ECE 300
Signals and Systems
Homework 9

Due Date: ***Monday*** February 12 at 5 PM
Note: Exam 3 Tuesday February 13, Lab Practical Wednesday February 14

Note: Use the Fourier transform table given out in class. Also, you do not need to plot the figures in the problems where the text problem says to plot the figure.

Problems

1. Find the fraction of the total signal energy (as a percentage) contained between 100 and 300 Hz in the signal $x(t)$ given below:

$$x(t) = 5\,\mathrm{sinc}\left(\frac{t}{0.002}\right) + 5\,\mathrm{sinc}\left(\frac{t}{0.001}\right)$$ *Answer* 56%

2. K & H, Problem 3.21 (**a**,**b**,**c** only)

3. K & H, Problem 3.24

4. K & H, Problem 5.14

5. K & H, Problem 5.16 (**a, b, c** only)

6. Consider a linear time invariant system with transfer function given by

$$H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & else \end{cases}$$

with input $x(t) = \frac{8}{\pi}\mathrm{sinc}^2\left(\frac{2(t-1)}{\pi}\right)$. The output of the system is $y(t)$.

a) Determine $X(\omega)$.

b) Sketch the spectrum of $X(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

c) Sketch the spectrum of $H(\omega)$ (magnitude and phase) accurately labeling the axes and important points.

d) Determine $y(t)$, the output of the system.

Answer $y(t) = \frac{20}{\pi}\mathrm{sinc}\left[\frac{2}{\pi}(t-3)\right] + \frac{10}{\pi}\mathrm{sinc}^2\left[\frac{1}{\pi}(t-3)\right]$

(#1) Problem Set 9 ECE-300

$x(t) = 5\,\mathrm{sinc}\left(\frac{t}{0.002}\right) + 5\,\mathrm{sinc}\left(\frac{t}{0.001}\right)$

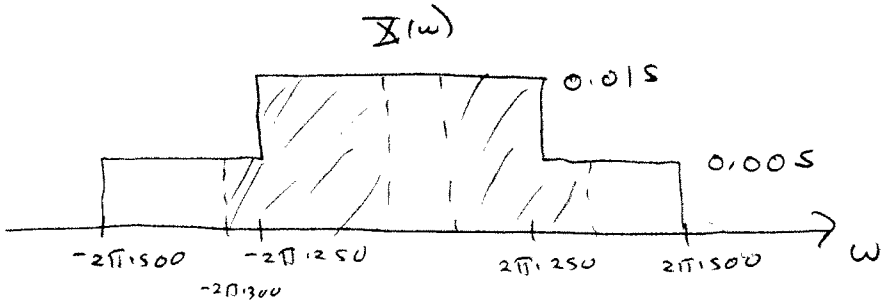
compute the % of energy between 100 and 300 Hz

for $\mathrm{sinc}(Wt) \leftrightarrow \frac{1}{W}\,\mathrm{rect}\left(\frac{w}{2\pi W}\right)$

here $W = \frac{1}{0.002} = 500$ or $W = \frac{1}{0.001} = 1000$

$$X(w) = \frac{5}{500}\,\mathrm{rect}\left(\frac{w}{2\pi\cdot 500}\right) + \frac{5}{1000}\,\mathrm{rect}\left(\frac{w}{2\pi\cdot 1000}\right)$$

$$= 0.01\,\mathrm{rect}\left(\frac{w}{2\pi\cdot 500}\right) + 0.005\,\mathrm{rect}\left(\frac{w}{2\pi\cdot 1000}\right)$$



$$E_{total} = \frac{1}{2\pi}\int_{-2\pi\cdot 500}^{2\pi\cdot 500} |X(w)|^2 dw = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 500}^{-2\pi\cdot 250}(0.005)^2 dw + 2\int_{-2\pi\cdot 250}^{0}(0.015)^2 dw\right]$$

$$= \frac{1}{2\pi}\left[2\cdot(2\pi\cdot 250)(0.005)^2 + 2\cdot(2\pi\cdot 250)(0.015)^2\right]$$

$$= 2\cdot 250\cdot(0.005)^2 + 2\cdot 250\cdot(0.015)^2 = \boxed{0.125 = E_{total}}$$

$$E_{Band} = \frac{1}{2\pi}\left[2\cdot\int_{-2\pi\cdot 300}^{-2\pi\cdot 250}(0.005)^2 dw + 2\int_{-2\pi\cdot 250}^{-2\pi\cdot 100}(0.015)^2 dw\right]$$

$$= \frac{1}{2\pi}\left[2\cdot(2\pi\cdot 50)(0.005)^2 + 2(2\pi\cdot 150)(0.015)^2\right]$$

