) -4, -6, -6, -8

c) -1+j, -1-j, -2, -2

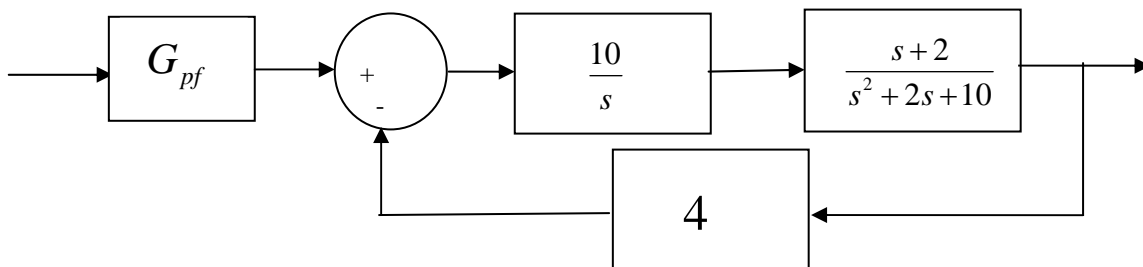
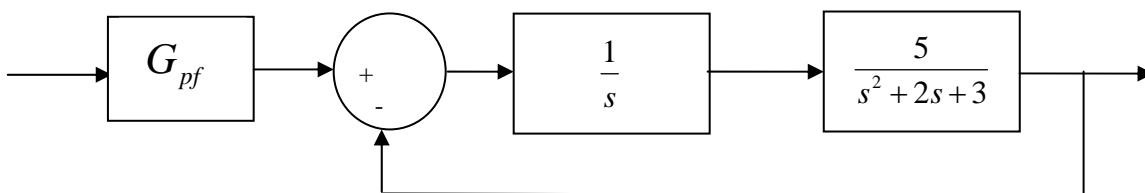
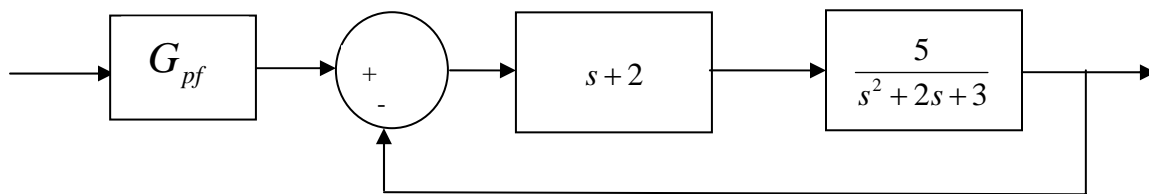
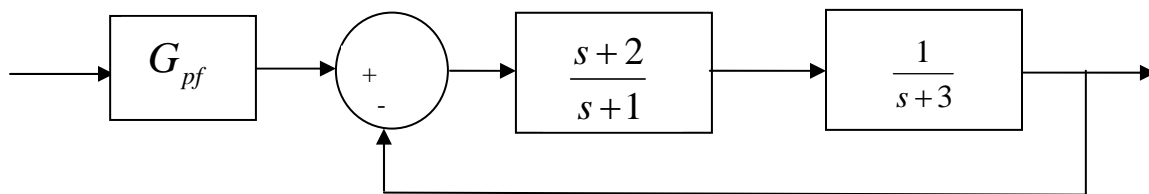
d) -3-2j, -3+2j, -4+j, -4-j

