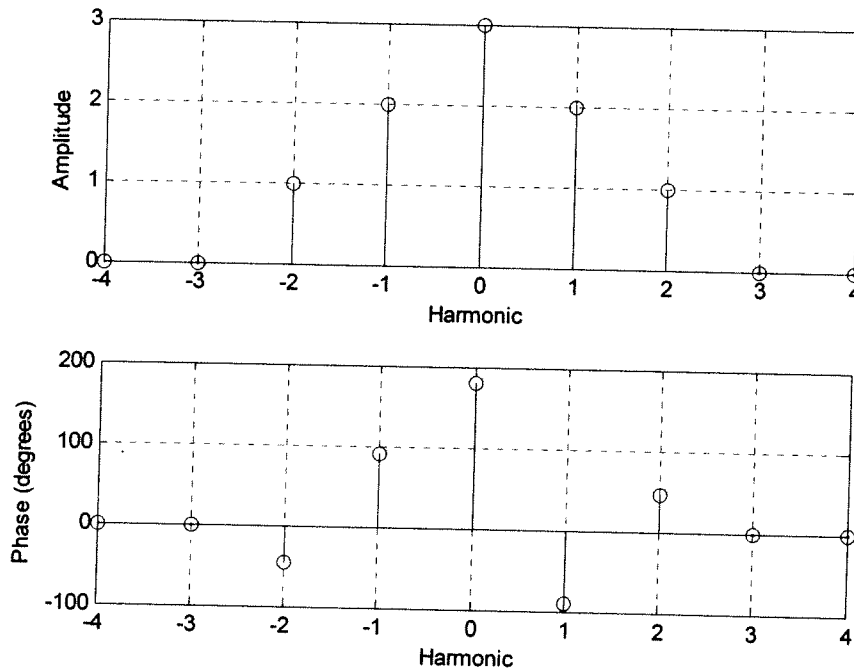


ECE 300
Signals and Systems
Homework 5

Due Date: Tuesday January 16, at 2:40 PM

Problems:

1. A signal $x(t)$, which has a fundamental period of 2 seconds, has the following spectrum (all phases are multiples of 45



degrees)

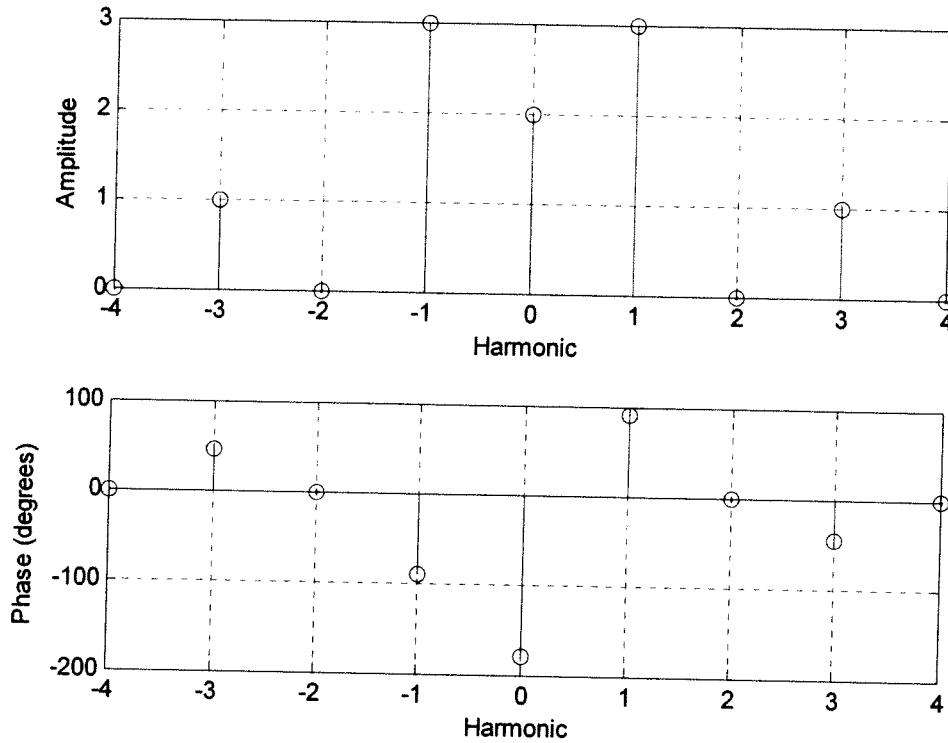
a) What is $x(t)$? Your expression must be real.

b) What is the average value of $x(t)$?

c) What is the average power in $x(t)$?

2. K & H, Problem 3.13. For part c you should get $c_k^y = c_{k-1}^x$, use Euler's identity for part d.

3. A signal $x(t)$, which has a fundamental period of 3 seconds, has the following spectrum (all phases are multiples of 45 degrees)



- What is $x(t)$? Your expression must be real.
- What is the average power in $x(t)$?
- Sketch the single sided power spectrum for $x(t)$.

4. Simplify each of the following into the form $c_k = \alpha(k)e^{-j\beta(k)}\text{sinc}(\lambda k)$

a) $c_k = \frac{e^{j7k\pi} - e^{-j2k\pi}}{k\pi j}$

b) $c_k = \frac{e^{-j2\pi k} - e^{-j5\pi k}}{jk}$

c) $c_k = \frac{e^{j5k} - e^{j2k}}{k}$

Scrambled Answers $c_k = 3\pi e^{-j\frac{7\pi k}{2}} \text{sinc}\left(\frac{3k}{2}\right)$, $c_k = 3e^{j(\frac{7}{2}k + \frac{\pi}{2})} \text{sinc}\left(\frac{3k}{2\pi}\right)$, $c_k = 9e^{j\frac{5}{2}k\pi} \text{sinc}\left(k\frac{9}{2}\right)$

5. An LTI system has the transfer function $H(s) = \frac{10}{s+3}$. Assume the input to this system is $x(t) = 2 \cos(3t + 30^\circ)$

- Determine the steady state output of the system.
- Determine the bandwidth (-3dB point) of the system. Be sure to include the proper units!
- Write the output of the system in terms of a time delay between the input and output signals. That is, the output should have the form

$$y(t) = A \cos(3t + 30^\circ - \theta) = A \cos(3(t - t_d) + 30^\circ)$$

Determine the time delay of the system, t_d .

