

**ECE 300**  
**Signals and Systems**  
Homework 4

**Due Date:** Tuesday January 9 at 2:40 PM

**Problems:**

1. Find the Fourier series representation for the signal indicated using hand analysis. Clearly indicate the values of  $\omega_0$  and the  $c_k$ . *Hints: (1) Draw the signal, and then use the sifting property to calculate the  $c_k$ . (2) If you understand how to do this, there is very little work involved.*

$$x(t) = \sum_{p=-\infty}^{\infty} \delta(t-3p)$$

2. For the periodic square wave  $x(t)$  with period  $T_o = 0.5$  and

$$x(t) \begin{cases} 1 & 0 \leq t < 0.25 \\ -1 & 0.25 \leq t < 0.5 \end{cases}$$

show that the Fourier series coefficients are given by

$$c_k = \begin{cases} \frac{-2j}{k\pi} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

where  $x(t) = \sum_k c_k e^{jk4\pi t}$

**Special Note:** We will be using the code you write in the next part for the next few homeworks and labs, so be sure you do this and understand what is going on!

3. Read the **Appendix** and then do the following:

a) Copy the file **Trigonometric\_Fourier\_Series.m** to file **Complex\_Fourier\_Series.m**.

b) Modify **Complex\_Fourier\_Series.m** so it computes the average value  $c_0$

c) Modify **Complex\_Fourier\_Series.m** so it also computes  $c_k$  for  $k=1$  to  $k=N$

d) Modify **Complex\_Fourier\_Series.m** so it also computes the Fourier series estimate using the formula

$$x(t) \approx c_0 + \sum_{k=1}^N 2|c_k| \cos(k\omega_0 t + \angle c_k)$$

You will probably need to use the Matlab functions **abs** and **angle** for this.

e) Using the code you wrote in part **d**, find the complex Fourier series representation for the following functions (defined over a single period)

$$f_1(t) = e^{-t}u(t) \quad 0 \leq t < 3$$

$$f_2(t) = \begin{cases} t & 0 \leq t < 2 \\ 3 & 2 \leq t < 3 \\ 0 & 3 \leq t < 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & -2 \leq t < -1 \\ 1 & -1 \leq t < 2 \\ 3 & 2 \leq t < 3 \\ 0 & 3 \leq t < 4 \end{cases}$$

These are the same functions you used for the trigonometric Fourier series. Use  $N = 10$  and turn in your plots for each of these functions. Also, turn in your Matlab program for one of these. *Note that the values of **low** and **high** will be different for each of these functions!*

## Appendix

In the majority of this course we will be using the complex (or exponential) form of the Fourier series, since it is really easier to do various mathematical things with it once you get used to it.

**Exponential Fourier Series** If  $x(t)$  is a periodic function with fundamental period  $T$ , then we can represent  $x(t)$  as a Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$$

where  $\omega_o = \frac{2\pi}{T}$  is the fundamental period,  $c_o$  is the average (or DC, i.e. zero frequency) value, and

$$c_o = \frac{1}{T} \int_0^T x(t) dt$$
$$c_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_o t} dt$$

If  $x(t)$  is a real function, then we have the relationships  $|c_k| = |c_{-k}|$  (the magnitude is even) and  $\angle c_{-k} = -\angle c_k$  (the phase is odd). Using these relationships we can then write

$$x(t) = c_o + \sum_{k=1}^{\infty} 2|c_k| \cos(k\omega_o t + \angle c_k)$$

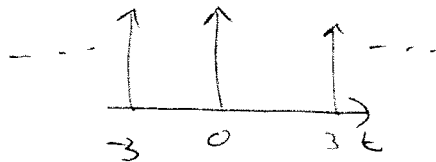
This is usually a much easier form to deal with, since it lends itself easily to thinking of a phasor representation of  $x(t)$ . This will be particularly useful when we starting filtering periodic signals.

Problem set #4

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$$\textcircled{\#1} \quad x(t) = \sum_{p=-\infty}^{\infty} \delta(t-3p)$$

$$T_0 = 3$$



$$C_k = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0} = \frac{1}{3}$$

$$x(t) = \sum_k \frac{1}{3} e^{jk \frac{2\pi}{3} t}$$

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22-142 100 SHEETS  
22-144 200 SHEETS

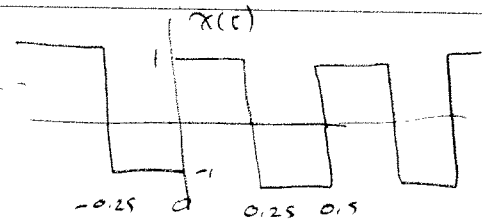
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Problem set 4

