ECE-320: Linear Control Systems
Homework 9

Due: Tuesday February 17 at the beginning of class

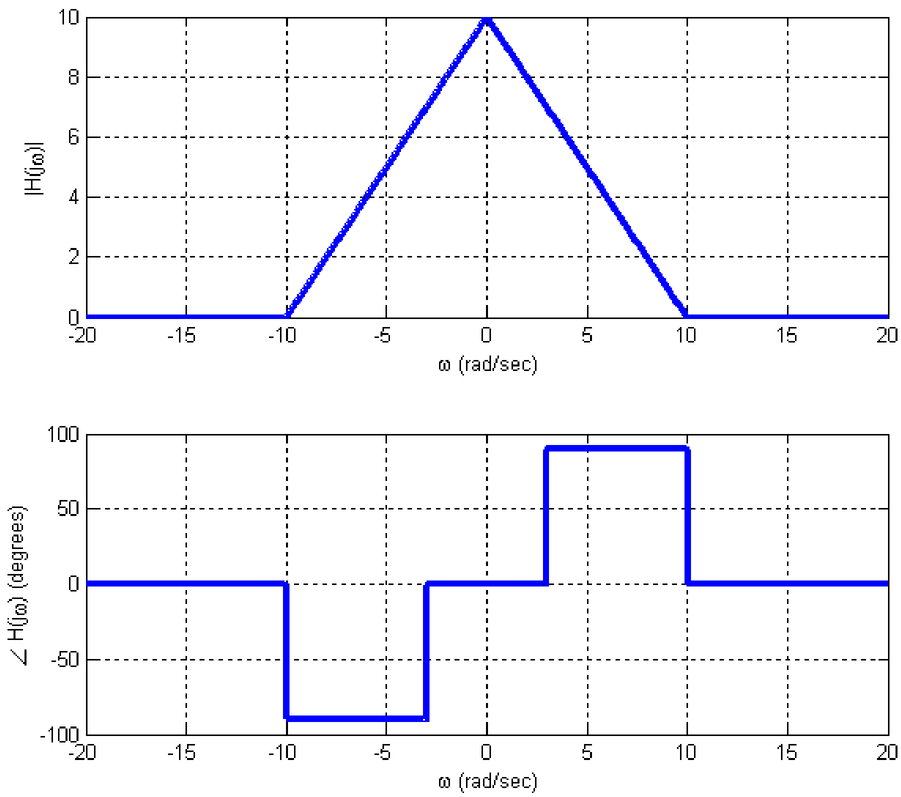
1) Assume $x(t) = 3 + 2\cos(2t - 3)$ is the input to an LTI system with transfer function

$$H(j\omega) = \begin{cases} 2e^{-j\omega} & |\omega| < 3 \\ 3e^{-j2\omega} & |\omega| \geq 3 \end{cases}$$

The **steady state output** will be

a) $y(t) = 6 + 4\cos(2t - 5)$ b) $y(t) = 4\cos(2t - 5)$ c) $y(t) = [3 + 2\cos(2t - 3)][2e^{-j\omega}]$
d) $y(t) = 6 + 4\cos(2t - 3)e^{-j2}$ e) $y(t) = 3 + 4\cos(2t - 5)$ f) none of these