6. Pre-Lab/Matlab Exercises (to be done by all students. Turn this in with your homework and bring a copy of this with you to lab!

Note: The function we use for numerical integration, `quadl`, has a default tolerance. This tolerance level is used so the function can determine when its estimate of the integral is "good enough". When dealing with either small valued functions, or very short intervals, you may need to change this tolerance. In the following problems, you will need to change the tolerance to get the proper results (use `help quadl`). Read the **Appendix** and then do the following:

- Plot
 - the Fourier series representation,
 - the spectrum,
 - the single sided power spectrum in dBmV

for each of the waveforms listed below. Use 9 terms for each function. It is probably easiest to modify your program `Complex_Fourier_Series.m`.

Some useful hints:

- It is probably easiest for you to construct two arrays in Matlab

```
C_single = [c0 c];  
C_double = [fliplr(conj(c)) c0 c]; % conj takes the complex conjugate, fliplr does what?
```

- You should use the `log10` command to compute the base 10 logarithm. You may want to use the Matlab function `length` to determine the length of `c`. The functions `angle` and `abs` will also be useful.

You should also use the `subplot` command so you can plot both a signal and its Fourier series representation in one graph, its spectrum in the next two graphs, and its single-sided power spectrum in a subsequent graph within the same window. Utilize the `stem` command in Matlab to do the plotting.

For the power spectrum plot, the y-axis should be labeled $Power(dBmV)$, the x-axis labeled $Harmonic$. Your plots for function $x_2(t)$ should look like those in Figure 1. (Note the axis on the top graph is limited using the command `axis('tight')`).

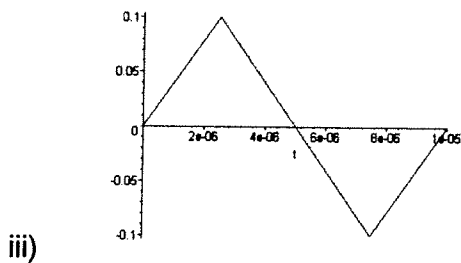
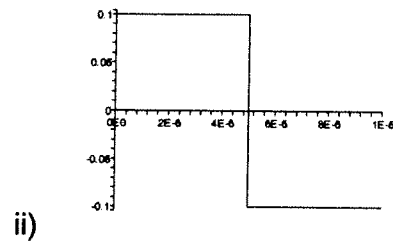
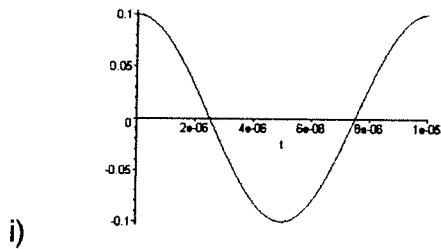
The waveforms, each having zero DC offset, are:

i) $x_1(t) = 0.1 \cos(2\pi 100 \times 10^3 t)$ V

ii) $x_2(t)$ is a square wave of period $10 \mu s$ and peak-to-peak amplitude 0.2 V.

iii) $x_3(t)$ is a triangle wave of period $10 \mu s$ and peak-to-peak amplitude 0.2 V.

Just to be sure there is no confusion regarding the waveforms, they are displayed below.



