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ECE-320 Linear Control Systems

Winter 2014, Exam 2

No calculators or computers allowed.

Problem 1	**__________/19**
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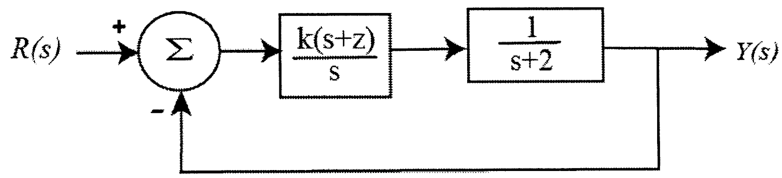
1) Consider the following simple PI control system.



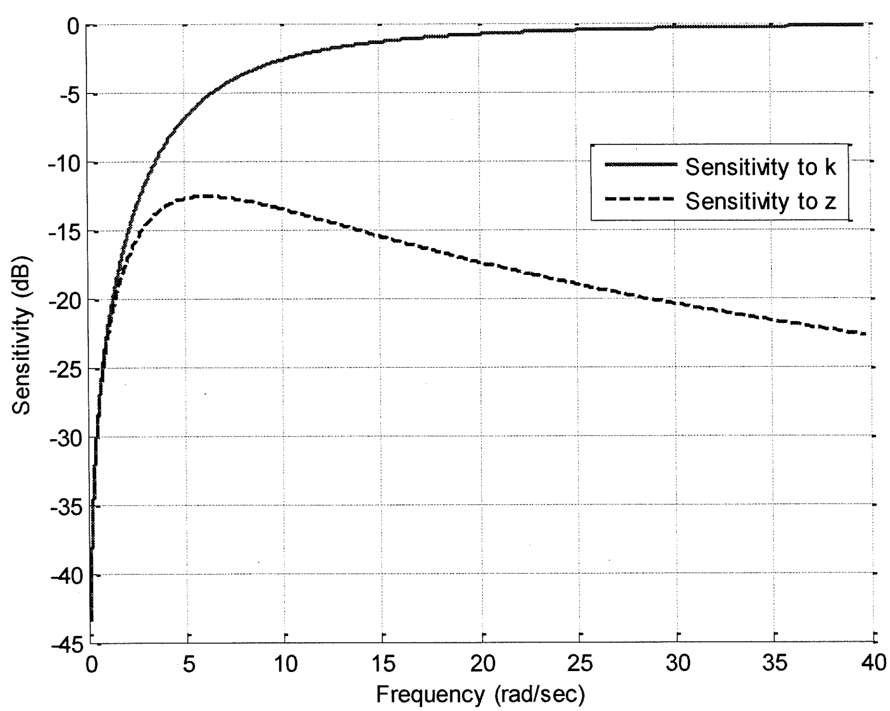
The nominal value of k is 10 and the nominal value of z is 3.

a) Determine an expression for the closed loop transfer function Go(s).

b) Determine an expression for the sensitivity of the closed loop transfer function to changes in k. You may leave your answer in terms of s, but it must be simplified as much as possible (i.e., it should be a ratio of polynomials and all numbers except for the letter s).

c) Determine an expression for the sensitivity of the closed loop transfer function to changes in z. You may leave your answer in terms of s, but it must be simplified as much as possible (i.e., it should be a ratio of polynomials and all numbers except for the letter s, the denominator may be the product of two polynomials).

d) The graph below shows a plot of the sensitivities to each of these parameters. Over this frequency range, the system is more sensitive to which parameter?



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a) $$G_0(s) = \frac{\frac{K(s+z)}{s}\,\frac{1}{s+2}}{1+\frac{K(s+z)}{s}\,\frac{1}{s+2}} = \frac{K(s+z)}{s^2+2s+Ks+Kz} = \boxed{\frac{10(s+3)}{s^2+12s+30}}$$

b) $$S_K^{G_0} = \frac{K}{N_0}\frac{\partial N_0}{\partial K} - \frac{K}{D_0}\frac{\partial D_0}{\partial K} = \frac{K}{K(s+z)}(s+z) - \frac{K}{s^2+(K+2)s+Kz}(s+z)$$

$$= 1 - \frac{10(s+3)}{s^2+12s+30} = \boxed{\frac{s^2+2s}{s^2+12s+30} = S_K^{G_0}}$$

c) $$S_z^{G_0} = \frac{z}{N_0}\frac{\partial N_0}{\partial z} - \frac{z}{D_0}\frac{\partial D_0}{\partial z} = \frac{z}{K(s+z)}(K) - \frac{z}{s^2+(K+2)s+Kz}(K)$$

$$= \frac{30}{10(s+3)} - \frac{30}{s^2+12s+30} = \frac{30\left[s^2+12s+30-10s-30\right]}{10(s+3)(s^2+12s+30)}$$