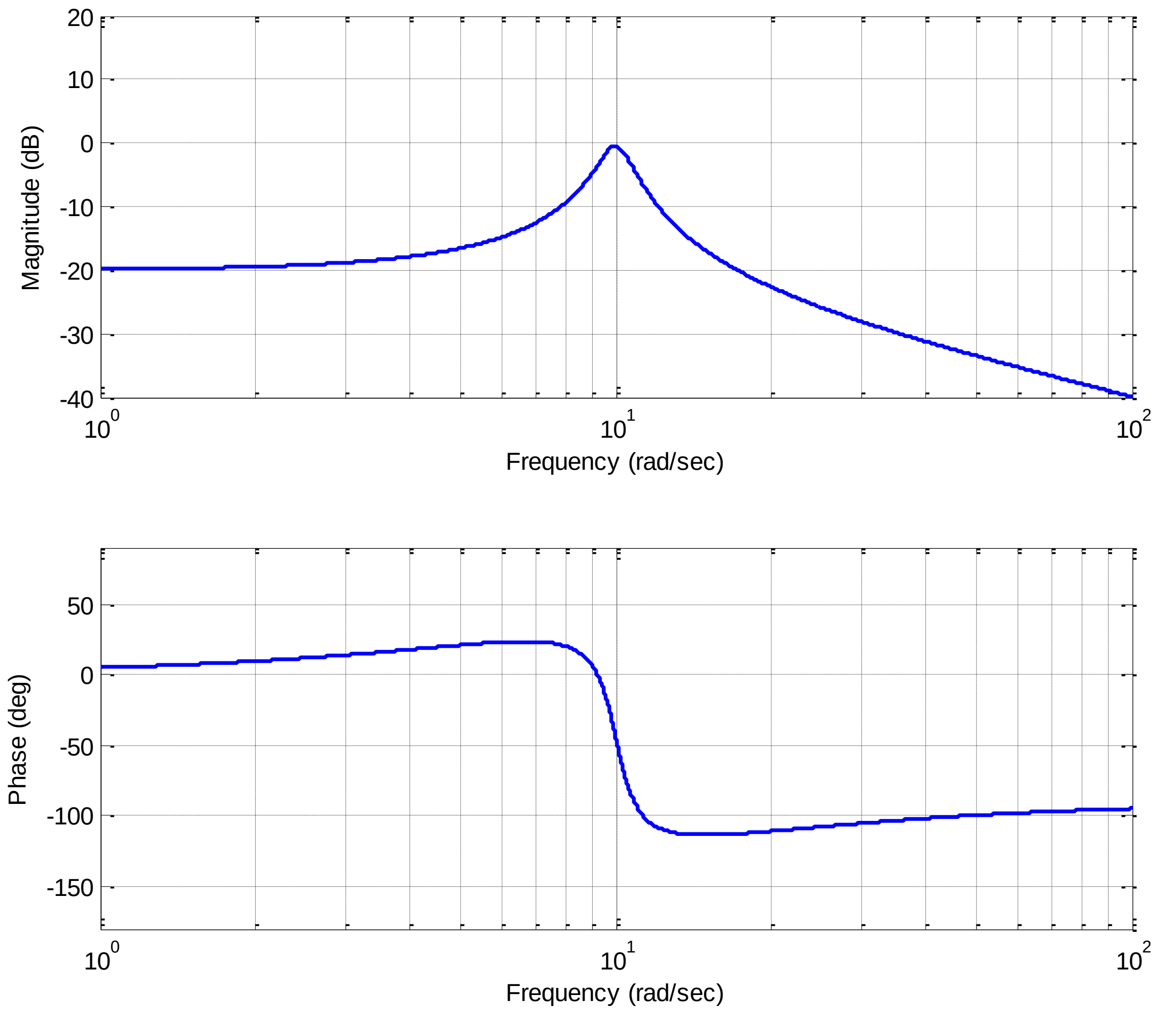
2) Assume $x(t) = 2 + \sin(5t) + 3\cos(8t + 30^o)$ is the input to an LTI system with transfer function shown below



The **steady state output** of this system will be

a) $y(t) = 20 + 5\sin(5t + 90^o) + 6\cos(8t + 90^o)$ b) $y(t) = 2 + 5\sin(5t + 90^o) + 6\cos(8t + 90^o)$
c) $y(t) = 20 + 5\sin(5t + 90^o) + 6\cos(8t + 120^o)$ d) $y(t) = 10 + 5\sin(5t + 90^o) + 6\cos(8t + 120^o)$
e) none of these

Problems 3 and 4 refer to a system whose frequency response is represented by the Bode plot below



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3) If the input to the system is $x(t)=5\cos(10t+30^o)$, then the steady state output is best estimated as

a) $y_{ss}(t)=0$ b) $y_{ss}(t)=5\cos(10t+30^o)$

c) $y_{ss}(t)=5\cos(10t-20^o)$ d) $y_{ss}(t)=5\cos(10t-50^o)$

4) If the input to the system is $x(t)=50\sin(100t)$, then the steady state output is best estimated as