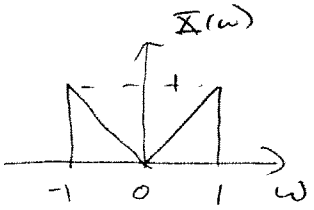
$$= 2\cdot(50)(0.005)^2 + 2(150)(0.015)^2 = \boxed{0.070 = E_{Band}}$$

$$\text{ratio} = \frac{0.070}{0.125} = 0.560 \quad (56\%)$$

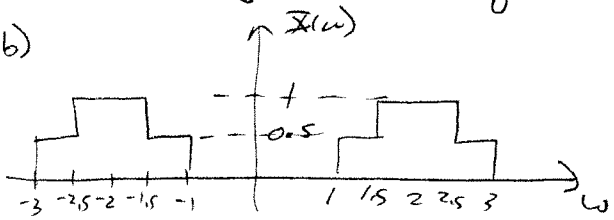
#2

3.21 Compute the inverse Fourier transforms of the following

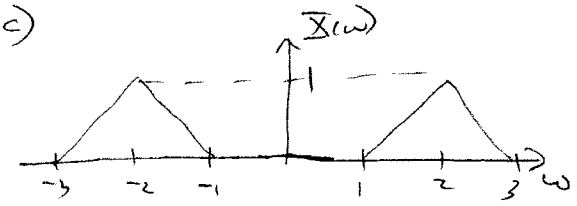
a)



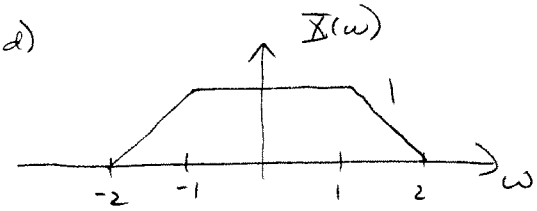
b)



c)



d)



a) $X(\omega) = \text{rect}\left(\frac{\omega}{2}\right) - \Lambda\left(\frac{\omega}{2}\right)$

$$\frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow \text{sinc}(Wt)$$

$$\text{rect}\left(\frac{\omega}{2\pi W}\right) = \text{rect}\left(\frac{\omega}{2}\right) \qquad \pi W = 1 \quad W = 1/\pi$$

$$\pi\,\text{rect}\left(\frac{\omega}{2}\right) \longleftrightarrow \text{sinc}\left(\frac{t}{\pi}\right) \qquad \text{rect}\left(\frac{\omega}{2}\right) \Leftrightarrow \frac{1}{\pi}\text{sinc}\left(\frac{t}{\pi}\right)$$

$$\frac{1}{B}\Lambda\left(\frac{\omega}{4\pi B}\right) \longleftrightarrow \text{sinc}^2(Bt)$$

$$\Lambda\left(\frac{\omega}{4\pi B}\right) = \Lambda\left(\frac{\omega}{2}\right) \rightarrow B = 1/2\pi$$

$$2\pi\,\Lambda\left(\frac{\omega}{2}\right) \longleftrightarrow \text{sinc}^2\left(\frac{t}{2\pi}\right) \qquad \Lambda\left(\frac{\omega}{2}\right) \Leftrightarrow \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)$$

$$\boxed{x(t) = \frac{1}{\pi}\text{sinc}\left(\frac{t}{\pi}\right) - \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)}$$

b) $X(\omega) = 0.5\,\text{rect}\left(\frac{\omega-2}{2}\right) + 0.5\,\text{rect}\left(\frac{\omega-2}{1}\right) + 0.5\,\text{rect}\left(\frac{\omega+2}{2}\right) + 0.5\,\text{rect}\left(\frac{\omega+2}{1}\right)$

$$\frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow \text{sinc}(Wt)$$

$$\text{rect}\left(\frac{\omega}{2}\right) = \text{rect}\left(\frac{\omega}{2\pi W}\right) \quad W = 1/\pi \qquad \text{rect}\left(\frac{\omega}{2}\right) \Leftrightarrow \frac{1}{\pi}\text{sinc}\left(\frac{t}{\pi}\right)$$

$$\text{rect}\left(\frac{\omega}{1}\right) = \text{rect}\left(\frac{\omega}{2\pi W}\right) \quad W = \frac{1}{2\pi} \qquad \text{rect}\left(\frac{\omega}{1}\right) \Leftrightarrow \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right)$$

$$\boxed{x(t) = \left[\frac{1}{\pi}\text{sinc}\left(\frac{t}{\pi}\right) + \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right)\right]\cos(2t)}$$