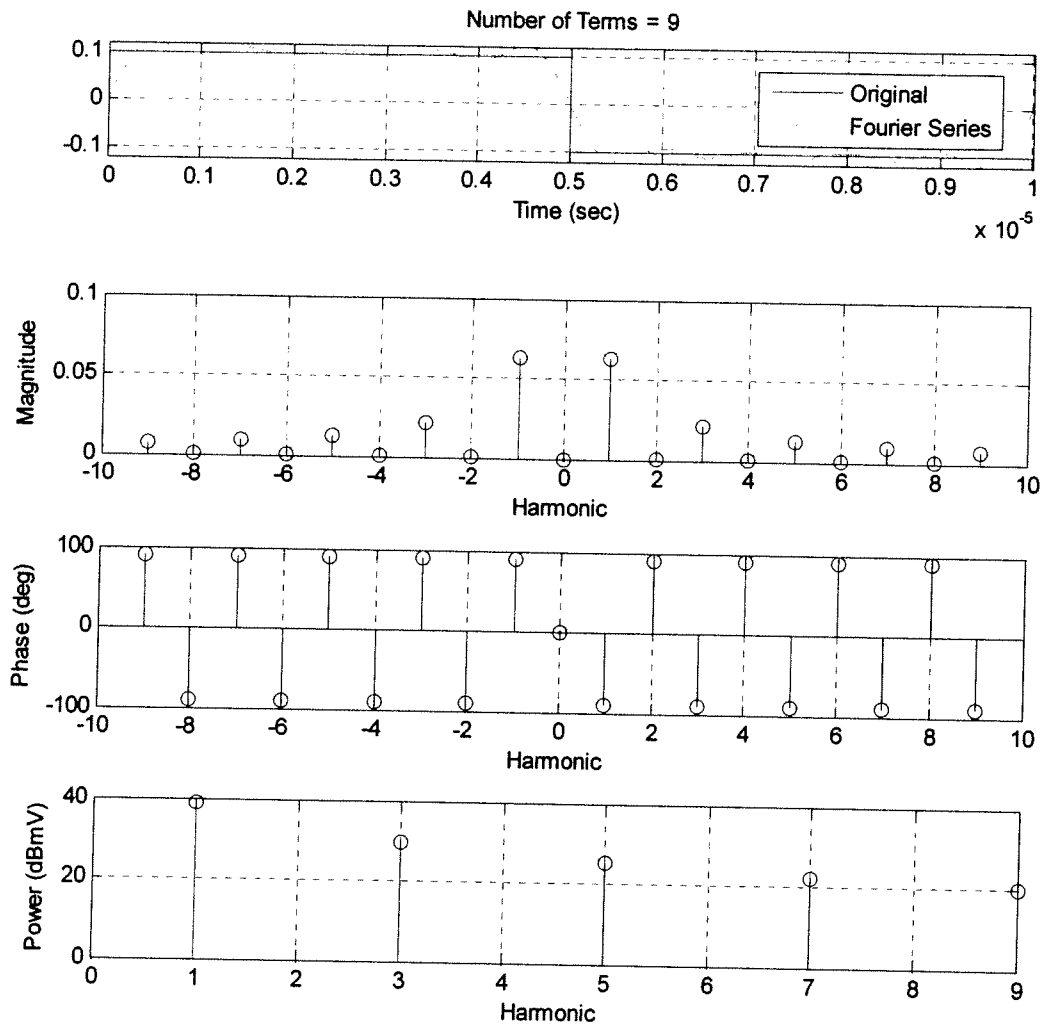


Figure 1: Results for problem 6a for function $x_2(t)$.

b) For each waveform, create a table containing a column of harmonics, corresponding frequency (in Hz), and values of predicted decibel levels (dBmV) for the corresponding c_k . Do not read these from your plots, but get them directly from your program. Note that if you do not put a semicolon after a statement in Matlab, the values in the array will be printed to the screen. (For $x_2(t)$ you should get numbers like $-\text{inf}$, 38.0, -157, 29.5, -161, 25...)

Appendix

There are a number of different ways of graphically presenting the information in the Fourier series representation of a periodic function. The most common methods of presenting this information are by plotting the spectrum of the signal, or the power spectrum of the signal. In the following examples we will use the function

$$f(t) = \begin{cases} 0 & -2 \leq t < -1 \\ t & -1 \leq t < 1 \\ 1 & 1 \leq t < 2 \end{cases}$$

Spectrum of a Periodic Signal The spectrum of a periodic signal is a plot of the magnitude $|c_k|$ against the corresponding frequency $k\omega_0$, and the phase $\angle c_k$ against the corresponding frequency. Since our signal is periodic we only have values at discrete frequencies ($k\omega_0$), we do not know what happens in between these discrete frequencies so we do not "connect the dots". Since ω_0 is common to all of the discrete frequencies, we often plot against the harmonic k , rather than the frequency $k\omega_0$. Figure 2 shows the spectrum of $f(t)$.

Power Spectrum of a Periodic Signal The power spectrum of a signal tells us how the power in the signal is distributed in frequency. To plot the power spectrum of a signal, we consider each term of the complex Fourier series, $c_k e^{jk\omega_0 t}$. The average power in this term is $|c_k|^2$. The power spectrum indicates the power associated with each frequency, and is thus a plot of $|c_k|^2$ against frequency $k\omega_0$ or just the harmonic k . In this plot we need to include both positive and negative frequencies. Figure 3 shows the power spectrum of $f(t)$.

Single-Sided Power Spectrum of a Periodic Signal Because the magnitudes of the coefficients are even ($|c_k| = |c_{-k}|$), the powers associated with negative frequencies are the same as those associated with positive frequencies. Since only positive frequencies are realizable, we often just want to plot the power against positive frequencies. This gives us a single sided power spectrum, which is a plot of $|c_0|^2$ $2|c_1|^2$ $2|c_2|^2$... $2|c_N|^2$ versus the corresponding frequency 0 ω_0 $2\omega_0$... $N\omega_0$. Since the fundamental frequency ω_0 is common to all of the frequency terms, we often just plot against the harmonics 0 1 2 ... N . Figure 4 shows the single sided power spectrum of $f(t)$.

Spectrum Analyzer (SA) Display The SA displays a one-sided spectrum, but instead of showing the value of $2|c_k|^2$ at each frequency, the spectrum analyzer shows average power in decibels with respect to a one millivolt RMS reference. For the sinusoid at frequency kf_0 , the average power in decibels is given by

$$P_k|_{dB} = 10 \log_{10} \frac{P_k}{P_{ref}},$$

where the power P_k represents the power spectrum coefficient $2|c_k|^2$, and the power P_{ref} is the average power delivered to a one-ohm resistor by a one millivolt RMS sinusoid. We have

$$P_k|_{dBmV} = 10 \log_{10} \left(\frac{2|c_k|^2}{(0.001)^2} \right) \text{ dBmV} .$$

The units “dBmV” indicate that the reference for the decibels is a one millivolt RMS sinusoid.

