ECE-320: Linear Control Systems
Homework 4

Due: **Tuesday January 13** at the beginning of class

1) (Easy) Show that $\sum_{l=-\infty}^{l=n} \delta(l) = u(n)$ and $\sum_{l=-\infty}^{l=n} \delta(l-k) = u(n-k)$

2) (Easy) For impulse response $h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$ and input

$x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-2)$, determine the output $y(n)$ (this should be easy).

3) For impulse response $h(n) = \left(\frac{1}{2}\right)^{n} u(n)$ and input $x(n) = u(n)$, show that the system output is

$y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$

a) by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$

b) by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$

Note that this is the unit step response of the system.

4) For impulse response $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$ and input $x(n) = \left(\frac{1}{2}\right)^{n} u(n-1)$, show that the system output is $y(n) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right] u(n-2)$ by evaluating the convolution sum

$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$

5) For impulse response $h(n) = \left(\frac{1}{2}\right)^{n-3} u(n-1)$ and input $x(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$, show that the system output is $y(n) = \left[\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{4}\right)^{n-1}\right] u(n-3)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$

6) For impulse response $h(n) = \left(\frac{1}{3}\right)^{n+1} u(n-2)$ and input $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n+1)$, show that the system output is $y(n) = \frac{16}{9}\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n\right]u(n-1)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$

7) For $h(n) = \left(\frac{1}{a}\right)^n u(-n) + a^n u(n)$ with $|a| < 1$,using the two-sided z transform to show that

$$H(z) = \frac{1}{1-az} + \frac{1}{1-az^{-1}}$$

and the region of convergence is $|a| < |z| < \frac{1}{|a|}$.

8) For the z -transform $X(z) = \frac{3}{z-2}$

a) Show that, by multiplying and dividing by z and then using partial fractions, the corresponding discrete-time sequence is

$$x(n) = -\frac{3}{2}\delta(n) + \frac{3}{2}2^k u(n)$$

b) By starting with the z -transform

$$G(z) = \frac{3z}{z-2}$$

where $X(z) = z^{-1}G(z)$, determine $g(n)$ and use the delay property to show that

$$x(n) = 3 \times 2^{n-1} u(n-1)$$

9) For impulse response $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$ and input $x(n) = \left(\frac{1}{2}\right)^n u(n-1)$, use z-transforms of the input and impulse response to show the output is

$$y(n) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-2) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right]u(n-1)$$

Hint: Assume $Y(z) = z^{-1}G(z)$, determine $g(n)$ and then $y(n)$

10) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n+1}u(n-2)$, use z-transforms of the input and impulse response to show the system output is $y(n)=\left[\left(\frac{1}{2}\right)^{n}-\left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$

Hint: Assume $Y(z)=z^{-2}G(z)$,