Due: Wednesday January 7 at 5 PM (note the change in time)

Exam 1: Thursday January 8

1) For the following problem, assume we are using the following control system
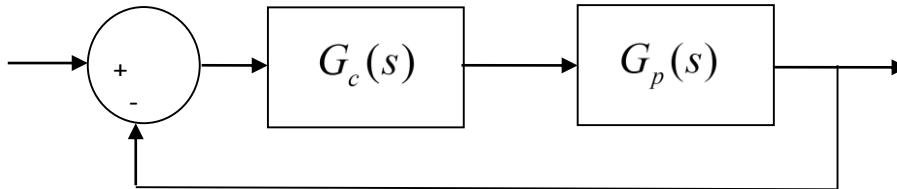
where the plant is given by

$$G_p(s) = \frac{1}{s^2 + 4s + 29} = \frac{1}{(s+2-5j)(s+2+5j)}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \dfrac{k}{s}$ (an integral (I) controller)

c) $G_c(s) = \dfrac{k(s+z)}{s}$ (a proportional + integral (PI) controller) Write the centroid σ_c as a function of z. For what values of z will the two asymptotes be in the right half plane? *(For plotting purposes, assume z is equal to 2.)*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *(For plotting purposes, assume z is equal to 2.)*

e) $G_c(s) = \dfrac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) Sketch this for the case where both zeros are real and then when both zeros are complex conjugates.

f) $G_c(s) = \dfrac{k(s+z)}{(s+p)}$ (a lead controller, $p > z$) Write an expression for σ_c as a function of the distance between the pole and the zero, $l = p - z$. What happens to the asymptotes as l gets larger? *(For plotting purposes, assume p is 5 and z is 1.)*

2) For the following problem, assume we are using the following control system
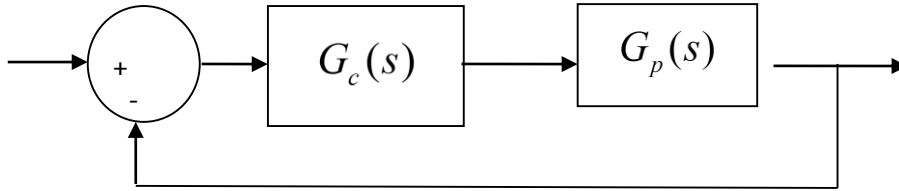
where the plant is given by

$$G_p(s) = \frac{1}{s+3}$$

For the following controllers, sketch the root locus with arrows showing the direction of travel as k increases. If there are any poles going to zeros at infinity, you need to compute the centroid of the asymptotes (σ_c) and the angles of the asymptotes.

You may (and should) check your answers with Matlab (use the **rlocus** command), but you need to do this by hand.

a) $G_c(s) = k$ (proportional (P) controller)

b) $G_c(s) = \dfrac{k}{s}$ (an integral (I) controller)

c) $G_c(s) = \dfrac{k(s+z)}{s}$ (a proportional + integral (PI) controller) *Sketch this for the case when z is equal to 2 and then assume z is equal to 4; there will be two plots.*

d) $G_c(s) = k(s+z)$ (a proportional+derivative (PD) controller) *Sketch this for the case where z is equal to 2 and then assume z = 4; there will be two plots.*

e) $G_c(s) = \dfrac{k(s+z_1)(s+z_2)}{s}$ (a proportional+integral+derivative (PID) controller) *Sketch this for the case where there are zeros at $-4 \pm 4j$ and when they are at -6 and -8; there will be two plots.*

2

3) One of the things that we need to worry about in practical systems is the limitation of the amplitude of the control signal, or the *control effort*. This is also a problem for most practical systems. In this problem we will do some simple analysis to better understand why Matlab's sisotool won't give us a good estimate of the control effort for some types of systems.