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#2

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 0.25 \\ -1 & 0.25 \leq t \leq 0.5 \end{cases}$$



$$T_0 = \frac{1}{2} \quad \omega_0 = \frac{2\pi}{T_0} = 4\pi$$

$$C_0 = 0 \quad (\text{average value is zero})$$

$$C_k = \frac{1}{T_0} \int_{-0.25}^0 (-1) e^{-jk\omega_0 t} dt + \frac{1}{T_0} \int_0^{0.25} (1) e^{-jk\omega_0 t} dt$$

$$= 2 \int_{-0.25}^0 (-1) e^{-jk4\pi t} dt + 2 \int_0^{0.25} (1) e^{-jk4\pi t} dt$$

$$= -2 \frac{e^{-jk4\pi t}}{-jk4\pi} \Big|_{-0.25}^0 + 2 \frac{e^{-jk4\pi t}}{-jk4\pi} \Big|_0^{0.25}$$

$$= \frac{2}{jk4\pi} [1 - e^{-jk\pi}] - \frac{2}{jk4\pi} [e^{+jk\pi} - 1]$$

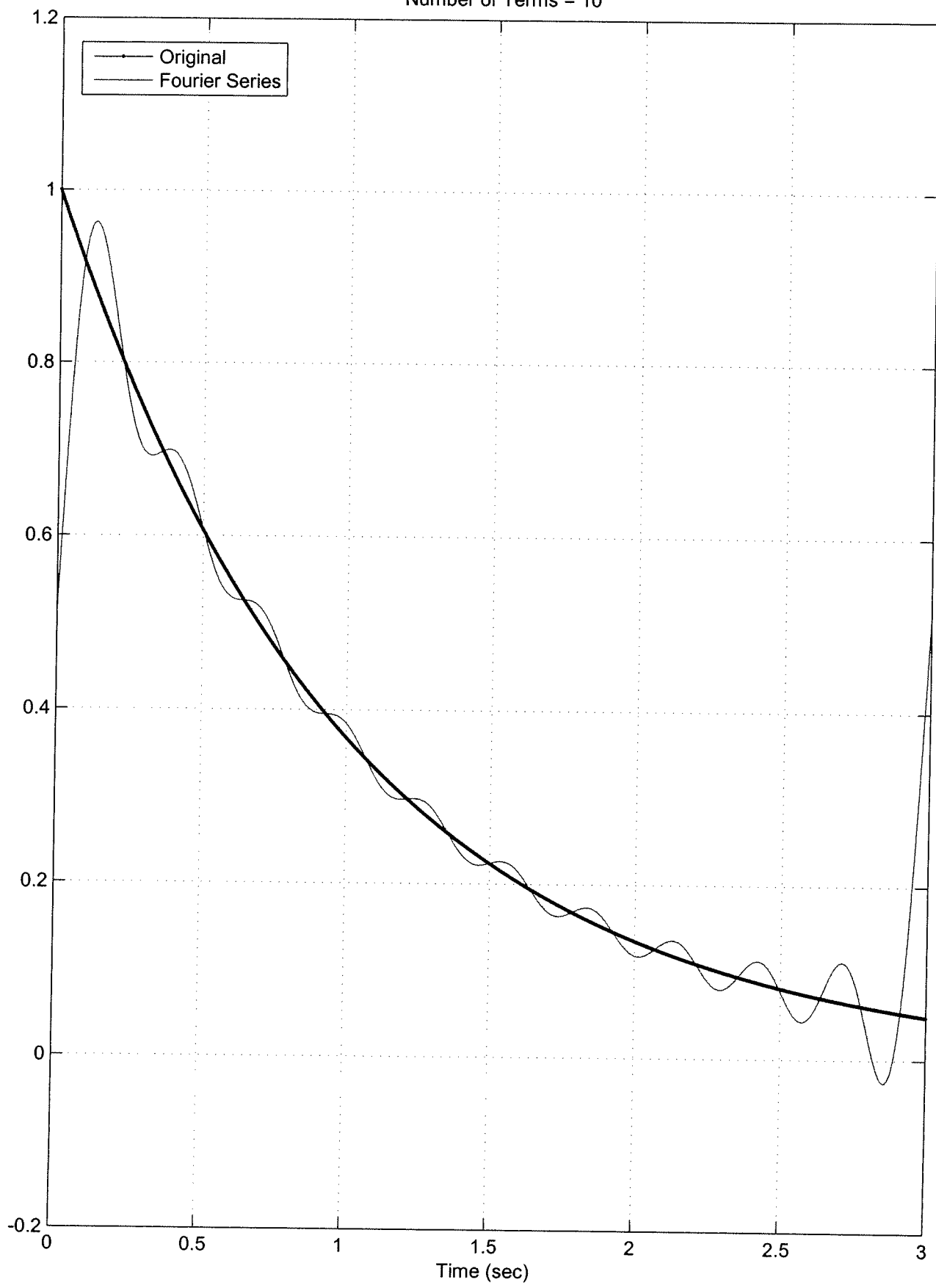
$$= \frac{4}{jk4\pi} [1 - e^{jk\pi}] = \frac{1}{jk\pi} [1 - (-1)^k]$$