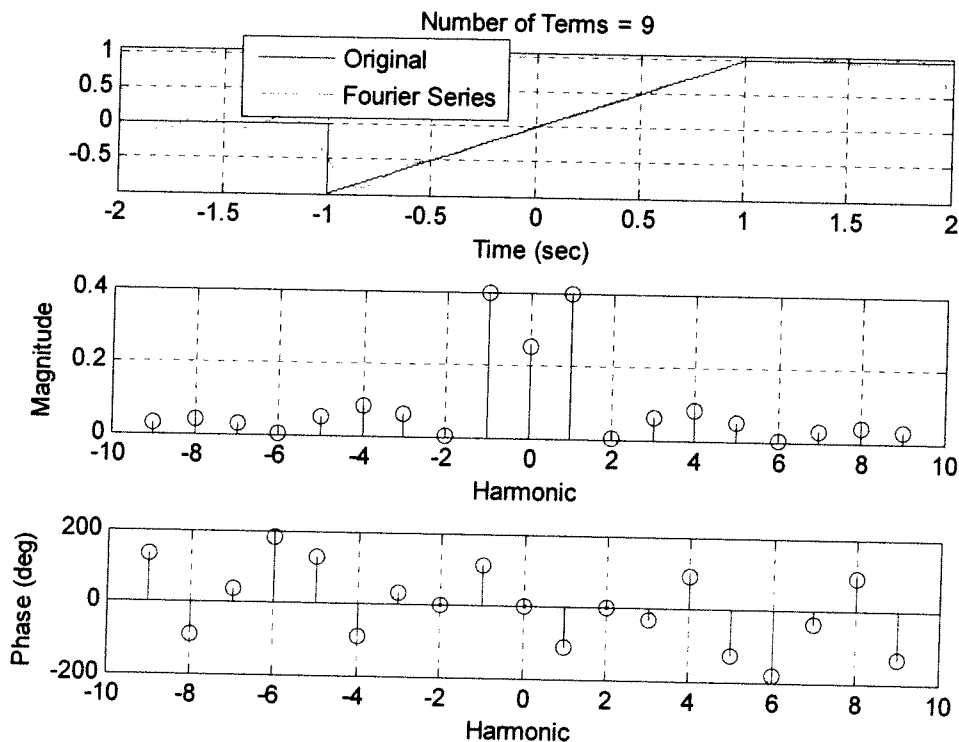


Figure 2: The spectrum of $f(t)$.

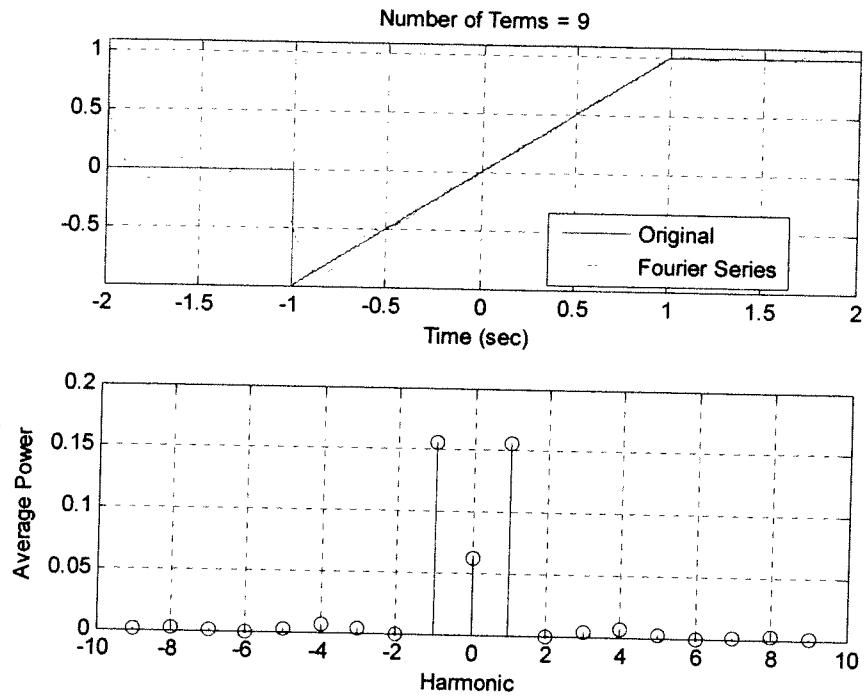


Figure 3: The power spectrum of $f(t)$.

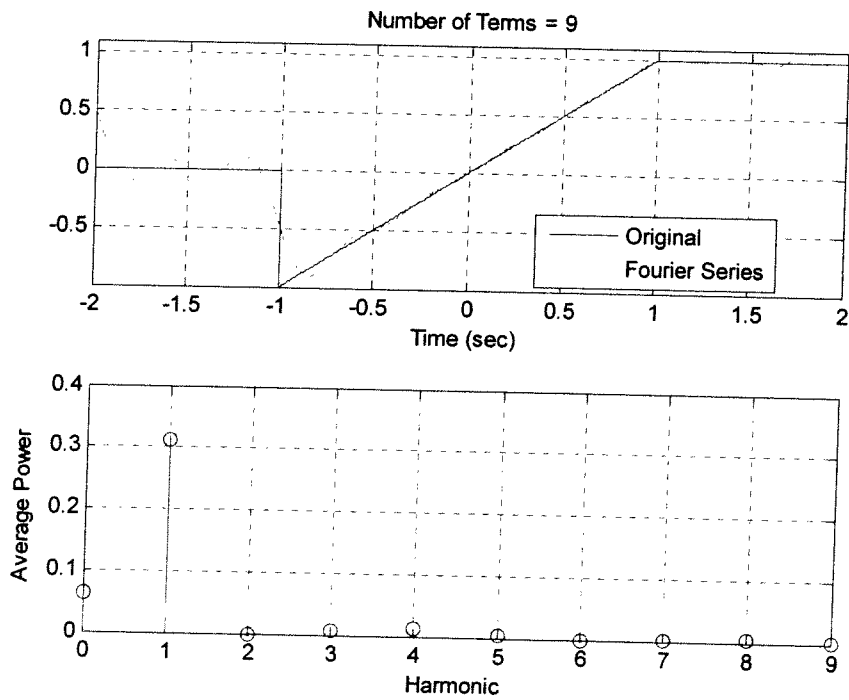
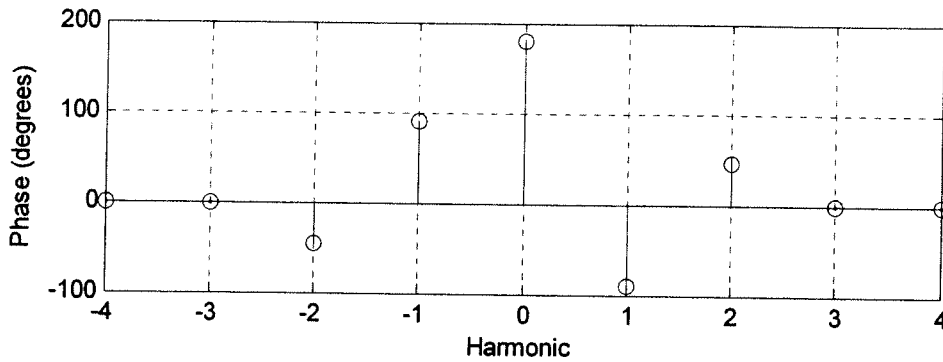
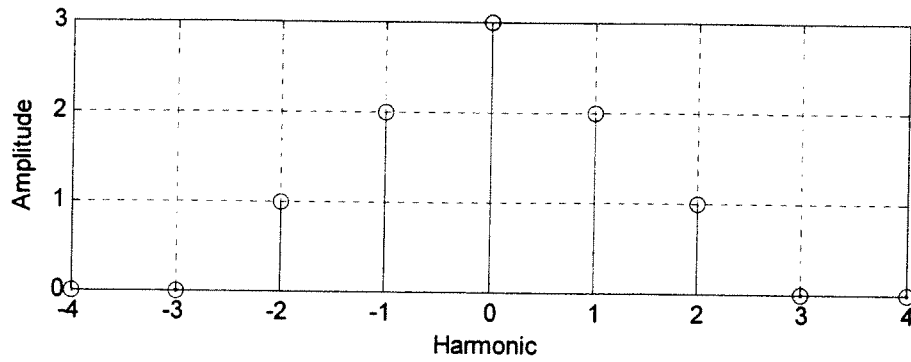