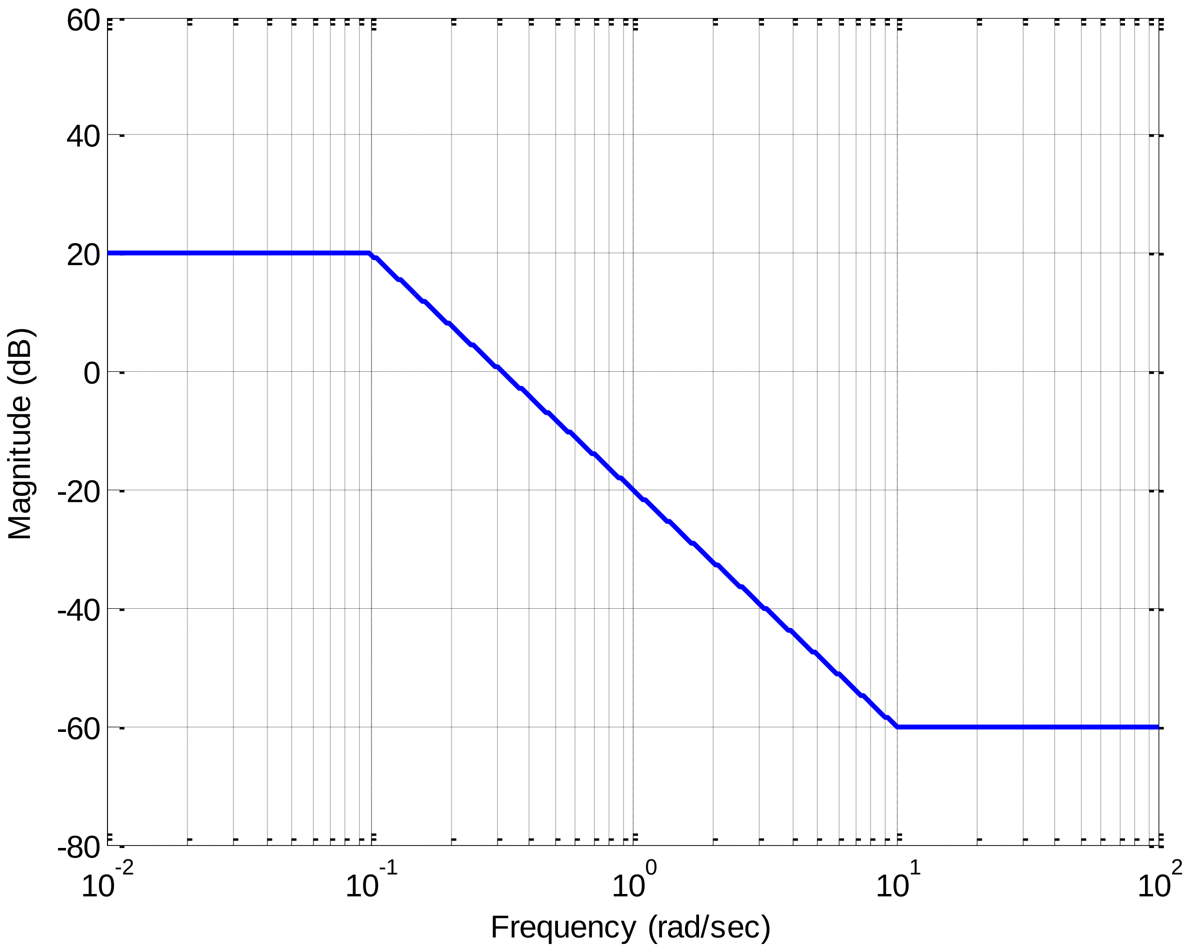
a) $y_{ss}(t)=-2000\sin(100t-100^o)$ b) $y_{ss}(t)=0.5\sin(100t-100^o)$

c) $y_{ss}(t)=2000\sin(100t-100^o)$ d) $y_{ss}(t)=5\sin(100t-100^o)$

5) For the straight line approximation to the magnitude portion of a Bode plot shown below, the best estimate of the corresponding transfer function is

a) $H(s) = \dfrac{20\left(\frac{1}{10}s+1\right)}{10s+1}$ b) $H(s) = \dfrac{10\left(\frac{1}{10}s+1\right)}{10s+1}$

