## ECE-320 Linear Control Systems Fall 2004, Exam 2

## You must show all work and justify your answers!

 $\boxed{1}$  (20 points)

a) Consider the PID controller

$$
G_c(s) = \frac{100s + 2s^2 + 5}{s}
$$

determine  $k_p$ ,  $k_i$ , and  $k_d$ .

b) Consider the following unity feedback system



- $\bullet\,$  the poles of the plant  $G_p(s)$  are at -1 and -2
- $\bullet\,$  the pole of the controller  $G_c(s)$  is at -15
- $\bullet\,$  the poles of the closed loop system are at -7.1, -5.43 $\pm3.98j$

Estimate the settling time of the closed loop system.(You can leave your answer as a fraction.)

c) Consider the following closed loop system



Determine a lag controller that will produce an  $e<sub>v</sub>$  of 0.1 without appreciably changing the systems' transient behavior.

d) The unit step response of a system is given below.



Estimate the percent overshoot for this system. (You can leave your answer as a fraction.)

2 (40 points) Consider the system shown below, with plant  $G_p(s) = \frac{1}{s+1}$ , controller  $G_c(s)$ , and prefilter  $G_{pf}$ .



In what follows you may find the following convenient:

$$
(s+3)2 = s2 + 6s + 9
$$
  
\n
$$
(s+3)3 = s3 + 9s2 + 27s + 27
$$
  
\n
$$
(s+3)4 = s4 + 12s3 + 54s2 + 108s + 81
$$

a) By solving the Diophantine equations, determine the minimum order controller  $G_c(s)$  so that the closed loop poles are all at -3.

b) Determine the steady state errors for a unit step and a unit rampe for this system assuming  $G_{pf} = 1.$ 

c) Determine the value of the prefilter  $G_{pf}$  so that the steady state error for a unit step is zero for the system (with your controller from part a).

d) By solving the Diophantine equations, determine the minimum order controller  $G_c(s)$  so that the system is a type one system and all closed loop poles are at -3.

e) Determine the steady state error for a unit ramp assuming  $G_{pf} = 1$ .

f) Find, if possible, a value of the prefilter  $G_{pf}$  so that the steady state error for a unit ramp is zero. If this is not possible say so.

 $3 \overline{3}$  (40 points) Consider the following control system, with plant  $G_p(s) = \frac{1}{s(s+1)}$  and controller  $G_c(s)$ .



We want to design a controller so that  $T_s \leq 2$ . For each of the following controller types sketch the root locus so the closed loop poles of the system meet the constraints, if possible. Specifically, your plots need to include

- $\bullet$  the direction the poles move as k increases
- the asymptotes (if any poles not on the real axis go off to infinity )
- the centroid of the asymptotes (if any poles not on the real axis go off to infinity)

Indicate if there are any conditions on z, p,  $z_1$ ,  $z_2$ , or l to meet the settling time constraints and if it is possible to meet the constraints with the specified controller. Note that all poles and zeros for your controllers must be in the left half plane!

a) a proportional controller  $G_c(s) = k_p$ 

b) an integral controller  $G_c(s) = k_i/s$ 

c) a PI controller  $G_c(s) = k(s + z)/s$ 

d) a PD controller  $G_c(s) = k(s + z)$ 

e) a PID controller with complex conjugate zeros  $G_c(s) = k(s + z_1)(s + z_2)/s$ 

f) a lead controller  $G_c(s) = k(s+z)/(s+p)$ ,  $p > z$  and  $p-z = l$  (you need a condition on z and l)