Figure 4: The single-sided power spectrum of $f(t)$.

Problem Set 5

ECE-300

#1



a) (The easy way) $x(t) = |c_0|e^{j\phi_0} + \sum 2|c_k| \cos(k\omega_0 t + \phi_k)$
 here $\omega_0 = \frac{2\pi}{2} = \pi$

$$x(t) = 3e^{j180^\circ} + 2|2| \cos(\pi t - 90^\circ) + 2|1| \cos(2\pi t + 45^\circ)$$

$$x(t) = -3 + 4 \cos(\pi t - 90^\circ) + 2 \cos(2\pi t + 45^\circ)$$

$$\text{or } x(t) = -3 + 4 \sin(\pi t) - 2 \cos(2\pi t + 45^\circ)$$

(The hard way)

$$\begin{aligned} x(t) &= |c_0|e^{j\phi_0} + |c_1|e^{j\phi_1}e^{j\omega_0 t} + |c_{-1}|e^{j\phi_{-1}}e^{-j\omega_0 t} + |c_2|e^{j\phi_2}e^{j2\omega_0 t} + |c_{-2}|e^{j\phi_{-2}}e^{-j2\omega_0 t} \\ &= 3e^{j180^\circ} + 2e^{j(-90^\circ)}e^{j\omega_0 t} + 2e^{j90^\circ}e^{-j\omega_0 t} + 1e^{j45^\circ}e^{j2\omega_0 t} + 1e^{j(-45^\circ)}e^{-j2\omega_0 t} \\ &= -3 + 2 \left(e^{j(\omega_0 t - 90^\circ)} + e^{-j(\omega_0 t - 90^\circ)} \right) + 1 \left(e^{j(2\omega_0 t + 45^\circ)} + e^{-j(2\omega_0 t + 45^\circ)} \right) \\ &= -3 + 4 \cos(\omega_0 t - 90^\circ) + 2 \cos(2\omega_0 t + 45^\circ) \\ &= -3 + 4 \cos(\pi t - 90^\circ) + 2 \cos(2\pi t + 45^\circ) \end{aligned}$$

b) $\bar{x} = -3 = 3 \angle 180^\circ = \bar{x}$

c) $P_{\text{ave}} = |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 = |3|^2 + 2|2|^2 + 2|1|^2 = 19 \text{ W} = P_{\text{ave}}$

#2

Problem 3.13 Given: $x(t) = \sum_{k=-\infty}^{\infty} C_k^x \exp(jk\omega_0 t)$ $\omega_0 = \frac{2\pi}{T}$

Compute C_k^y for periodic signal $y(t)$ above

(a) $y(t) = x(t-1)$ (b) $y(t) = \frac{dx(t)}{dt}$ (c) $y(t) = x(t) \exp\left[j\left(\frac{2\pi}{T}\right)t\right]$

(d) $y(t) = x(t) \cos\left(\frac{2\pi}{T}t\right)$

(a) $y(t) = x(t-1)$

$$C_k^x = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$C_k^y = \frac{1}{T_0} \int_{T_0} x(t-1) e^{-jk\omega_0 t} dt$$

let $\lambda = t-1$ $t = \lambda+1$

$$= \frac{1}{T_0} \int_{T_0} x(\lambda) e^{-jk\omega_0(\lambda+1)} d\lambda$$

$$= e^{-jk\omega_0} \frac{1}{T_0} \int_{T_0} x(\lambda) e^{-jk\omega_0 \lambda} d\lambda = e^{-jk\omega_0} C_k^x$$

$$C_k^y = e^{-jk\omega_0} C_k^x$$

#2 (continued)

(b)
$$v(t) = \frac{d x(t)}{dt}$$

$$x(t) = \sum C_k^x e^{j k \omega_0 t}$$

$$v(t) = \frac{d x}{dt} = \sum C_k^x (j k \omega_0) e^{j k \omega_0 t} = \sum C_k^v e^{j k \omega_0 t}$$

$$C_k^v = j k \omega_0 C_k^x$$

(c)
$$v(t) = x(t) e^{j \frac{2\pi}{T_0} t} = x(t) e^{j \omega_0 t}$$

$$C_k^v = \frac{1}{T_0} \int_{T_0} v(t) e^{-j k \omega_0 t} dt = \frac{1}{T_0} \int_{T_0} x(t) e^{j \frac{2\pi}{T_0} t} e^{-j k \omega_0 t} dt$$

$$= \int_{T_0} x(t) e^{-j k \omega_0 t} e^{j \omega_0 t} dt = \int_{T_0} x(t) e^{-j (k-1) \omega_0 t} dt = C_{k-1}^x$$

