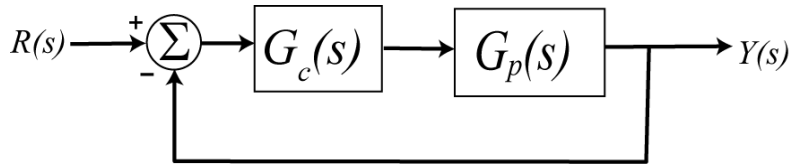**ECE-320**: Linear Control Systems

Homework 2

Due: Tuesday December 16 at the beginning of class

1) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{2}{s+4}$. The input is a unit step.

a) Determine the settling time and steady state error of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the steady state error for a unit step is 0.1, and the corresponding settling time for the system.

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the settling time is 0.5 seconds, and the corresponding steady state error.

e) Assuming an integral controller, $G_c(s) = k_i / s$, determine closed loop transfer function, $G_0(s)$

f) Assuming an integral controller, $G_c(s) = k_i / s$, determine the value of k_i so the steady state error for a unit step is less than 0.1 and the system is stable.

Partial Answers: $T_s = 1, e_{ss} = 0.5, k_p = 18, k_p = 2, T_s = 0.1, e_{ss} = 0.5, k_i > 0$

2) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \dfrac{4}{s+7}$.
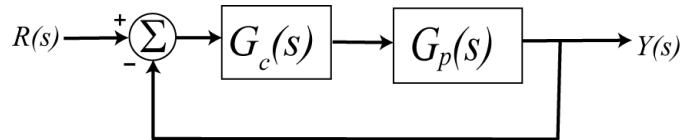
a) What is the bandwidth of the plant alone (assuming there is no feedback)

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_0(s)$

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of k_p so the bandwidth of the closed loop system is 27 rad/sec.

d) Assuming the proportional controller from problem c, determine the settling time and the steady state error for a unit step.

Partial Answers: 7, 5, 7/27, 4/27

3) An ideal second order system has the transfer function $G_o(s)$. The system specifications for a step input are as follows:

a) Percent Overshoot < 5%

b) Settling Time < 4 seconds (2% criteria)

c) Peak Time < 1 second

Sketch, in the complex plane, the permissible area for the poles of $G_o(s)$ in order to achieve the desired response.

4) For systems with the following transfer functions:

$$H_a(s) = \frac{1}{s+2} \quad H_b(s) = \frac{s+6}{(s+2)(s+3)}$$

a) Determine the unit step and unit ramp response for each system using Laplace transforms. Your answer should be time domain functions $y_a(t)$ and $y_b(t)$.

b) From these time domain functions, determine the steady state errors for a unit step and unit ramp input.

The following Matlab code can be used to estimate the step and ramp response for 5 seconds for transfer function $H_b(s)$.

```
H = tf([1 6],[1 5 6]);              % enter the transfer function
t = [0:0.01:5];                     % t goes from 0 to 5 by increments of 0.01
ustep = ones(1,length(t));          % the step input is all ones, u(t) = 1;
uramp = t;                          % the ramp input is has the input u(t) = t;
ystep = lsim(H,ustep,t);            % find the step response
yramp = lsim(H,uramp,t);            % find the ramp response
figure;                             % make a new figure
orient tall                         % or orient landscape, use more of the page
subplot(2,1,1);                     % put two graphs on one piece of paper
plot(t,ustep,'.-',t,ystep,'-');     % plot input/output with different line types
grid;                               % put a grid on the graph
legend('Step Input','Step Response',4);   % put a legend on the graph
subplot(2,1,2);                     % second of two graphs on one piece of paper
plot(t,uramp,'.-',t,yramp,'-');     % plot input/output with different line types
grid;                               % put a grid on the graph
legend('Ramp Input','Ramp Response',4);   % put a legend on the graph
```

c) Plot the step and ramp response for both systems (a and b) and indicate the steady state errors on the graph. Draw on the graph to show you know what the steady state errors are.

Ans. Steady state errors for a unit step input: 0.5,0; for a unit ramp input : infinity and 0.666

5) For the following systems

a) Determine the system type (0, 1, 2, ...)

b) If the system is type 0 assume $G_{pf} = 1$ and determine the position error constant K_p and the steady state error for a unit step input. Then determine the value of G_{pf} to make this error zero. If the system is type 1, assume $G_{pf} = 1$ and determine the steady state error for a unit step, the velocity error constant K_v, and the steady state error for a unit ramp.