(3.29 continued)

c) $X(\omega) = \Lambda\left(\frac{\omega-2}{2}\right) + \Lambda\left(\frac{\omega+2}{2}\right)$

$$\frac{1}{B}\Lambda\left(\frac{\omega}{4\pi B}\right) \longleftrightarrow \text{sinc}^2(Bt)$$

$$4\pi B = 2 \quad B = \frac{1}{2\pi}$$

$$\Lambda\left(\frac{\omega}{2}\right) \longleftrightarrow \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)$$

$$\boxed{x(t) = \frac{1}{\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)\cos(2t)}$$

d) $X(\omega) = 2\Lambda\left(\frac{\omega}{4}\right) - \Lambda\left(\frac{\omega}{2}\right)$

$$\frac{1}{B}\Lambda\left(\frac{\omega}{4\pi B}\right) \longleftrightarrow \text{sinc}^2(Bt)$$

$$\Lambda\left(\frac{\omega}{2}\right) \longleftrightarrow \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right) \quad B = \frac{1}{2\pi}$$

$$\Lambda\left(\frac{\omega}{4}\right) = \Lambda\left(\frac{\omega}{4\pi B}\right) \quad B = \frac{1}{\pi}$$

$$\Lambda\left(\frac{\omega}{4}\right) \longleftrightarrow \frac{1}{\pi}\text{sinc}^2\left(\frac{t}{\pi}\right)$$

$$\boxed{x(t) = \frac{2}{\pi}\text{sinc}^2\left(\frac{t}{\pi}\right) - \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)}$$

#3

3.24 Given $X(\omega) = \frac{1}{j\omega+b}$

Find the fourier transform of the following

a) $V(t) = X(5t-4)$

b) $V(t) = t^2 x(t)$

c) $V(t) = x(t) e^{j2t}$

d) $V(t) = x(t) \cos(4t)$

e) $V(t) = \frac{d^2}{dt^2} x(t)$

f) $V(t) = x(t) * x(t)$

g) $V(t) = x(t)^2$

h) $V(t) = \frac{1}{jt-b}$

Note $x(t) = e^{-bt} u(t)$

a) $V(t) = X(5t-4) = X(5(t-4/5))$

if $x(t) \longleftrightarrow X(\omega) = \frac{1}{j\omega+b}$

$x(5t) \longleftrightarrow \frac{1}{5} X(\frac{\omega}{5}) = \frac{1/5}{j(\frac{\omega}{5})+b} = \frac{1}{j\omega+5b}$

$x(5(t-4/5)) \longleftrightarrow \frac{1}{5} X(\frac{\omega}{5}) e^{-j\omega 4/5} = \boxed{\frac{e^{-j\omega 4/5}}{j\omega+5b} = V(\omega)}$

b) $V(t) = t^2 x(t)$

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$\frac{d}{d\omega} X(\omega) = \int_{-\infty}^{\infty} -jt\, x(t) e^{-j\omega t} dt$

$\frac{d^2}{d\omega^2} X(\omega) = \int_{-\infty}^{\infty} (-jt)^2 x(t) e^{-j\omega t} dt$

$\mathcal{F}\{-t^2 x(t)\} = \frac{d^2}{d\omega^2} X(\omega)$

$\mathcal{F}\{t^2 x(t)\} = -\frac{d^2}{d\omega^2} X(\omega)$

$\frac{d}{d\omega} X(\omega) = \frac{d}{d\omega} (j\omega+b)^{-1} = \frac{-j}{(j\omega+b)^2}$

$\frac{d^2}{d\omega^2} X(\omega) = \frac{(-j)(-2j)}{(j\omega+b)^3} = \frac{-2}{(j\omega+b)^3}$