$$C_k^v = C_{k-1}^x$$

(d)
$$v(t) = x(t) \cos\left(\frac{2\pi}{T_0} t\right) = x(t) \cos(\omega_0 t)$$

$$= x(t) \frac{e^{j \omega_0 t}}{2} + x(t) \frac{e^{-j \omega_0 t}}{2}$$

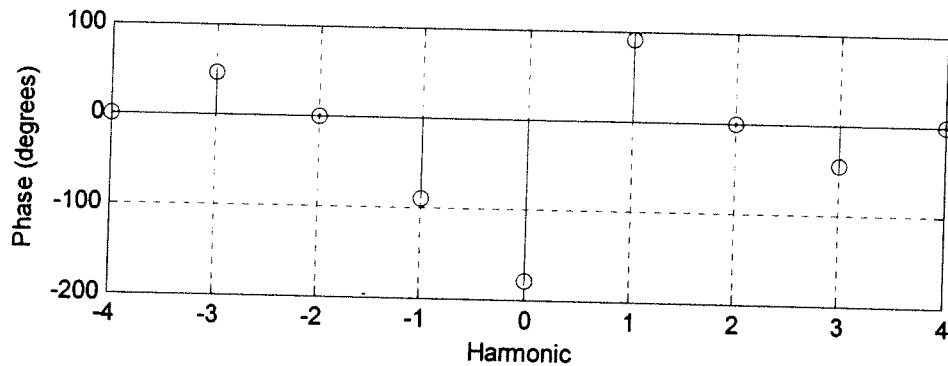
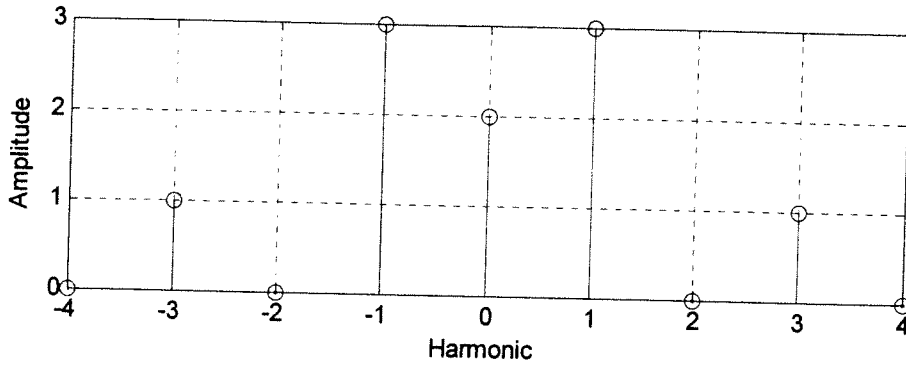
From part (c), this implies

$$C_k^v = \frac{C_{k-1}^x + C_{k+1}^x}{2}$$

Problem Let 5

$\epsilon/\epsilon \rightarrow 0$

#3



a) (The easy way) $x(t) = |c_0|e^{j\delta c_0} + \sum 2|c_k| \cos(k\omega_0 t + \delta c_k)$

here $\omega_0 = 2\pi/5 = 2\pi/3$

$x(t) = |c_0|e^{j\delta c_0} + 2|c_1| \cos(\omega_0 t + \delta c_1) + 2|c_2| \cos(2\omega_0 t + \delta c_2) + 2|c_3| \cos(3\omega_0 t + \delta c_3)$

$x(t) = 2e^{j(-180^\circ)} + 2|3| \cos(\omega_0 t + 90^\circ) + 0 + 2|1| \cos(3\omega_0 t - 45^\circ)$

$x(t) = -2 + 6 \cos\left(\frac{2\pi}{3}t + 90^\circ\right) + 2 \cos(2\pi t - 45^\circ)$

(The hard way)

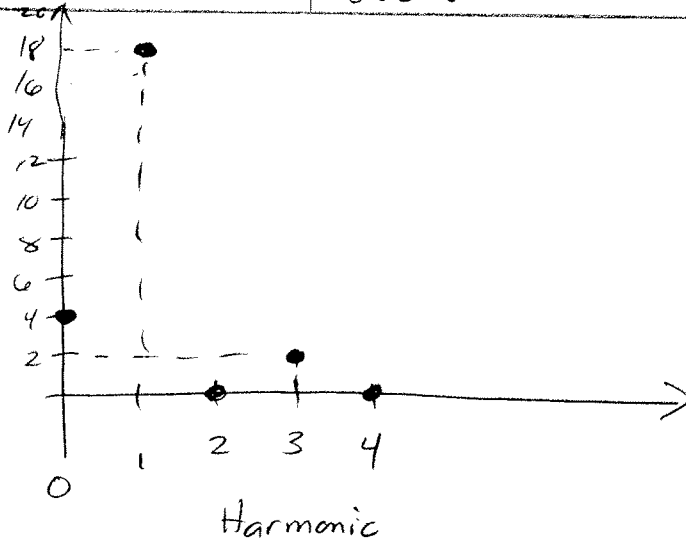
$$\begin{aligned}
 x(t) &= |c_0|e^{j\delta c_0} + |c_1|e^{j\delta c_1}e^{j\omega_0 t} + |c_1|e^{j\delta c_1}e^{-j\omega_0 t} + |c_3|e^{j\delta c_3}e^{j3\omega_0 t} + |c_3|e^{j\delta c_3}e^{-j3\omega_0 t} \\
 &= 2e^{j(-180^\circ)} + 3e^{j90^\circ}e^{j\omega_0 t} + 3e^{j(-90^\circ)}e^{-j\omega_0 t} + 1e^{j(-45^\circ)}e^{j3\omega_0 t} + 1e^{j45^\circ}e^{-j3\omega_0 t} \\
 &= -2 + 3\left(e^{j(\omega_0 t + 90^\circ)} + e^{-j(\omega_0 t + 90^\circ)}\right) + 1\left(e^{j(3\omega_0 t - 45^\circ)} + e^{-j(3\omega_0 t - 45^\circ)}\right) \\
 &= -2 + 6 \cos\left(\frac{2\pi}{3}t + 90^\circ\right) + 2 \cos(2\pi t - 45^\circ)
 \end{aligned}$$

b) $P_{ave} = |c_0|^2 + 2|c_1|^2 + 2|c_2|^2 + 2|c_3|^2 = |2|^2 + 2|3|^2 + 2|0|^2 + 2|1|^2 = 24W = P_{ave}$