$$= \boxed{\frac{3(s^2+2s)}{(s+3)(s^2+12s+30)}}$$

d) more sensitive to K

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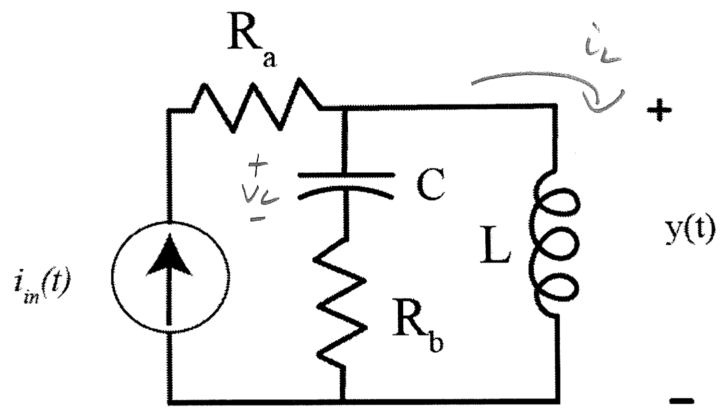
2) For the following circuit, the state variables are the current through the inductor and the voltage across the capacitor. Determine a state variable model for this system. Specifically, you need to identify the A, B, C, and D matrices/vectors/scalars. You surely recall the useful relationships

$v(t) = L\frac{di(t)}{dt}$, $i(t) = C\frac{dv(t)}{dt}$



$$i_{in}(t) = C\frac{dV_c(t)}{dt} + i_L(t) \qquad \frac{dV_c(t)}{dt} = -\frac{1}{C} i_L(t) + \frac{1}{C} i_{in}(t)$$

$$L\frac{di_L(t)}{dt} = V_c(t) + R_b\left[i_{in}(t) - i_L(t)\right] \qquad \frac{di_L(t)}{dt} = \frac{1}{L} V_c(t) - \frac{R_b}{L} i_L(t) + \frac{R_b}{L} i_{in}(t)$$

$$y(t) = V_c(t) + \left[i_{in}(t) - i_L(t)\right]R_b$$

$$\frac{d}{dt}\begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_b}{L} \end{bmatrix}}_{A} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{C} \\ \frac{R_b}{L} \end{bmatrix}}_{B} i_{in}(t)$$

$$y(t) = \overbrace{\begin{bmatrix} 1 & -R_b \end{bmatrix}}^{C} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} + \overbrace{\begin{bmatrix} R_b \end{bmatrix}}^{D} i_{in}(t)$$

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3) For the state variable model

$$\dot{q} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} q + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} q + [0] u$$

Determine the closed loop transfer function with ***state variable feedback***, $u(t) = G_{pf} r(t) - Kq(t)$. Note that you need to write out the determinant, but you don't need to simplify it.

$$\tilde{A} = A - BK = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -K_1 & 3-K_2 \end{bmatrix}$$

$$sI - \tilde{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -K_1 & 3-K_2 \end{bmatrix} = \begin{bmatrix} s-1 & -2 \\ K_1 & s+K_2-3 \end{bmatrix}$$

$$(sI - \tilde{A})^{-1} = \frac{1}{(s-1)(s+K_2-3)+2K_1} \begin{bmatrix} s+K_2-3 & 2 \\ -K_1 & s-1 \end{bmatrix}$$

$$G_o(s) = C(sI-\tilde{A})^{-1}\tilde{B} = \frac{\begin{bmatrix} 0 & 1 \end{bmatrix}}{\Delta(s)} \begin{bmatrix} s+K_2-3 & 2 \\ -K_1 & s-1 \end{bmatrix} \begin{bmatrix} 0 \\ G_{pf} \end{bmatrix} = \frac{(s-1)G_{pf}}{\Delta(s)}$$

$$\boxed{G_o(s) = \frac{(s-1)G_{pf}}{(s-1)(s+K_2-3)+2K_1}}$$

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4) For impulse response $h(n) = \left(\frac{1}{3}\right)^{n-1} u(n)$ and input $x(n) = \left(\frac{1}{2}\right)^{n+2} u(n-1)$, use z-transforms of the input and impulse response to determine the system output $y(n)$

$h(n) = \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^{-1} u(n) = 3\left(\frac{1}{3}\right)^n u(n) \qquad H(z) = \frac{3z}{z - 1/3}$

$x(n) = \left(\frac{1}{2}\right)^{n-1} \left(\frac{1}{2}\right)^3 u(n-1) = \frac{1}{8}\left(\frac{1}{2}\right)^{n-1} u(n-1) \qquad X(z) = \frac{1}{8} z^{-1} \frac{z}{z - 1/2} = \frac{1/8}{z - 1/2}$