$-\frac{d^2}{d\omega^2} X(\omega) = \boxed{\frac{2}{(j\omega+b)^3} = V(\omega)}$

3.24 (continued)

c) $v(t) = x(t)e^{j2t}$ $\quad V(\omega) = X(\omega - 2) = \boxed{\frac{1}{j(\omega-2)+b} = V(\omega)}$

d) $v(t) = x(t)\cos(4t)$ $\quad V(\omega) = \frac{1}{2}X(\omega+4) + \frac{1}{2}X(\omega-4)$

$$= \boxed{\frac{1}{2}\frac{1}{j(\omega+4)+b} + \frac{1}{2}\frac{1}{j(\omega-4)+b} = V(\omega)}$$

e) $v(t) = \frac{d^2}{dt^2}x(t)$

$\frac{d^2}{dt^2}x(t) \longleftrightarrow (j\omega)^2 X(\omega) \Rightarrow \boxed{V(\omega) = \frac{-\omega^2}{j\omega+b}}$

f) $v(t) = x(t) * x(t)$

$V(\omega) = X(\omega)X(\omega) = \boxed{\frac{1}{(j\omega+b)^2} = V(\omega)}$

g) $v(t) = x(t)^2$

$V(\omega) = \frac{1}{2\pi} X(\omega) * X(\omega)$ This is easier in the time domain

$x(t) = e^{-bt}u(t) \quad x^2(t) = e^{-2bt}u(t)$

$\boxed{V(\omega) = \frac{1}{j\omega+2b}}$

h) $v(t) = \frac{1}{jt-b}$ $\quad x(t) = e^{-bt}u(t) \longleftrightarrow X(\omega) = \frac{1}{j\omega+b}$

First use duality $X(t) \longleftrightarrow 2\pi x(-\omega)$

$x_1(t) = \frac{1}{jt+b} \longleftrightarrow 2\pi e^{b\omega}u(-\omega) = X_1(\omega)$

$x_1(-t) \longleftrightarrow X_1(-\omega)$ and $-x_1(-t) = -X_1(-\omega)$

$\frac{-1}{jt+b} \longleftrightarrow -2\pi e^{-b\omega}u(\omega)$

so $\frac{1}{jt-b} \longleftrightarrow \boxed{-2\pi e^{-b\omega}u(\omega) = V(\omega)}$

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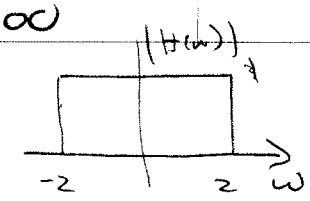
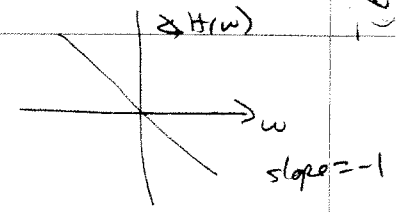
#4 5.14 $H(\omega) = \begin{cases} e^{-j\omega} & -2 < \omega < 2 \\ 0 & \text{else} \end{cases}$





Compute $g(t)$ for each of the following inputs

a) $x(t) = 5\,\text{sinc}\left(\frac{3t}{2\pi}\right)$ b) $x(t) = 5\,\text{sinc}\left(\frac{t}{2\pi}\right)\cos(2t)$

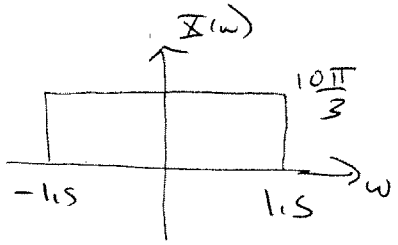
c) $x(t) = \text{sinc}^2\left(\frac{t}{2\pi}\right)$ d) $x(t) = \sum_{k=1}^{\infty} \frac{1}{k}\cos\left(\frac{k\pi}{2}t + 30^\circ\right)$

a) $x(t) = 5\,\text{sinc}\left(\frac{3t}{2\pi}\right)$ $\quad \text{sinc}(Wt) \leftrightarrow \frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right)$

here $W = \frac{3}{2\pi}$ $\quad \text{sinc}\left(\frac{3t}{2\pi}\right) \leftrightarrow (5)\frac{2\pi}{3}\,\text{rect}\left(\frac{\omega}{2\pi \cdot \frac{3}{2\pi}}\right)$