32-141 50 SHEETS
22-112 100 SHEETS
32-141 240 SHEETS



#3



#4 (a)
$$C_k = \frac{e^{j\frac{1}{2}k\pi} - e^{-j\frac{1}{2}k\pi}}{jk\pi} = e^{j\frac{1}{2}k\pi} \frac{e^{j\frac{1}{2}k\pi} - e^{-j\frac{1}{2}k\pi}}{jk\pi}$$

$$= \frac{e^{j\frac{1}{2}k\pi}}{k\pi} \cdot 2 \left[\frac{e^{j\frac{1}{2}k\pi} - e^{-j\frac{1}{2}k\pi}}{2j} \right]$$

$$= \frac{2e^{j\frac{1}{2}k\pi}}{k\pi} \sin\left(\frac{1}{2}k\pi\right)$$

$$= \frac{2e^{j\frac{1}{2}k\pi}}{\pi k \left(\frac{1}{2}\right) \left(\frac{2}{1}\right)} = \boxed{e^{j\frac{1}{2}k\pi} \operatorname{sinc}\left(\frac{k}{2}\right) = C_k}$$

(b)
$$C_k = \frac{e^{-j\frac{1}{2}k\pi} - e^{-j\frac{3}{2}k\pi}}{jk} = e^{-j\frac{1}{2}k\pi} \frac{e^{+j\frac{1}{2}k\pi} - e^{-j\frac{3}{2}k\pi}}{jk}$$

$$= \frac{2e^{-j\frac{1}{2}k\pi}}{k} \sin\left(\frac{3}{2}k\pi\right) = 2e^{-j\frac{1}{2}k\pi} \frac{\sin\left(\frac{3}{2}k\pi\right)}{k \cdot \left(\frac{3}{2}\right) \left(\frac{2}{3\pi}\right)}$$

$$= 2e^{-j\frac{1}{2}k\pi} \operatorname{sinc}\left(\frac{3k}{2}\right) \cdot \frac{3\pi}{2}$$

$$\boxed{C_k = 3\pi e^{-j\frac{1}{2}k\pi} \operatorname{sinc}\left(\frac{3k}{2}\right)}$$

(c)
$$C_k = \frac{e^{j\frac{1}{2}k\pi} - e^{j\frac{3}{2}k\pi}}{k} = e^{j\frac{1}{2}k\pi} \frac{e^{j\frac{1}{2}k\pi} - e^{j\frac{3}{2}k\pi}}{k}$$

$$= \frac{2j}{k} e^{j\frac{1}{2}k\pi} \left(\frac{e^{j\frac{1}{2}k\pi} - e^{j\frac{3}{2}k\pi}}{2j} \right) = \frac{2j}{k} e^{j\frac{1}{2}k\pi} \sin\left(\frac{3}{2}k\pi\right)$$

$$= \frac{2e^{j\left(\frac{1}{2}k\pi + \frac{\pi}{2}\right)}}{k} \sin\left(\frac{3}{2}k\pi\right) = \frac{2}{k} e^{j\left(\frac{1}{2}k\pi + \frac{\pi}{2}\right)} \sin\left(\pi \cdot \frac{3k}{2\pi}\right)$$

$$= \frac{2e^{j\left(\frac{1}{2}k\pi + \frac{\pi}{2}\right)}}{\frac{\pi \cdot \frac{3k}{2\pi} \cdot \frac{3\pi}{3\pi}}{\pi \cdot \frac{3k}{2\pi} \cdot \frac{3\pi}{3\pi}}} \sin\left(\pi \cdot \frac{3k}{2\pi}\right)$$

$$\boxed{C_k = 3e^{j\left(\frac{1}{2}k\pi + \frac{\pi}{2}\right)} \operatorname{sinc}\left(\frac{3k}{2\pi}\right)}$$