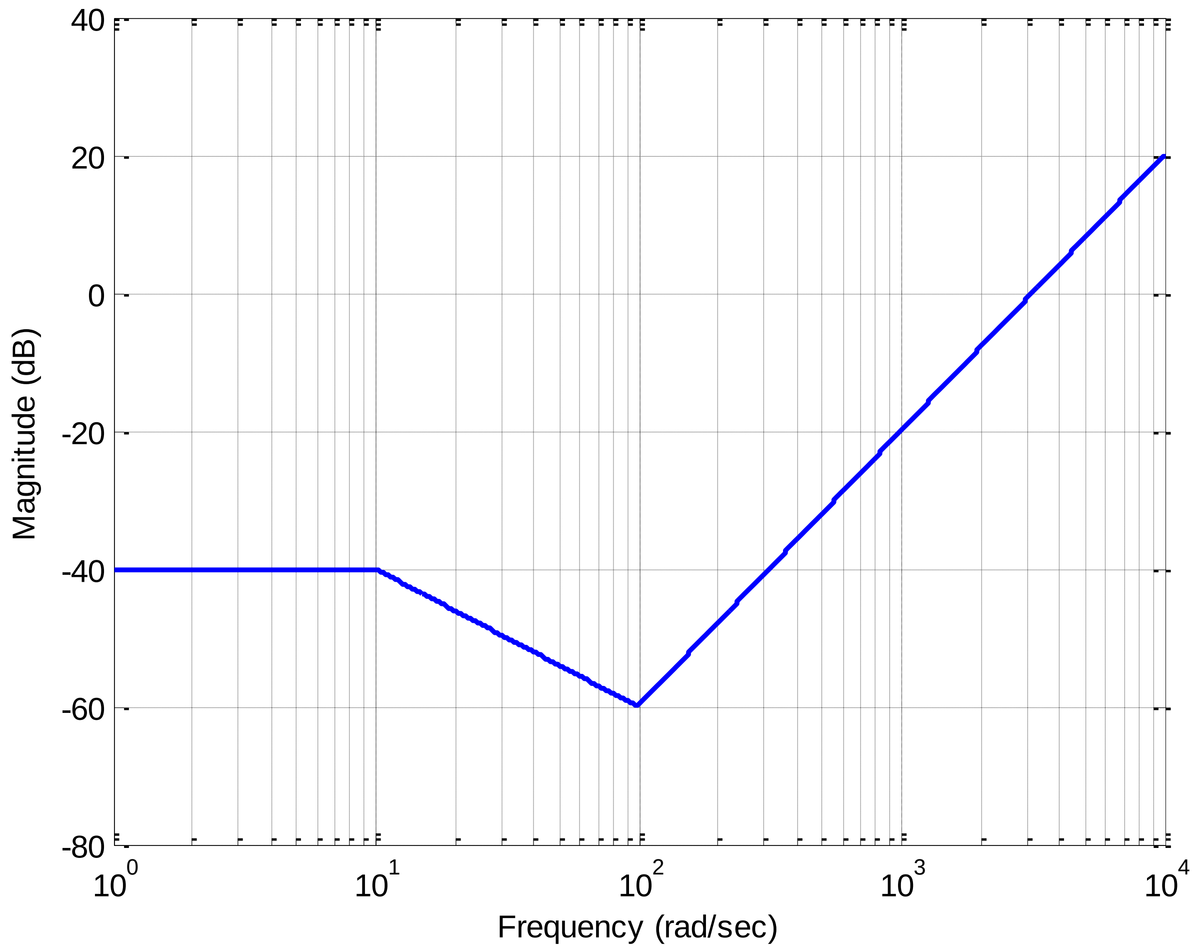
c) $H(s) = \dfrac{10\left(\frac{1}{10}s+1\right)}{(10s+1)^2}$ d) $H(s) = \dfrac{10\left(\frac{1}{10}s+1\right)^2}{(10s+1)^2}$