$= \frac{10\pi}{3}\,\text{rect}\left(\frac{\omega}{3}\right)$

$X(\omega) = \frac{10\pi}{3}\,\text{rect}\left(\frac{\omega}{3}\right)$



the entire signal passes

$Y(\omega) = H(\omega)X(\omega) = 1e^{-j\omega}X(\omega)$

$y(t) = 5\,\text{sinc}\left(\frac{3(t-1)}{2\pi}\right)$

$E_{in} = E_{out} = \frac{1}{2\pi}\int_{-1.5}^{1.5}\left(\frac{10\pi}{3}\right)^2 = \left(\frac{10\pi}{3}\right)^2 \cdot 3 \cdot \frac{1}{2\pi} = \frac{100\pi^2}{9} \cdot 3 \cdot \frac{1}{2\pi}$

$= \frac{50\pi}{3} = E_{in} = E_{out}$

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5.14

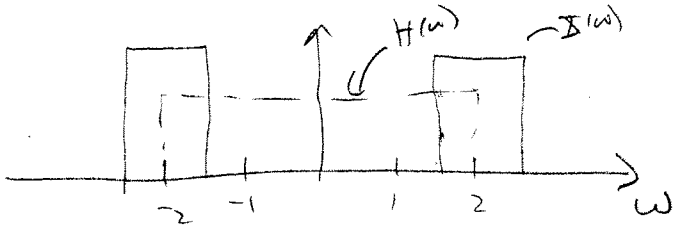
b) $x(t) = 5\,\text{sinc}\left(\frac{t}{2\pi}\right)\cos(2t)$

$$\text{sinc}(Wt) \longleftrightarrow \frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right)$$

$$\text{sinc}\left(\frac{t}{2\pi}\right) \longleftrightarrow 2\pi\,\text{rect}(\omega)$$

$$5\,\text{sinc}\left(\frac{t}{2\pi}\right) \longleftrightarrow 10\pi\,\text{rect}(\omega)$$

$$5\,\text{sinc}\left(\frac{t}{2\pi}\right)\cos(2t) \longleftrightarrow 5\pi\,\text{rect}(\omega+2) + 5\pi\,\text{rect}(\omega-2) = X(\omega)$$



$$Y(\omega) = X(\omega)\,H(\omega) = 5\pi\,\text{rect}\left(\frac{\omega-1.75}{0.5}\right)e^{-j\omega} + 5\pi\,\text{rect}\left(\frac{\omega+1.75}{0.5}\right)e^{-j\omega}$$

$$\frac{1}{W}\,\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow \text{sinc}(Wt)$$

$$\text{rect}\left(\frac{\omega}{0.5}\right) = \text{rect}\left(\frac{\omega}{2\pi W}\right) \qquad 2\pi W = \frac{1}{2} \quad W = \frac{1}{4\pi}$$

$$\text{rect}\left(\frac{\omega}{0.5}\right) \longleftrightarrow \frac{1}{4\pi}\,\text{sinc}\left(\frac{t}{4\pi}\right)$$

$$5\pi\,\text{rect}\left(\frac{\omega}{0.5}\right) \longleftrightarrow \frac{5}{4}\,\text{sinc}\left(\frac{t}{4\pi}\right)$$

$$5\pi\,\text{rect}\left(\frac{\omega-1.75}{0.5}\right) + 5\pi\,\text{rect}\left(\frac{\omega+1.75}{0.5}\right) \longleftrightarrow \frac{5}{2}\,\text{sinc}\left(\frac{t}{4\pi}\right)\cos(1.75t)$$

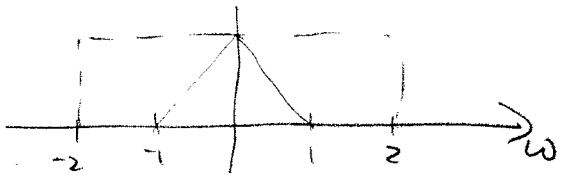
$$e^{-j\omega}\left[5\pi\,\text{rect}\left(\frac{\omega-1.75}{0.5}\right) + 5\pi\,\text{rect}\left(\frac{\omega+1.75}{0.5}\right)\right] \longleftrightarrow \boxed{\frac{5}{2}\,\text{sinc}\left(\frac{t-1}{4\pi}\right)\cos\left(1.75(t-1)\right) = x(t)}$$