#5 $H(s) = \frac{10}{s+3}$ $x(t) = 2\cos(3t+30^\circ)$

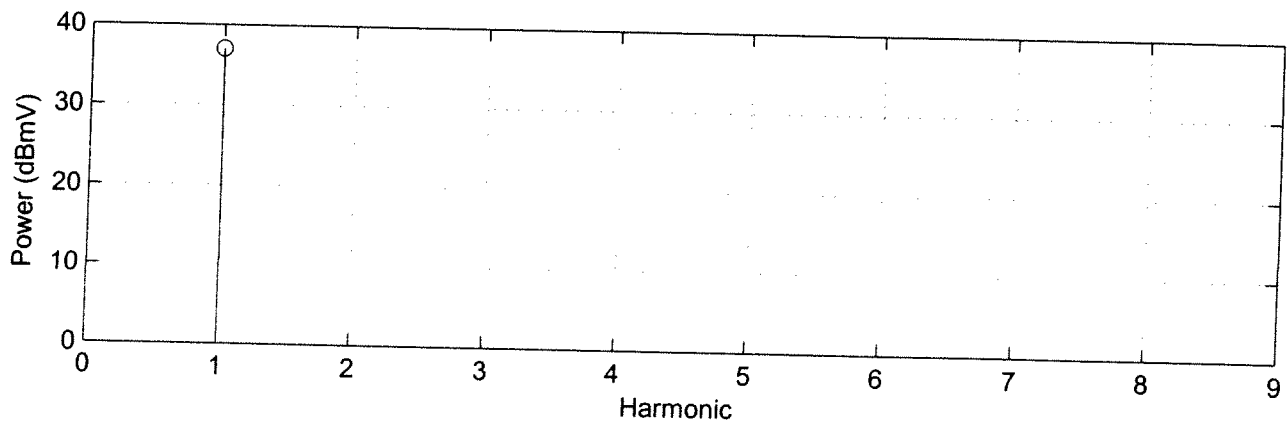
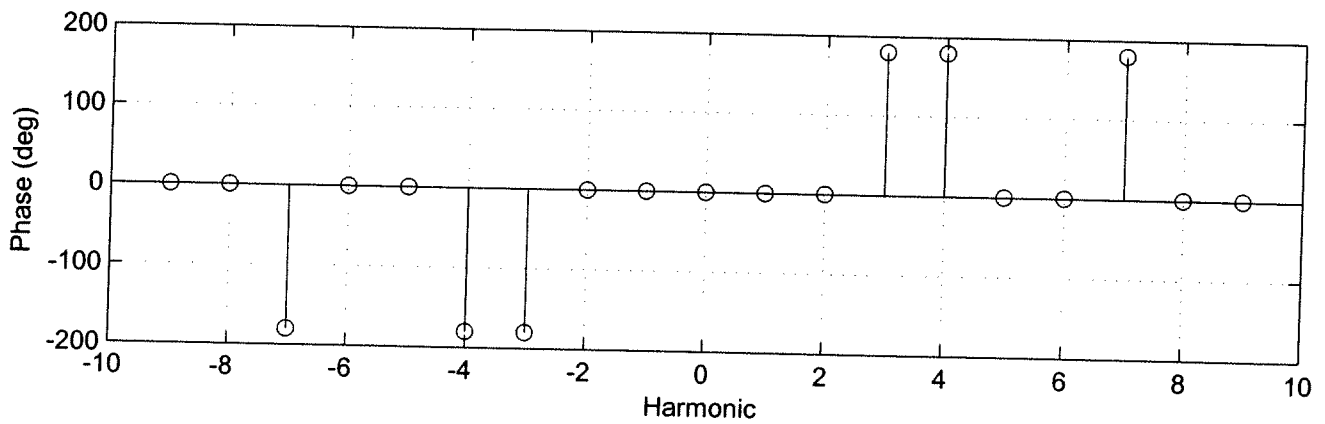
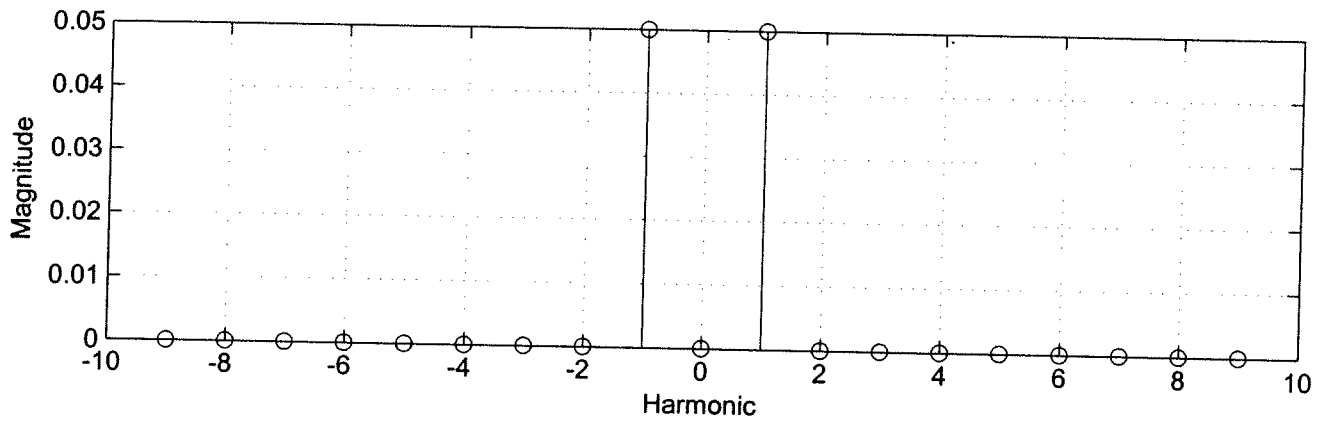
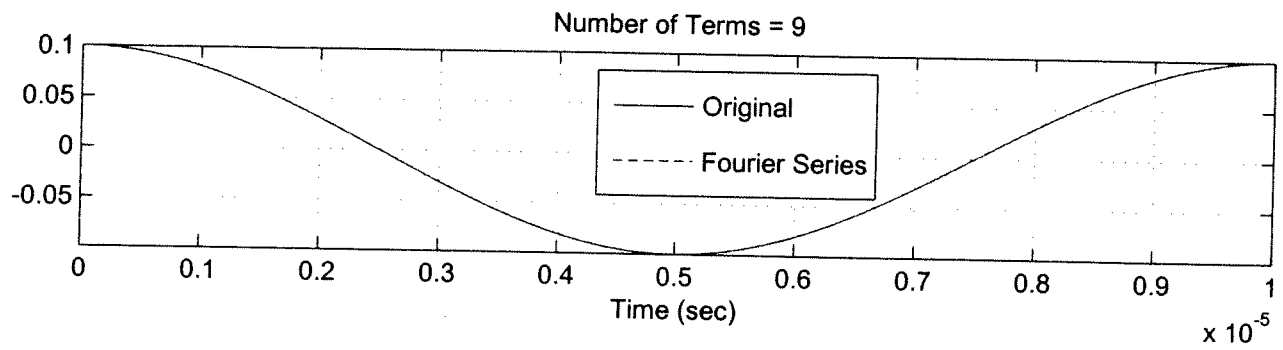
$$H(j3) = \frac{10}{j3+3} = \frac{10}{\sqrt{3^2+3^2}} e^{-j45^\circ} = \frac{10}{\sqrt{18}} e^{-j45^\circ}$$
$$= 2.3570 e^{-j0.7854}$$

a) $y(t) = 2(2.3570)\cos(3t - 0.7854 + 30^\circ)$

$$= 4.7140\cos(3(t - 0.7854/3) + 30^\circ)$$
$$= 4.7140\cos(3(t - 0.2618) + 30^\circ)$$

$t_d = 0.2618$

b) $\omega_n = 3 \text{ rad/sec}$



Number of Terms = 10