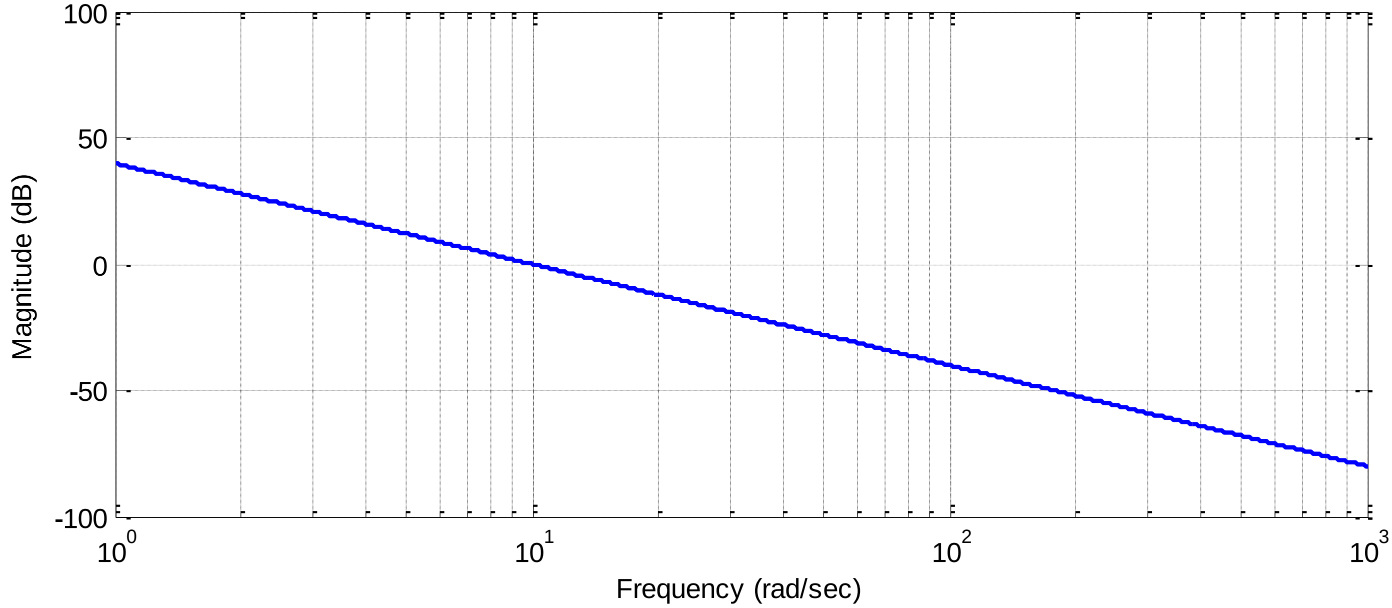
6) For the straight line approximation to the magnitude portion of a Bode plot shown below, the best estimate of the corresponding transfer function is

a) $H(s) = \dfrac{0.01\left(\frac{1}{100}s+1\right)^2}{\left(\frac{1}{10}s+1\right)}$ b) $H(s) = \dfrac{-40\left(\frac{1}{100}s+1\right)^2}{\left(\frac{1}{10}s+1\right)}$

c) $H(s) = \dfrac{0.01\left(\frac{1}{100}s+1\right)^3}{\left(\frac{1}{10}s+1\right)}$ d) $H(s) = \dfrac{0.01\left(\frac{1}{100}s+1\right)^3}{\left(\frac{1}{10}s+1\right)^2}$



7) The following three figures display the magnitude of six transfer functions. All of the poles and zeros of these transfer functions are in the left half plane (these are minimum phase transfer functions). All of the magnitudes, poles, and zeros are either zero or simple powers of 10 ($10^{-1}, 1, 10^{1}, 10^{2}$ etc). Estimate the transfer functions.