5.14

c) $x(t) = \text{sinc}^2\left(\frac{t}{2\pi}\right)$

$$\text{sinc}^2(Bt) \leftrightarrow \frac{1}{B}\Lambda\left(\frac{\omega}{4\pi B}\right) \qquad B = 1/2\pi$$

$$\text{sinc}^2(2\pi t) = 2\pi\,\Lambda\left(\frac{\omega}{2}\right)$$



The entire signal gets through

$$Y(\omega) = X(\omega)\,H(\omega) = 2\pi\,\Lambda\left(\frac{\omega}{2}\right)e^{-j\omega}$$

$$\boxed{y(t) = \text{sinc}^2\left(\frac{t-1}{2\pi}\right)}$$

d) $x(t) = \sum_K \frac{1}{K}\cos\left(\frac{K\pi}{2}t + 30°\right) \qquad \omega_0 = \frac{\pi}{2} = 1.57$

K	$K f_0 = F$	
1	1.57	← only this frequency gets through
2	3.14	

$$H(\omega) = H\left(\tfrac{\pi}{2}\right) = e^{-j\frac{\pi}{2}}$$

$$y(t) = \frac{1}{(1)}\cos\left(\tfrac{\pi}{2}t + 30° + \left(-\tfrac{\pi}{2}\right)\right)$$

↑ $K=1$ ↑ phase of $H(\omega)$

$$\boxed{y(t) = \cos\left(\tfrac{\pi}{2}(t-1) + 30°\right)}$$

#5 S.16 $H(\omega) = \begin{cases} 1+\cos(2\pi\omega) & -0.5 < \omega < 0.5 \\ 0 & \text{else} \end{cases}$

a) find $h(t)$

b) find $y(t)$ for $x(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$

c) find $y(t)$ for $x(t) = \text{sinc}\left(\frac{t}{4\pi}\right)$

a) $H(\omega) = \left[1+\cos(2\pi\omega)\right] \text{rect}\left(\frac{\omega}{1}\right)$

$\frac{1}{W}\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow \text{sinc}(Wt)$

$\text{rect}\left(\frac{\omega}{2\pi W}\right) = \text{rect}\left(\frac{\omega}{1}\right) \qquad W = \frac{1}{2\pi}$

$\text{rect}\left(\frac{\omega}{1}\right) \longleftrightarrow \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right)$

$\cos(2\pi\omega)\text{rect}\left(\frac{\omega}{1}\right) = \frac{e^{j2\pi\omega}}{2}\text{rect}\left(\frac{\omega}{1}\right) + \frac{e^{-j2\pi\omega}}{2}\text{rect}\left(\frac{\omega}{1}\right)$

$\longleftrightarrow \frac{1}{4\pi}\text{sinc}\left(\frac{t+2\pi}{2\pi}\right) + \frac{1}{4\pi}\text{sinc}\left(\frac{\omega-2\pi}{2\pi}\right)$

so $\boxed{h(t) = \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right) + \frac{1}{4\pi}\text{sinc}\left(\frac{t+2\pi}{2\pi}\right) + \frac{1}{4\pi}\text{sinc}\left(\frac{t-2\pi}{2\pi}\right)}$

(b) $x(t) = \text{sinc}\left(\frac{t}{2\pi}\right) \qquad X(\omega) = 2\pi\,\text{rect}\left(\frac{\omega}{1}\right)$

$Y(\omega) = H(\omega)X(\omega) = 2\pi H(\omega)$

$\boxed{y(t) = \text{sinc}\left(\frac{t}{2\pi}\right) + \frac{1}{2}\text{sinc}\left(\frac{t+2\pi}{2\pi}\right) + \frac{1}{2}\text{sinc}\left(\frac{t-2\pi}{2\pi}\right)}$

(c) $x(t) = \text{sinc}\left(\frac{t}{4\pi}\right) \qquad X(\omega) = 4\pi\,\text{rect}\left(\frac{\omega}{0.5}\right)$

$Y(\omega) = H(\omega)X(\omega) = 4\pi\left[1+\cos(2\pi\omega)\right]\text{rect}\left(\frac{\omega}{0.5}\right)$