*Ans. 4/3, 4, 4, 1 (though not in that order)*

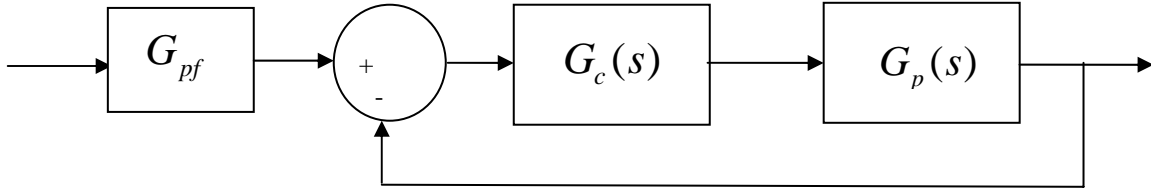
3) For the following systems

a) Determine the value of the prefilter gain  $G_{pf}$  so that the steady state error for a unit step input of the closed loop system is zero. (*Hint: Think about the easiest way to do this, you should never have to write out the closed loop transfer function as the ratio of two polynomials.*) Ans. 1,4,1.3,2.5 (though not in that order)

b) Simulate each system in Matlab (not Simulink) for a step response. Run the simulation until the system comes to steady state. Use **subplot** to put all four plots on one page. You may want to type **orient tall** before your first plot so Matlab will use more of the page.



4) For the following system, with prefilter  $G_{pf}$ , plant  $G_p(s) = \frac{1}{s+1}$ , and controller  $G_c(s)$



a) Assume the prefilter is absent (i.e., it's a 1), and determine the controller so that the closed loop system matches a second order ITAE optimal system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^2}{s^2 + 1.4\omega_0 s + \omega_0^2}$$

Ans.  $G_c(s) = \frac{\omega_0^2(s+1)}{s(s+1.4\omega_0)}$ , note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.

b) Show that the damping ratio for this system is 0.7, the closed loop poles of this system are at  $-0.7\omega_0 \pm j0.714\omega_0$ . For faster response should  $\omega_0$  be large or small?

c) If we want the steady state error for a step input to be zero, what should the value of the prefilter be?

d) Assume the prefilter is absent (i.e., it's a 1) and determine the controller so that the closed loop system matches a third order deadbeat system, i.e., so that the closed loop transfer function is

$$G_0(s) = \frac{\omega_0^3}{s^3 + 1.90\omega_0 s^2 + 2.20\omega_0^2 s + \omega_0^3}$$

Ans.  $G_c(s) = \frac{\omega_0^3(s+1)}{s(s^2 + 1.9\omega_0 s + 2.20\omega_0^2)}$ , note that there is a pole/zero cancellation between the controller and the plant and there is a pole at zero in the controller.

e) If we want the steady state error for a step input to be zero, what should the value of the prefilter be?

5) For a system with plant

$$G_p(s) = \frac{s+3}{s(s-1)}$$

a) Show that the quadratic optimal closed loop transfer function for  $q = 100$  is

$$G_0(s) = \frac{10(s+3)}{s^2 + 12.7s + 30}$$

and the controller is

$$G_c(s) = \frac{10(s-1)}{s+2.7}$$

b) Show that with this controller, the steady state errors for unit step and unit ramp inputs are 0.0 and 0.09, respectively.

6) For a system with plant

$$G_p(s) = \frac{s-1}{s(s-2)}$$

a) Show that the quadratic optimal closed loop transfer function for  $q = 100$  is

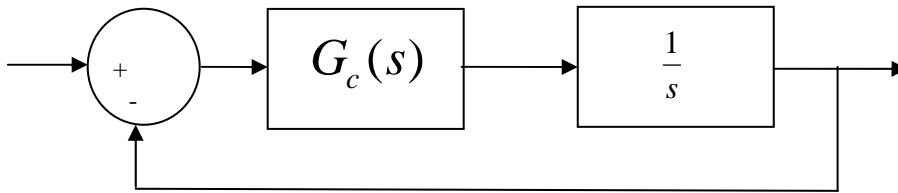
$$G_0(s) = \frac{-10(s-1)}{s^2 + 11.1s + 10}$$

and the controller is

$$G_c(s) = \frac{-10s + 20}{s + 21.14}$$

b) Show that with this controller, the steady state errors for unit step and unit ramp inputs are 0.0 and 2.11, respectively.