$Y(z) = \frac{3/8\, z}{(z - 1/3)(z - 1/2)}$

$\frac{Y(z)}{z} = \frac{3/8}{(z - 1/3)(z - 1/2)} = \frac{A}{z - 1/3} + \frac{B}{z - 1/2}$

$A = \frac{3/8}{-1/6} = \frac{-18}{8} = \frac{-9}{4}$

$B = \frac{3/8}{1/6} = \frac{18}{8} = \frac{9}{4}$

$Y(z) = -\frac{9}{4} \frac{z}{z - 1/3} + \frac{9}{4} \frac{z}{z - 1/2}$

$$\boxed{y(n) = \frac{9}{4}\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n\right] u(n)}$$

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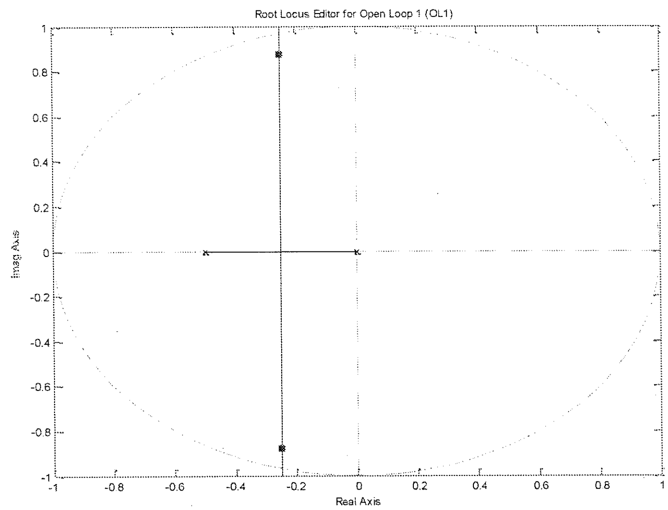
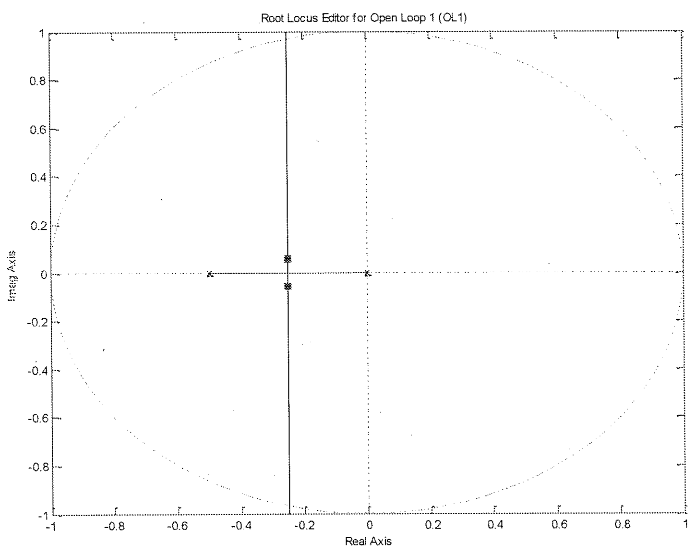
Problems 5 and 6 refer to the following two root locus plot for a discrete-time system





5) For which system is the settling time likely to be smallest?

a) The system on the top b) the system on the bottom c) the settling time will be the same

6) Is this a type 1 system?

a) yes b) no c) not enough information

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7) Which of the following transfer functions represents an (asymptotically) unstable systems? (circle all of them)

a) $G(z)=\dfrac{z}{z+0.8}$ b) $G(z)=\dfrac{z}{z-0.8}$ c) $G(z)=\dfrac{z}{z+1.2}$ d) $G(z)=\dfrac{z}{z-1.2}$

8) Which of the following systems will have a smaller settling time?

a) $G(z)=\dfrac{z}{z-0.9}$ b) $G(z)=\dfrac{z}{z-0.7}$ c) $G(z)=\dfrac{z}{z+0.5}$ d) $G(z)=\dfrac{z}{z+0.1}$

9) Is the following system ***controllable***?

$$G(s)=\frac{8G_{pf}}{s^2+12s+(k_1+k_2+20)}$$

a) Yes b) No c) impossible to determine

10) Is the following system **controllable**?

$$G(s)=\frac{G_{pf}}{s^2+(k_2+k_1-1)s+(k_2+2)}$$

a) Yes b) No c) impossible to determine

11) Assume *a, b, c, and d* are real-valued numbers. Write and expression for the magnitude of the following:

$$Z=\frac{a+b-j\omega c}{d+j\omega}$$

$$|Z| = \sqrt{\frac{(a+b)^2+(\omega c)^2}{d^2+\omega^2}}$$

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