$$= \frac{-j}{k\pi} [1 - (-1)^k]$$

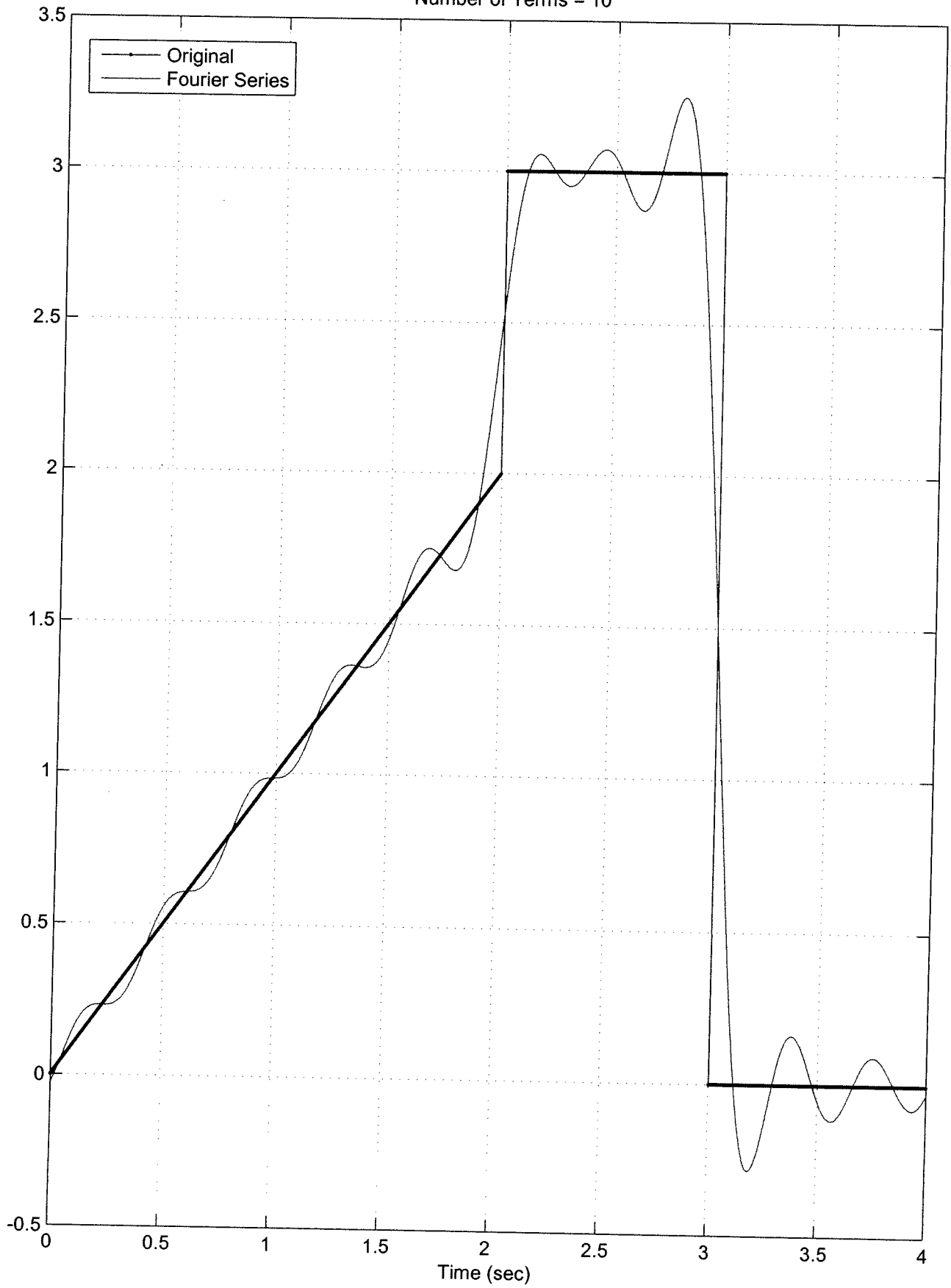
$$C_k = \begin{cases} -\frac{2j}{k\pi} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

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Number of Terms = 10



Number of Terms = 10



Number of Terms = 10