7) Consider the following control system:



We are going to use quadratic optimal control to determine the closed loop transfer function  $G_o(s)$  and the controller  $G_c(s)$ . For quadratic optimal control we determine the closed loop transfer function  $G_o(s)$  to minimize

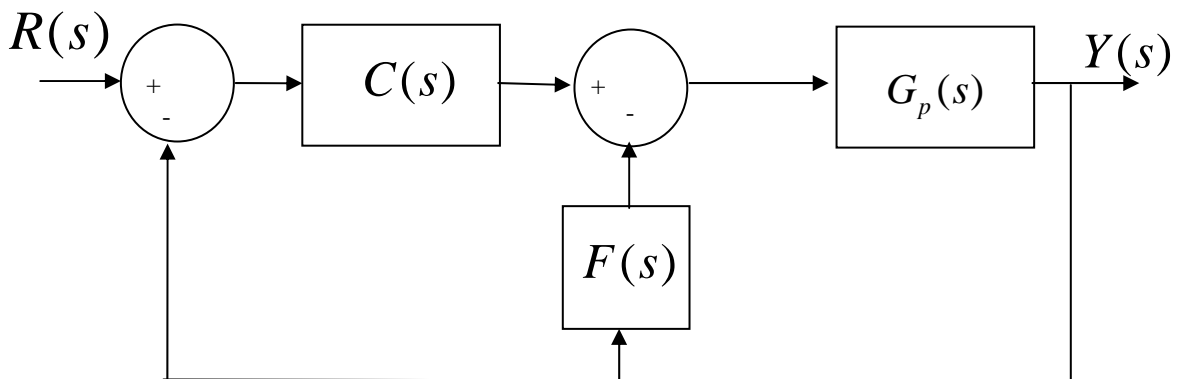
$$J = \int_0^{\infty} [q(r(t) - y(t))^2 + u^2(t)] dt$$

a) Determine the value of  $q$  so that the (2%) settling time of the system is (approximately) 1 second. *Hint: Solve for  $G_o(s)$  in terms of  $q$ , then figure out what  $q$  should be to meet the settling time requirements.*

b) Determine the controller  $G_c(s)$ .

c) What is the steady state error for a unit step input for this system?

8) For the following block diagram,



show that the transfer function from input to output is given by

$$\frac{Y(s)}{R(s)} = \frac{C(s)G_p(s)}{1 + F(s)G_p(s) + C(s)G_p(s)}$$

### *Preparation for Lab 3.*

a) Download the file **solve\_quadratic.m** from the class website. The input arguments to this function are (1) the transfer function of the plant  $G_p$ , and (2) the value of  $q$ . The routine returns the optimal closed loop transfer function  $G_o(s)$ . To use this function with problem 5 we would type something like the following:

```
Gp = tf([1 3],[1 -1 0]);  
q = 100;  
Go = solve_quadratic(Gp,q)  
Gc = mineral(Go/(Gp*(1-Go)))
```

Check your answers to problems 5 and 6 and be sure you understand how to use this function.

b) Modify **closedloop\_driver.m** to implement a quadratic optimal controller. Be organized and neat! You will need to be able to figure out what you have done later! You should have one section of code for determining the plant, one section of code for finding the controller, one section of code for determining the prefilter, and one section of code for plotting the results! Each lab partner should do this separately (though you can help each other).

Modify your code to simulate one of the 1 degree of freedom models you developed in lab 2. That is, we are going to change the plant  $G_p$  by loading a different state variable model. (This lab will go much faster if each lab partner simulates a different system!).

Comment out any controller you currently have (don't delete!), and implement the quadratic optimal controller as shown above. (Your  $G_p$  will come from your model.)

Determine the value of the prefilter so the steady state error for a unit step is 0. This should be coded in Matlab based on the closed loop transfer function. Do not just use a number here (unless it's a 1).

For ECP-210 systems assume the input is 1 cm, for ECP-205 system assume the input is 15 degrees (be sure to convert to radians for the input, but plot the output in degrees), Plot the appropriate step responses for  $q=0.001$ ,  $q = 0.01$ , and  $q = 0.1$ . How dose changing  $q$  affect the response and the maximum control effort?

Turn in your Matlab code and all three plots.