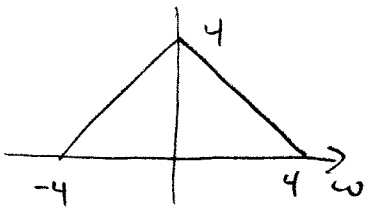
$\boxed{y(t) = \text{sinc}\left(\frac{t}{4\pi}\right) + \frac{1}{2}\text{sinc}\left(\frac{t+2\pi}{4\pi}\right) + \frac{1}{2}\text{sinc}\left(\frac{t-2\pi}{4\pi}\right)}$

#6 $x(t) = \frac{8}{\pi}\text{sinc}^2\left(2\frac{(t-1)}{\pi}\right)$ $\quad H(\omega) = \begin{cases} 5e^{-j2\omega} & |\omega| \le 2 \\ 0 & \text{else} \end{cases}$

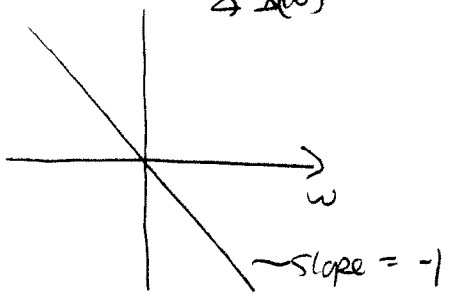
a) for $x(t) = \text{sinc}^2\left(\frac{2t}{\pi}\right) \longleftrightarrow X(\omega) = \frac{\pi}{2}\Lambda\left(\frac{\omega}{8}\right)$

so $\boxed{X(\omega) = 4\Lambda\left(\frac{\omega}{8}\right)e^{-j\omega}}$

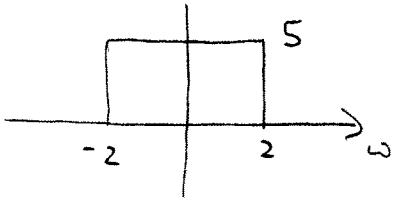
b) $|X(\omega)|$



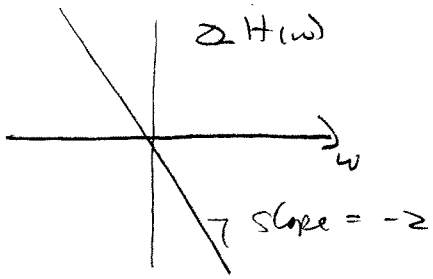
$\measuredangle X(\omega)$



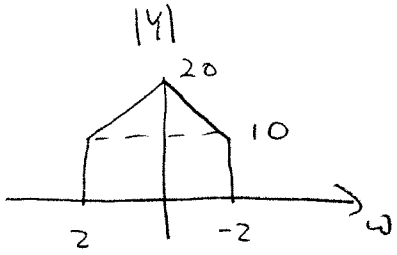
c) $|H(\omega)|$



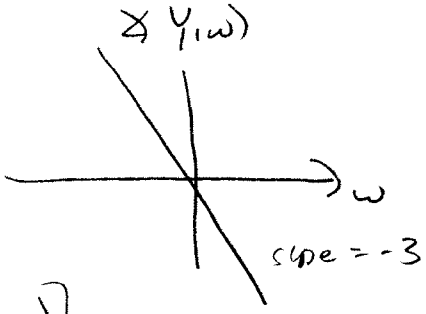
$\measuredangle H(\omega)$



d) $|Y|$



$\measuredangle Y(\omega)$



$$Y(\omega) = \left[10\,\text{rect}\left(\frac{\omega}{4}\right) + 10\Lambda\left(\frac{\omega}{4}\right)\right]e^{-j3\omega}$$

$$\text{rect}\left(\frac{\omega}{2\pi W}\right) \longleftrightarrow W\text{sinc}(Wt) \qquad W = \frac{2}{\pi}$$

$$\Lambda\left(\frac{\omega}{4\pi B}\right) \longleftrightarrow B\text{sinc}^2(Bt) \qquad B = 1/\pi$$

$$\boxed{y(t) = \frac{20}{\pi}\text{sinc}\left(\frac{2}{\pi}(t-3)\right) + \frac{10}{\pi}\text{sinc}^2\left(\frac{1}{\pi}(t-3)\right)}$$