Ans. (steady state errors) $-\dfrac{3}{2}, \dfrac{3}{13}, -\dfrac{3}{5}, \dfrac{1}{2};$ (prefilers) $\dfrac{2}{5}, \dfrac{13}{10}$
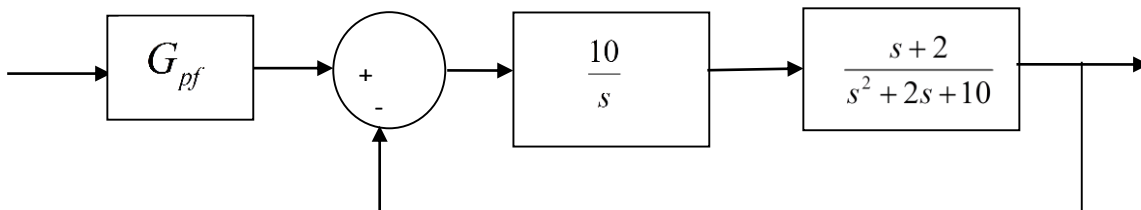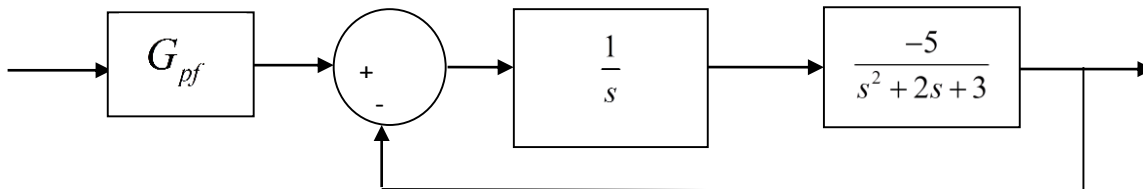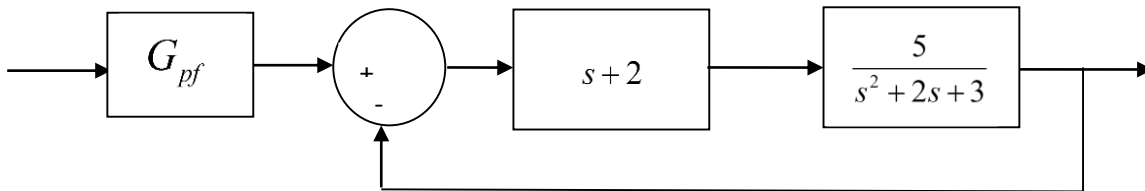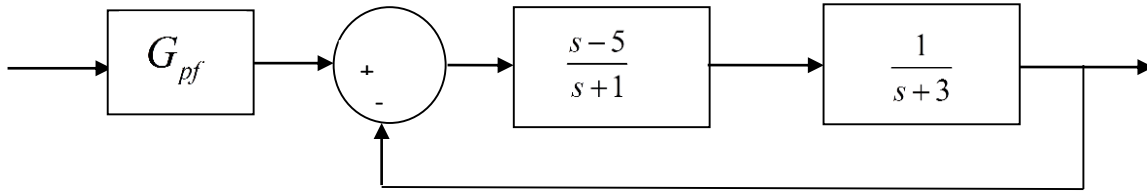
6) Consider the following control system:
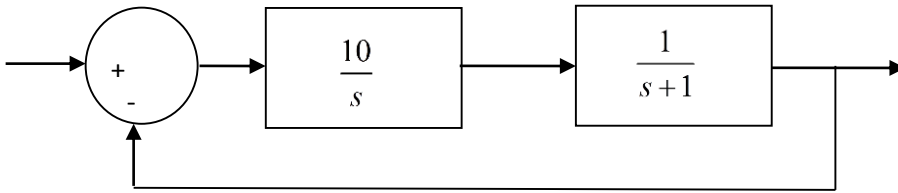
a) If the input to the system is $r(t) = 8u(t)$, what is the steady state output?

b) If the input to the system is $r(t) = 8\sin(3t)u(t)$, what is the output in steady state?
What is the time lag between the input signal and the output signal?

Hint: (1) use phasors, (2) you can write $\omega t - \theta = \omega(t - t_d)$ if θ is measured in radians.

Answers: $y(t) = 8$, $y(t) = 8\sqrt{10}\sin(3t - 71.57°), t_d = 0.416\sec$

7) The differential equation model of an ideal second order system is

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

Here K is the static gain, ω_n is the natural frequency, and ζ is the damping ratio. These are the parameters we need to determine for these models.

You will need to download the and uncompress the file **Homework2 Files.rar**. The Matlab file **second_order_driver.m** reads in data from a file (the measured data files for an unknown system) and extracts the time, input, and output values for the system. After this, you are to modify your guesses for the damping ratio, the natural frequency, and the static gain. The Matlab script **second_order_driver.m** then runs the Simulink model file **Second_Order_System.mdl** and plots the results of your model with the measured data.

You are to change the parameters in the Matlab driver file to try and get your model to match the step response of the unknown system as well as possible. Once you are done, save the graph with the step responses of the two systems and put it in your memo.

You will need to do this for the data files **measured_1**, **measured_2**, **measured_3**, and **measured_4**.

Your homework should contain four figures for this problem.