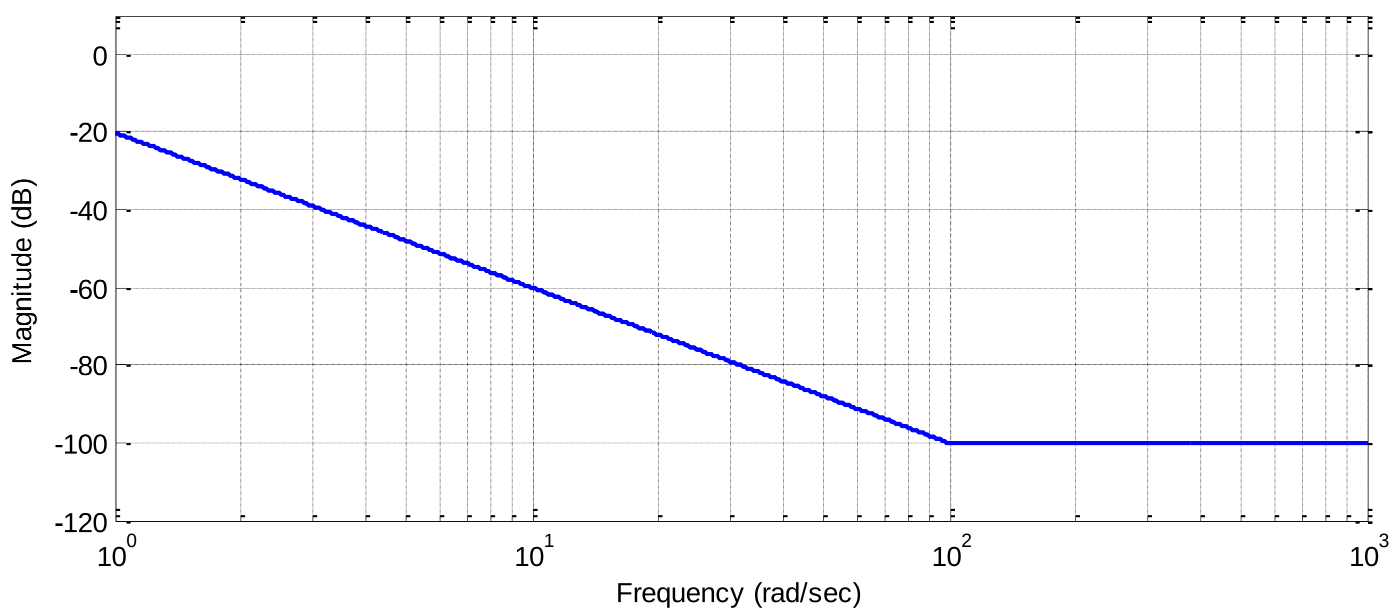


Figure 1: Problem 7, Systems a and b

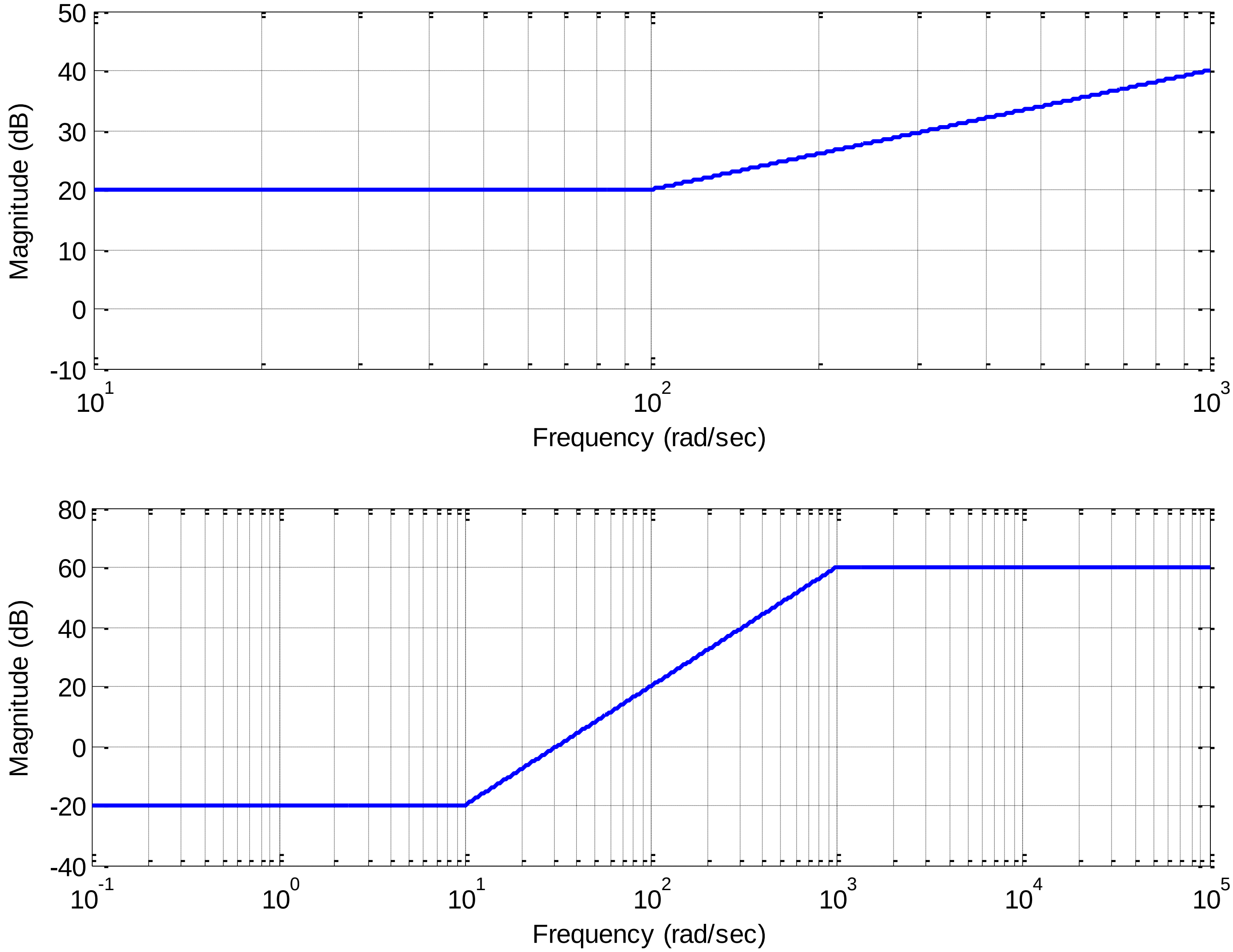


Figure 2: Problem 7, Systems c and d

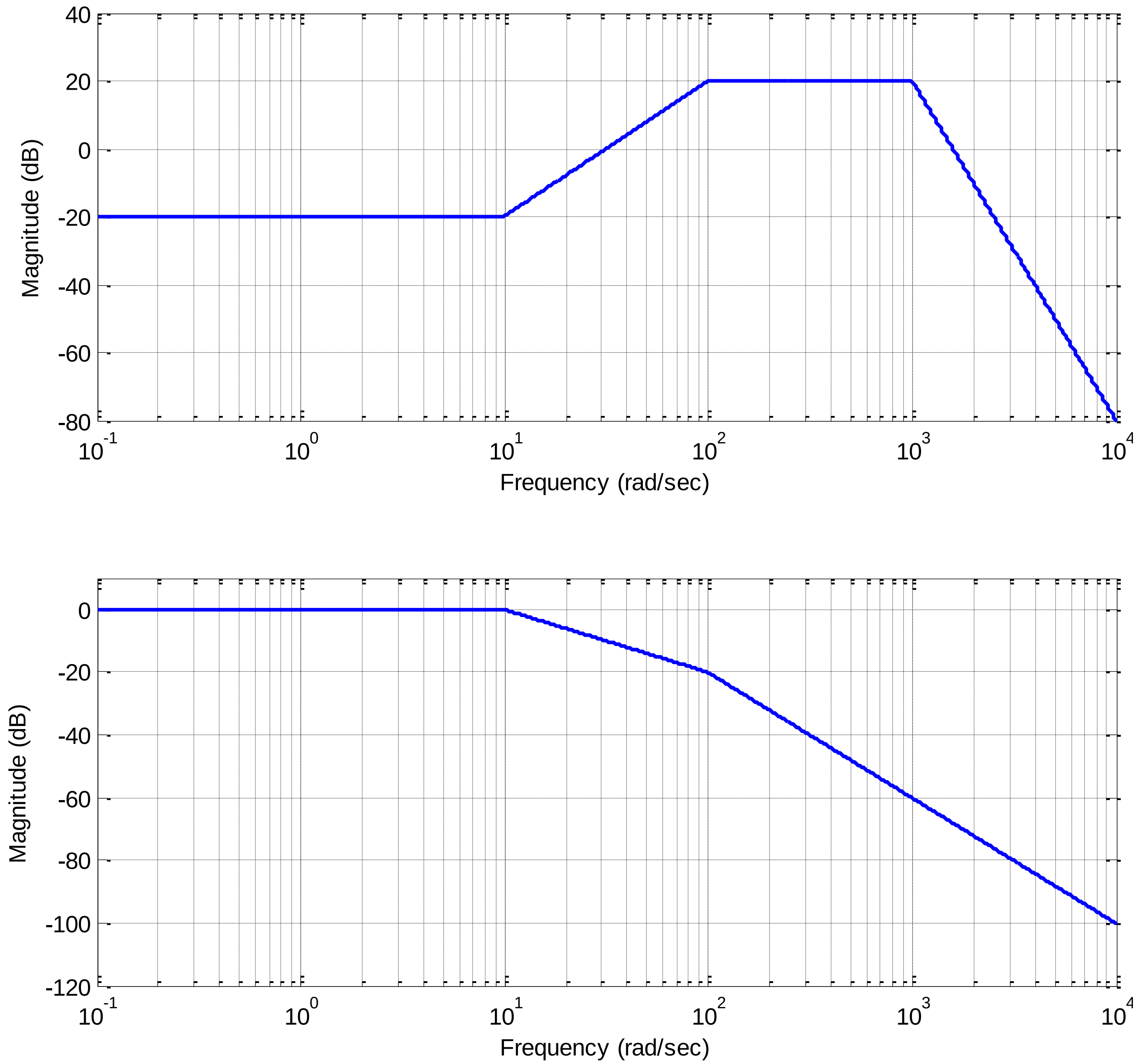


Figure 3: Problem 7, Systems e and f

Answers:

$$H(s)=\frac{100}{s^2},\quad H(s)=\frac{0.1\left(\frac{1}{100}s+1\right)^2}{s^2},\quad H(s)=10\left(\frac{1}{100}s+1\right)$$

$$H(s)=\frac{0.1\left(\frac{1}{10}s+1\right)^2}{\left(\frac{1}{1000}s+1\right)^2},\quad H(s)=\frac{0.1\left(\frac{1}{10}s+1\right)^2}{\left(\frac{1}{100}s+1\right)^2\left(\frac{1}{1000}s+1\right)^5},\quad H(s)=\frac{1}{\left(\frac{1}{10}s+1\right)\left(\frac{1}{100}s+1\right)}$$

8) Assume we want to construct a Bode plot for an LTI system.

We assume the input is $x(t)=0.087\cos(2\pi ft)$ and the corresponding steady state output is $y_{ss}(t)=Bcos(2\pi ft+\phi)=B\cos(2\pi f(t-t_d))$

We have made the following measurements,

f	B	t_d
1.40	0.68	0.07
1.50	1.20	0.18
1.60	0.72	0.26
1.75	0.40	0.26
1.90	0.30	0.25
2.0	0.26	0.24
2.5	0.19	0.19
3.0	0.16	0.16

On the attached pages, plot the magnitude and phase of the transfer function. (Note that the frequency axis is not logarithmic to make this easier to plot.)