a) For the system below,
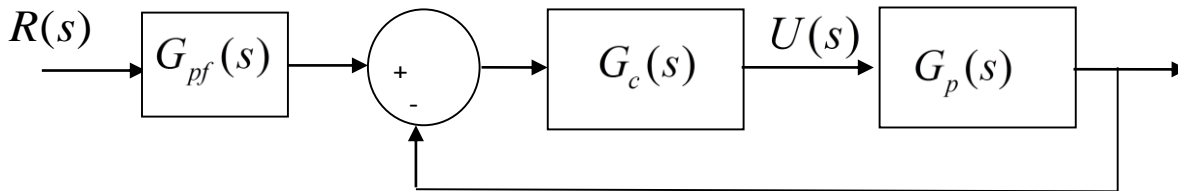
show that U(s) and R(s) are related by

$$U(s) = \frac{G_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}R(s)$$

b) For many types of controllers, the maximum value of the control signal is just after the step is applied, at $t = 0^+$. Although most of the time we are concerned with steady state values and use the final value Theorem in the *s*-plane, in this case we want to use the initial value Theorem, which can be written as

$$\lim_{t \to 0^+} u(t) = \lim_{s \to \infty} sU(s)$$

If the system input is a step of amplitude *A*, show that

$$u(0^+) = \lim_{s \to \infty} \frac{AG_c(s)G_{pf}(s)}{1+G_p(s)G_c(s)}$$

This result shows very clearly that the initial control signal is directly proportional to the amplitude of the input signal, which is pretty intuitive.

c) Now let's assume

$$G_c(s) = \frac{N_c(s)}{D_c(s)} \quad G_p(s) = \frac{N_p(s)}{D_p(s)} \quad G_{pf}(s) = \frac{N_{pf}(s)}{D_{pf}(s)}$$

If we want to look at the initial value for a unit step, we need to look at

3

$$u(0^+) = \lim_{s \to \infty} \frac{sG_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)} \frac{1}{s} = \lim_{s \to \infty} \frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

Let's also then define

$$\tilde{U}(s) = \frac{G_c(s)G_{pf}(s)}{1+G_c(s)G_p(s)}$$

so that

$$u(0^+) = \lim_{s \to \infty} \tilde{U}(s)$$

Show that

$$\tilde{U}(s) = \frac{N_{pf}(s)}{\left(\dfrac{D_c(s)}{N_c(s)}\right)D_{pf}(s) + \left(\dfrac{N_p(s)}{D_p(s)}\right)D_{pf}(s)}$$

and

$$\deg \tilde{U} = \deg N_{pf} - \max\left[\deg D_c - \deg N_c + \deg D_{pf}, \deg N_p - \deg D_p + \deg D_{pf}\right]$$

where $\deg Y$ is the degree of polynomial Y.

d) Since we are going to take the limit as $s \to \infty$, we need the degree of $\tilde{U}(s)$ to be less than or equal to zero for a step input to have a finite $u(0^+)$. Why?

For the remainder of this problem assume we have a 1 dof system such as an ideal second order system, so we have $\deg N_p = 0$ *and* $\deg D_p = 2$.

e) If the prefilter is a constant, show that in order to have a finite $u(0^+)$ we must have

$$\deg D_c \geq \deg N_c$$

f) If the numerator of the prefilter is a constant, then in order to have a finite $u(0^+)$ we must have

$$\deg D_c - \deg N_c + \deg D_{pf} \geq 0 \text{ or } -2 + \deg D_{pf} \geq 0$$

g) For P, I, D, PI, PD, PID, and lead controllers, determine if $u(0^+)$ is finite if the prefilter is a constant.

Note: Although it may appear that the control effort is sometimes infinite, in practice this is not true since our motor cannot produce an infinite signal. This large initial control signal is referred to as a *set-point kick*. There are different ways to implement a PID controller to avoid this.

4) (*sisotool problem*) For the plant modeled by the transfer function

$$G_1(s) = \frac{6000}{s^2 + 4s + 400}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$PO \leq 10\%$$
$$T_s \leq 2.5 \sec$$
$$k_p \leq 0.5$$
$$k_i \leq 5$$
$$k_d \leq 0.01$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

5) (*sisotool problem*) For the plant modeled by the transfer function

$$G_2(s) = \frac{6250}{s^2 + 0.5s + 625}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$PO \leq 10\%$$
$$PI\ T_s \leq 15.0\ sec, PID\ T_s \leq 0.5\ sec$$
$$k_p \leq 0.5$$
$$k_i \leq 5$$
$$k_d \leq 0.01$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.